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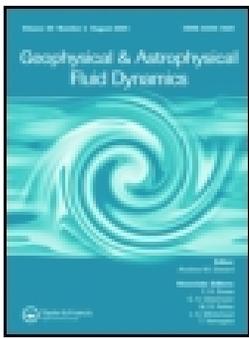
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Instabilities of low-latitude easterly jets in the presence of moist convection and topography and related cyclogenesis, in a simple atmospheric model

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1 A simple two-layer model, the moist-convective rotating shallow water, which al-
2 lows for low-cost high-resolution numerical simulations of the dynamics of the moist
3 atmosphere in the presence of topography, is used to identify and understand dy-
4 namical processes governing the evolution of easterly waves propagating on the
5 background of a low-latitude easterly jet crossing a land-sea boundary, a setup
6 crudely representing the African Easterly Jet over the West-African plateau and
7 the Atlantic ocean. We perform a thorough linear stability analysis and identify the
8 unstable modes of the jet, which we use then for initialisation of fully nonlinear
9 numerical simulations. In this way we determine nonlinear evolution of unstable
10 perturbations of the jet, both in the “dry” and moist-convective environments, and
11 highlight essential differences between the two cases. We identify a mechanism of
12 formation of intense lower-layer cyclonic vortices at the northern flank of the jet,
13 and determine the influence of the land-sea contrast upon this process.

14 **Keywords:** Baroclinic instability, Moist convection, Orographic effects, African Easterly Jet, Tropical
15 Cyclogenesis

16 1. Introduction

17 Understanding tropical cyclogenesis in the North Atlantic is of obvious importance. It is
18 well-known that the cyclogenesis is most often triggered by depressions due to the African
19 Easterly Waves (AEW) propagating over Atlantic, (e.g. [Bracken and Bosart 2000](#), [Brammer
20 and Thorncroft 2015](#), [Dieng *et al.* 2017](#)), and references therein. In turn, it is widely accepted
21 that AEW are related to instabilities of the African Easterly Jet (AEJ), a low-latitude atmo-
22 spheric easterly jet evolving over the West-African plateau and the Eastern Atlantic Ocean
23 ([Burpee 1972](#), [Hsie and Cook 2005, 2008](#), [Wu *et al.* 2012](#)). Understanding the instabilities of
24 AEJ is, thus, of primary importance, and their studies, which started almost half a century
25 ago ([Simmons 1977](#)) still continue ([Thorncroft and Hoskins 1994a](#), [Hall *et al.* 2006](#), [Diaz and
26 Aiyyer 2015](#)). It is known that humidity, and the contrast between the ocean and the continent
27 play an important role in the development and intensification of AEW (e.g. [Brammer and
28 Thorncroft 2015](#)).

29 In the present paper, we analyze the development and nonlinear evolution of the insta-
30 bilities of low-latitude easterly jets in the presence of moist convection and the land - sea
31 contrast with the help of a simple two-layer atmospheric model which, nevertheless, captures
32 essential dynamical mechanisms. An advantage of the model is that inclusion of topography
33 is simple and straightforward. The model allows for efficient low-cost high-resolution linear
34 and nonlinear analyses of the system, and we profit from this to establish fine details of lin-
35 ear stability diagrams, and of nonlinear saturation of instabilities. An early precursor of this
36 study is the work by [Kuo \(1978\)](#), where barotropic and baroclinic instabilities of easterly
37 jets were analyzed, with similar motivations, in the framework of a two-layer model of the
38 same kind, although only on the linear level and with inclusion of neither topography nor
39 dynamical effects of moisture and condensation. The model we are working with is the so-
40 called moist-convective rotating shallow water (mcRSW), which, in its two-layer version, was
41 derived in [Lambaerts *et al.* \(2011b\)](#), by using vertical averaging between isobaric surfaces and
42 Lagrangian conservation of the equivalent potential temperature in the hydrostatic primitive
43 equations with pseudo-height as vertical coordinate. This conceptual model proved to be use-
44 ful in studies of instabilities of large-scale Earth and planetary jets and vortices, and the effect
45 of diabatic effects upon them ([Lahaye and Zeitlin 2016](#), [Rostami and Zeitlin 2017](#), [Rostami
46 *et al.* 2017, 2018](#)). The main goal of the present paper is to show that, in spite of a highly
47 idealized character of the model, it allows to capture the fundamental dynamical mechanisms
48 governing the evolution of AEW. Our strategy below is similar to that of [Lambaerts *et al.*](#)

