Models of growth for system of cities: Back to the simple

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Modeling Urban Growth

_Growth in Urban Systems : multi-scalar, heterogeneous drivers, bifurcations and path-dependancy_

Role of spatial interactions in Urban Growth?

→ gravity-based flows influence population growth in a synergetic formulation [Sanders, 1992]

→ Simpop models (from Simpop1 to SimpopLocal) [Pumain, 2012]: agent-based approaches; more recently Marius [Cottineau et al., 2015] closer to system dynamics

→ Simple random growth (Gibrat model) becomes quickly complex by adding spatial interaction [Bretagnolle et al., 2000]; refined extension with waves of innovation in [Favaro and Pumain, 2011]
Between complex ABM and non-geographical models in economics/physics, what place for simple models of growth in Urban Systems?

Modulation of simple mechanisms to check for necessity/sufficiency: multi-modeling in models of simulation

Research Objective: Extend Gibrat simple model of growth in system of cities with spatial interactions and feedbacks through physical networks; Explore systematically and calibrate such families of models
Rationale: extend an interaction model for system of cities by including physical network as an additional carrier of spatial interactions (see [Raimbault, 2016b] for developed theoretical context)

→ Work under Gibrat independence assumptions, i.e. \(\text{Cov}[P_i(t), P_j(t)] = 0\). If \(\vec{P}(t + 1) = R \cdot \vec{P}(t)\) where \(R\) is also independent, then \(\mathbb{E}[\vec{P}(t + 1)] = \mathbb{E}[R] \cdot \mathbb{E}[\vec{P}](t)\). Consider expectancies only (higher moments computable similarly)

→ With \(\vec{\mu}(t) = \mathbb{E}[\vec{P}(t)]\), we generalize this approach by taking \(\vec{\mu}(t + 1) = f(\vec{\mu}(t))\)
Let $\bar{\mu}(t) = \mathbb{E}\left[\bar{P}(t)\right]$ cities population and $(d_{ij})$ distance matrix

Model specified by

$$f(\bar{\mu}) = r_0 \cdot \text{Id} \cdot \bar{\mu} + G \cdot 1 + N$$

with

- $G_{ij} = w_G \cdot \frac{V_{ij}}{\langle V_{ij} \rangle}$ and $V_{ij} = \left(\frac{\mu_i \mu_j}{\sum \mu_k^2}\right)^{\gamma_G} \exp\left(-d_{ij}/d_G\right)$

- $N_i = w_N \cdot \sum_{kl} \left(\frac{\mu_k \mu_l}{\sum \mu}\right)^{\gamma_N} \exp\left(-d_{kl,i}/d_N\right)$ where $d_{kl,i}$ is distance to shortest path between $k,l$ computed with slope impedance ($Z = (1 + \alpha / \alpha_0)^{n_0}$ with $\alpha_0 \simeq 3$)
Data: stylized facts


Non-stationarity of log-returns correlations function of distance
Physical transportation network abstracted through a geographical shortest path network
On the importance of visualization in spatial models: complementary implementations in NetLogo/R/Scala
Evidence of physical network effects: fit improve through feedback at fixed gravity
Results: model calibration

Model calibration using GA on computation grid, with software OpenMole [Reuillon et al., 2013]

Pareto front for full model calibration, objectives MSE and MSE on logs
Results: non-stationary model calibration
Quantifying overfitting: Empirical AIC

Not clear nor well theorized how to deal with overfitting in models of simulation. **Intuitive idea:** Approximate gain of information by approaching models of simulation by statistical models.

Let $M_k^* = M_k[\alpha_k^*]$ computational models heuristically fitted to the same dataset. With $S_k \simeq M_k^*$, we show that $\Delta D_{KL}(M_k^*, M_k') \simeq \Delta D_{KL}(S_k, S_k')$ if fits of $S_k$ are negligible compared to fit difference between computational models and models have same parameter number.

**Application** $M_1$: gravity only model with $(r_0 = 0.0133, w_G = 1.28e-4, \gamma_G = 3.82, d_G = 4e12)$; $M_2$: full model with $(r_0 = 0.0128, w_G = 1.30e-4, \gamma_G = 3.80, d_G = 8.4e14, w_N = 0.603, \gamma_N = 1.148, d_N = 7.474)$

Fitting of independent polynomial models ($\tilde{P}_i(t) = Q[\tilde{P}_i(t-1)]$) with 4 and 7 parameters) gives $\Delta D_{KL} \simeq 19.7 \rightarrow$ fit improvement without overfitting
Discussion

Theoretical and Methodological Implications
→ Indirect confirmation of known stylized facts (such as tunnel effect through non-stationary calibration)
→ For a better integration of theory, empirical and modeling on network aspects in evolutive urban theories
→ Methodology : first steps for empirical AIC in multi-modeling

Further Developments
→ Need to validate the approach on other system/subsystem of cities [Pumain et al., 2015]
→ Add Real Network in a static/dynamic way : towards models of co-evolution of cities and network [Raimbault, 2016b]
→ Coupling with growth models at other level, as e.g. mesoscopic reaction-diffusion model [Raimbault, 2016a]
Simple models of complex systems can have strong explanatory power, and be used to test hypothesis/confront a theory.

Crucial role of interdisciplinarity and integration theory/empirical and qualitative/quantitative.

- All code and data available at https://github.com/JusteRaimbault/CityNetwork/tree/master/Models/NetworkNecessity/InteractionGibrat
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Calibration with fixed gravity effects (iterative calibration)