49 (2012), where the model, for benchmarking purposes, was applied to the moist baroclinic in-
50 stability of the upper-tropospheric midlatitude westerly jet on the f - plane. Yet the important
51 differences, apart from the jet configuration itself, are the use of the beta-plane approximation
52 and addition of topography with a land-sea contrast. We choose a basic state which mimics
53 the AEJ, within the precision of the model, and perform, first, the linear stability analysis
54 in the “dry” case, with condensation switched off, and without topography. In this way we
55 identify the unstable normal modes of the jet, which we use then, either in a pure harmonic
56 form, or in a form of localized wave-packets, to initialize fully nonlinear high-resolution nu-
57 merical simulations, which are performed both in “dry” and moist-convective environments,
58 and also include an idealized step-like topography mimicking the West-African plateau and
59 the land - sea contrast. We are able in this way to highlight the role the moist convection
60 and topography are playing in intensification of the low-level depressions created by unstable
61 disturbances, and their eventual transformation into strong localized vortices.

62 The paper is organized as follows. In section 2 we present the model, introduce an appro-
63 priate scaling, and define the background jet configuration. In section 3 we give the results
64 of the linear stability analysis of the jet configuration in the “dry” version of the model, with
65 the condensation switched off. In section 4 we present results of numerical simulations of non-
66 linear evolution of the unstable disturbances in the “dry” and moist-convective environments,
67 both without and with topography. Section 5 contains summary and discussion. We give in
68 Appendix A the results of the stability analysis in the f -plane approximation, which was used
69 for benchmarking purposes.

70 2. The model, the scaling, and the jet configuration

71 2.1. The moist-convective rotating shallow water (mcRSW) model

We will work with the later version of the mcRSW model (Rostami and Zeitlin 2018) defined by the following set of equations:

$$\partial_t \mathbf{v}_1 + (\mathbf{v}_1 \cdot \nabla) \mathbf{v}_1 + f(y) \hat{\mathbf{z}} \times \mathbf{v}_1 = -g \nabla (h_1 + h_2) + \frac{\gamma C - \gamma^* V}{h_1} \cdot \frac{\mathbf{v}_1 - \mathbf{v}_2}{2}, \quad (2.1a)$$

$$\partial_t \mathbf{v}_2 + (\mathbf{v}_2 \cdot \nabla) \mathbf{v}_2 + f(y) \hat{\mathbf{z}} \times \mathbf{v}_2 = -g \nabla (h_1 + s h_2) + \frac{\gamma C - \gamma^* V}{h_2} \cdot \frac{\mathbf{v}_1 - \mathbf{v}_2}{2}, \quad (2.1b)$$

$$\partial_t h_1 + \nabla \cdot ((h_1 - b) \mathbf{v}_1) = \underline{-\gamma C + \gamma^* V}, \quad (2.1c)$$

$$\partial_t h_2 + \nabla \cdot (h_2 \mathbf{v}_2) = \underline{+\gamma C - \gamma^* V}, \quad (2.1d)$$

$$\partial_t Q_1 + \nabla \cdot (Q_1 \mathbf{v}_1) = \underline{-C + E}, \quad (2.1e)$$

$$\partial_t Q_2 + \nabla \cdot (Q_2 \mathbf{v}_2) = \underline{V}. \quad (2.1f)$$

$$\partial_t W_1 + \nabla \cdot (W_1 \mathbf{v}_1) = \underline{+(1 - \kappa) C - P}, \quad (2.1g)$$

$$\partial_t W_2 + \nabla \cdot (W_2 \mathbf{v}_2) = \underline{+\kappa C - V}, \quad (2.1h)$$

72 where $\mathbf{v}_i = (u_i, v_i)$ is the horizontal velocity field in the layers $i = 1, 2$ counted from the
 73 bottom, $f(y) = f_0 + \beta y$ is the Coriolis parameter, $\hat{\mathbf{z}}$ is a unit vector in the vertical direction,
 74 h_i are the thicknesses of the layers in the absence of topography, b represents the topography,
 75 g is the gravity acceleration, and $s = \theta_2/\theta_1 > 1$ is the stratification parameter, where θ_i are
 76 constant potential temperatures of the layers. Without the underlined terms (2.1) becomes the
 77 standard non-dissipative two-layer atmospheric shallow-water model (cf. Zeitlin 2018) with an
 78 addition of topography and two passive scalars layerwise: the specific humidity (water vapor
 79 content) integrated over the air column Q_i , and the bulk amount of liquid water W_i in the
 80 column. Both are conserved quantities if diabatic effects are switched off. We will call the
 81 system “dry” in the latter case. The underlined terms introduce a condensation sink C and
 82 a surface evaporation source E in the water vapor equation in the lower layer (2.1e), and a
 83 convective flux between the layers in the mass conservation equations (2.1c), (2.1d), due to
 84 the latent heat release. This flux is proportional to the condensation, with the proportionality
 85 coefficient γ which is defined by the underlying vertical stratification and thermodynamical
 86 parameters (cf. Lambaerts *et al.* 2011b). Similar parameterization is used for vaporization V ,
 87 which is a source of water vapor, and introduces cooling with a related downward convective
 88 flux which is proportional to vaporization, with a proportionality coefficient γ^* . We suppose

89 that the lower layer is close to saturation, and neglect vaporization in it and that the upper
 90 layer is far from the saturation and neglect condensation in it. Correspondingly, condensation
 91 (vaporization) is a source (sink) of liquid water. Entrainment of liquid water from lower to
 92 upper layer due to the upward convective flux is also introduced, and is controlled by the
 93 parameter κ . Precipitation P is a sink of liquid water. The upward and downward convective
 94 fluxes lead to the Rayleigh drag terms appearing in the right-hand sides of (2.1a), (2.1b).

95 Relaxational parameterizations are used for the condensation C and precipitation P , with
 96 corresponding relaxation times τ_c, τ_p :

$$C = \frac{Q - Q^s}{\tau_c} \mathcal{H}(Q - Q^s), \quad P = \frac{W - W_{cr}}{\tau_p} \mathcal{H}(W - W_{cr}), \quad (2.2)$$

97 where Q^s is the saturation threshold for water vapor, W_{cr} is the precipitation threshold, and
 98 \mathcal{H} denotes the Heaviside (step-) function. Bulk formulas are used for surface evaporation and
 99 vaporization:

$$E = \alpha_e \frac{|\mathbf{v}_1|}{|\mathbf{v}_{max}|} (Q^s - Q) \mathcal{H}(Q^s - Q), \quad V = \alpha_v \frac{(Q^s - Q)}{Q^s} W \mathcal{H}(Q^s - Q) \quad (2.3)$$

100 where $|\mathbf{v}_{max}|$ is the maximum value of the lower-layer velocity, and $\alpha_{e,v}$ are adjustable coeffi-
 101 cients. Although a threshold for evaporation can be taken to be different, we make the simplest
 102 choice and set it to be the same as in the condensation (2.2). The saturation threshold Q^s
 103 depends on pressure according to the Clausius-Clapeyron law. This dependence, within the
 104 approximations of the model, can be taken to be linear (cf. Bouchut *et al.* 2009) or exponen-
 105 tial, with a small exponent: $Q^s(h) = Q_0^s e^{-\epsilon\eta} \approx Q_0^s (1 - \epsilon\eta)$, where η is the thickness anomaly
 106 in the lower layer, see below. Evaporation (2.3) is the only process in the boundary layer we
 107 include in the model. Momentum dissipation due to the bottom friction can be easily included,
 108 and more involved parameterizations of the boundary layer can be used too (cf. Schecter and
 109 Dunkerton 2009), which we will not do, keeping the model as simple as possible and limiting
 110 the number of adjustable parameters. The parametrization of surface evaporation in (2.3) is
 111 well suited for the evolution of the atmosphere over the ocean (e.g. Katsaros 2001) but should
 112 be changed over land. Again, for simplicity, we simply switch it off whenever $b \neq 0$. We should
 113 emphasize that an advantage of this version of the model, as compared to the original one
 114 in Lambaerts *et al.* (2011b), is that it tracks condensed water, and thus is “cloud-resolving”,
 115 with the distribution of W giving a rough idea of the cloud cover layer-wise.

116 Radiative relaxation term of the form $-(h_1 - H_e)/\tau_r$, where τ_r is a characteristic relaxation
 117 time, can be added in the r.h.s. of (2.1c) in the condensation regions, to model the physical
 118 process of enhanced outgoing radiation by clouds. We test the influence of this term in the

119 numerical simulations below.

120 A detailed discussion of the conservation laws and energy budget in the model can be found
121 in [Rostami and Zeitlin \(2018\)](#) and we will not repeat it here.

122 2.2. *Scaling of the “dry” version of the model, and the background jet configuration*

123 We introduce unperturbed thicknesses of the layers H_{0i} , $i = 1, 2$ and the unperturbed total
124 thickness $H_0 = H_{01} + H_{02}$. The following scaling is used for the independent and dependent
125 variables in the “dry” version of the model:

$$(x, y) \sim L_d \equiv \sqrt{gH_0}/f_0, \quad t \sim (1/f_0), \quad (u, v) \sim \sqrt{gH_0}, \quad (h, b) \sim H_0, \quad (2.4)$$

126 where L_d denotes the barotropic deformation radius. For dimensional estimates we will place
127 the beta - plane at a low latitude Φ_0 , and take $H_0 = 10$ km, $f_0 = 2\Omega \sin \Phi_0 = 3.2717 \times 10^{-5} \text{ s}^{-1}$
128 at $\Phi_0 = 13^\circ \text{ N}$, which gives $L_d = 9568$ km, $\beta = 2.2243 \times 10^{-11} (\text{ms})^{-1}$, and $\sqrt{gH_0} = 313 \text{ ms}^{-1}$.

129 We recall that in the tangent-plane approximation

$$f = 2\Omega \sin \Phi = 2\Omega \sin \Phi_0 + \frac{2\Omega}{R_e} \cos \Phi_0 y = f_0 + \beta y = f_0 \left(1 + \frac{\beta L_d}{f_0} y^* \right) \equiv f_0 (1 + \bar{\beta} y^*), \quad (2.5)$$

where the asterisk denotes non-dimensional variables, and we introduced the non-dimensional
beta $\bar{\beta}$. Its value is $\bar{\beta} = 6.505$ at $\Phi = 13^\circ$. The non-dimensional “dry” equations on the
 β -plane, after dropping the asterisks become :

$$\partial_t u_i + u_i \partial_x u_i + v_i \partial_y u_i - (1 + \bar{\beta} y) v_i = -\partial_x (h_1 + s^{i-1} h_2), \quad (2.6a)$$

$$\partial_t v_i + u_i \partial_x v_i + v_i \partial_y v_i + (1 + \bar{\beta} y) u_i = -\partial_y (h_1 + s^{i-1} h_2), \quad (2.6b)$$

$$\partial_t h_i + \partial_x ((h_i - (2 - i)b) u_i) + \partial_y ((h_i - (2 - i)b) v_i) = 0, \quad i = 1, 2. \quad (2.6c)$$

130 It is easy to see that in the absence of nontrivial topography, i.e. with $b = \text{const}$, any zonal
131 jet $u_i = U_i(y)$, $h_i = H_{0i} + H_i(y)$, $v_i = 0$ in geostrophic equilibrium:

$$(1 + \bar{\beta} y) U_i = -\partial_y (H_1 + s^{i-1} H_2) \quad (2.7)$$

132 is an exact stationary solution of (2.6). If $b = b(x) \neq 0$, which will be our choice below, only
133 the upper-layer jets with quiescent lower layer $U_1 = 0$ are exact solutions. This will be the
134 case we consider. We choose the classical Bickley jet profile for U_2 , which we orient from East
135 to West:

$$U_2(y) = -U_0 \text{sech}^2[a(y - y_c)], \quad (2.8)$$

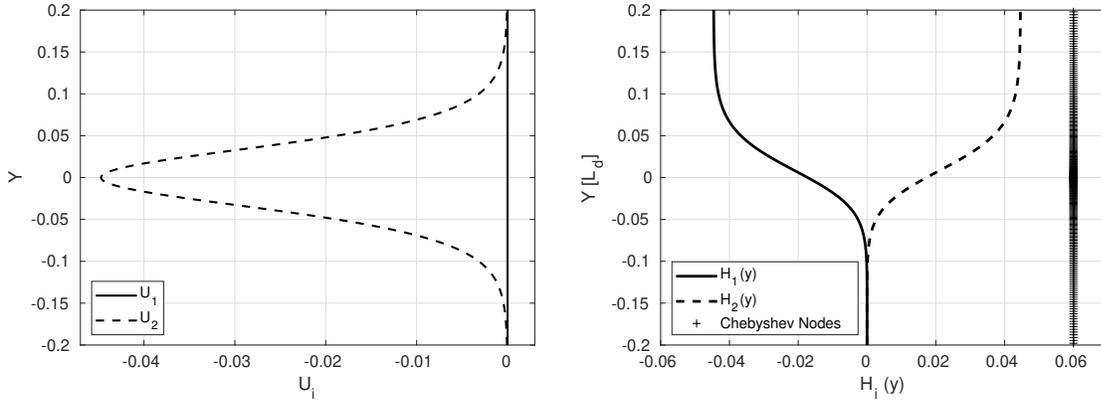


Figure 1. Meridional profiles of zonal velocities U_i (left panel) and thickness deviations $H_i(y)$ (right panel). Lower (upper) layer: solid (dashed). Chebyshev nodes, which are used for spatial discretization, are shown in the right panel. $Ro = 0.15$, $Bu = 11.35$, $U_0 = 0.045$, $a = 20$, $H_{01} = 1/3$, $H_{02} = 2/3$, $k = 20$, $\theta_2/\theta_1 = 1.1$, $\Phi = 13^\circ$ N.

136 where a controls the width of the jet. The Burger number of the jet is defined as $Bu = L_d^2/R^2$,
 137 where R is the width of the jet at the level $0.01 U_0$. The position of the jet axis y_c is variable.
 138 Instead of changing it, we choose $y_c = 0$ and vary instead the latitude $\bar{\Phi}_0$, and hence the value
 139 of $\bar{\beta}$ in (2.6), as it is known that this parameter is crucial for (in)stability properties (Kuo
 140 1978). The corresponding profiles of background thickness deviations H_i are recovered from
 141 the geostrophic balance

$$H_2(y) = - \int_{-\infty}^y \frac{1}{s-1} (1 + \bar{\beta}y) U_2(y) dy, \quad H_1(y) = -H_2(y). \quad (2.9)$$

142 An upper-layer jet solution, with the characteristics roughly corresponding to the AEJ (e.g.
 143 Hsie and Cook 2008) is presented in Figure 1.

144 3. Linear stability analysis of the upper-layer easterly jet in the absence of 145 topography

We introduce small perturbations of all variables, denoted by primes, with respect to the upper-layer jet solution (2.8), (2.9):

$$\begin{aligned} u_1 &= u'_1, & v_1 &= v'_1, & h_1 &= H_1(y) + \eta'_1 + H_{01}, \\ u_2 &= U_2(y) + u'_2, & v_2 &= v'_2, & h_2 &= H_2(y) + \eta'_2 + H_{02}, \end{aligned}$$

and linearize the equations (2.6) with $b \equiv 0$ about the stationary solution. Dropping the

primes, we thus get:

$$\partial_t u_1 - (1 + \bar{\beta}y)v_1 + \partial_x(\eta_1 + \eta_2) = 0, \quad (3.10a)$$

$$\partial_t v_1 + (1 + \bar{\beta}y)u_1 + \partial_y(\eta_1 + \eta_2) = 0, \quad (3.10b)$$

$$\partial_t \eta_1 + H_1 \partial_x u_1 + v_1 \partial_y H_1 + H_1 \partial_y v_1 = 0, \quad (3.10c)$$

$$\partial_t u_2 + U_2 \partial_x u_2 + v_2 \partial_y U_2 - (1 + \bar{\beta}y)v_2 + \partial_x(\eta_1 + s\eta_2) = 0, \quad (3.10d)$$

$$\partial_t v_2 + U_2 \partial_x v_2 + (1 + \bar{\beta}y)u_2 + \partial_y(\eta_1 + s\eta_2) = 0, \quad (3.10e)$$

$$\partial_t \eta_2 + U_2 \partial_x \eta_2 + H_2 \partial_x u_2 + v_2 \partial_y H_2 + H_2 \partial_y v_2 = 0. \quad (3.10f)$$

146 We look for harmonic solutions of (3.10) in the form: $(u_i, v_i, \eta_i) = \text{Re}[(\tilde{u}_i, i\tilde{v}_i, \tilde{\eta}_i) e^{i(kx - \omega t)}]$
 147 Complex eigenfrequencies $\omega = \omega_R + i\omega_I$ with positive imaginary part ($\omega_I > 0$), correspond to
 148 instabilities with linear growth rate $\sigma = \omega_I$. The resulting eigenproblem is:

$$L [\tilde{u}_1 \ \tilde{v}_1 \ \tilde{\eta}_1 \ \tilde{u}_2 \ \tilde{v}_2 \ \tilde{\eta}_2]^T = \omega [\tilde{u}_1 \ \tilde{v}_1 \ \tilde{\eta}_1 \ \tilde{u}_2 \ \tilde{v}_2 \ \tilde{\eta}_2]^T, \quad (3.11)$$

149

$$L = \begin{bmatrix} 0 & -(1 + \bar{\beta}y) & k & 0 & 0 & k \\ -(1 + \bar{\beta}y) & 0 & -D & 0 & 0 & -D \\ kH_1 & H_1'(y) + H_1(y)D & 0 & 0 & 0 & 0 \\ 0 & 0 & k & kU_2(y) & U_2'(y) - (1 + \bar{\beta}y) & ks \\ 0 & 0 & -D & -(1 + \bar{\beta}y) & kU_2(y) & -sD \\ 0 & 0 & 0 & kH_2(y) & H_2'(y) + H_2(y)D & kU_2(y) \end{bmatrix}, \quad (3.12)$$

150 where D denotes the operator of differentiation with respect to y , which will become the
 151 Chebyshev differentiation matrix after discretization, the superscript T denotes the transpose,
 152 and prime denotes the derivative of the corresponding functions with respect to their argument
 153 y .

154 We should emphasize at this point that the analysis of the spectrum of unstable modes
 155 of easterly jets on the beta-plane, as compared to their westerly counterparts, encounters
 156 difficulties related to the presence of critical levels $y = y_{cr}$ where the phase velocity of the
 157 eigenmode $c = \omega_R/k$ coincides with the jet velocity $U(y_{cr})$, which appear already in the sim-
 158 plest case of the barotropic jet in the quasi-geostrophic approximation (e.g. Maslowe 1991,
 159 Swaters 1999). This is why a theoretical analysis of this spectrum is a challenge. Instead of

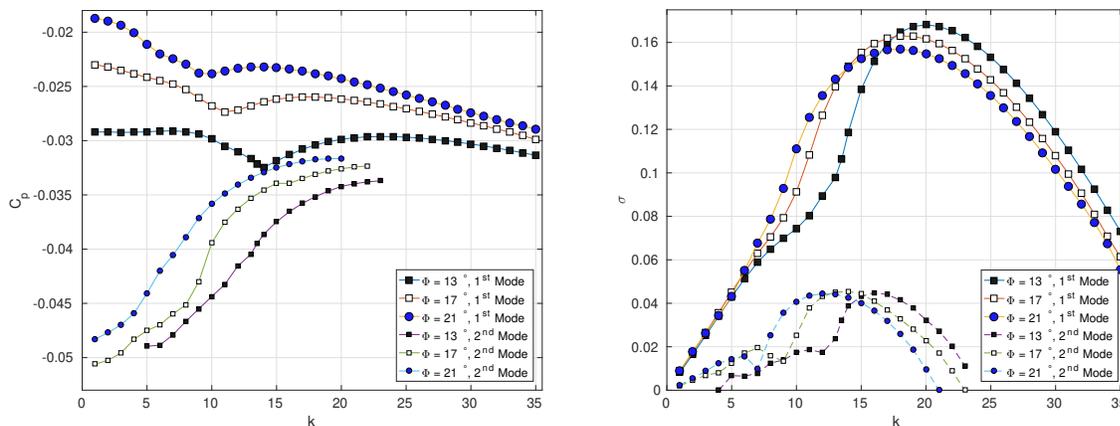


Figure 2. Stability diagrams of the upper-layer easterly Bickley jet on the β -plane at three different latitudes in terms of phase velocity c_p (left panel) and growth rate σ (right panel) as functions of zonal wavenumber k . $Ro = 0.15$, $Bu = 11.35$, $U_0 = 0.045 [f_0 L_d]$, $a = 20$, $H_{01} = 1/3$, $H_{02} = 2/3$ (Colour online).

160 tempting it, we adopt a pragmatic approach, consisting in a direct numerical solution of the
 161 eigenproblem (3.11), (3.12) by using pseudo-spectral collocation method with a non-uniform
 162 distribution of Chebyshev nodes which is denser inside the jet and is shown in the right panel
 163 of Figure 1. Critical levels should be treated carefully, as they can give rise to discontinu-
 164 ous numerical pseudo-modes. We identify them by increasing the numerical resolution, and
 165 discard. Typically, the resolution with $N = 400$ collocation points in y turns out to be fully
 166 sufficient. We first benchmarked the method on the f -plane, where the results are insensitive
 167 to the orientation of the jet on the plane, and had to reproduce those of Lambaerts *et al.*
 168 (2012) who studied stability of the same configuration, but in the case of a westerly jet. This
 169 is indeed the case, both for the stability diagram, and for the phase-portrait and meridional
 170 cross-section of the most unstable mode, as shown in the Appendix.

171 After having benchmarked the method, we perform the same analysis on the beta-plane.
 172 Stability diagram in a large range of zonal wavenumbers for three latitudinal positions of the
 173 jet, with corresponding changes of $\bar{\beta}$, is presented in Figure 2, where we display not only
 174 the first, most unstable, but also the second unstable mode. As follows from the Figure, the
 175 higher the latitude the closer, qualitatively, the results are to those on the f -plane, cf. Figure
 176 A1, although the growth rates on the beta-plane are higher. The curves of the phase velocity
 177 display a non-monotonicity in the interval $k \in [5, 20]$, which becomes more pronounced at
 178 lower latitudes. A similar non-monotonicity was reported in Kuo (1978), although that paper
 179 used a two-layer model with a rigid lid and explored a configuration with nonzero jet velocity
 180 in the lower layer, but also in Thorncroft and Hoskins (1994a), who worked with the primitive
 181 equation model on the sphere, like Simmons (1977). We should emphasize a very good qual-

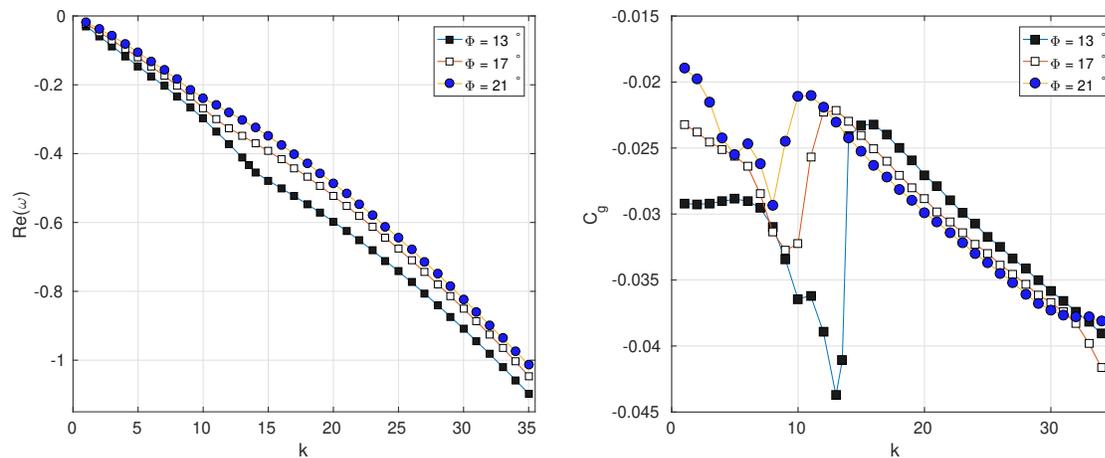


Figure 3. Real part of eigenfrequency (left panel) and group velocity (right panel) of unstable modes of Figure 2 (Colour online).

182 itative agreement of the stability diagram, as well as a good quantitative agreement of the
 183 zonal wavenumber, phase velocity, and growth rate of the most unstable mode in our simple
 184 model (taking into account our choice of scaling), with the results of [Thorncroft and Hoskins](#)
 185 ([1994a](#)). Note that only the most unstable mode was identified in this paper, while we find
 186 them all. What is important, the corresponding group velocity of the unstable modes, which
 187 is presented together with the real part of the eigenfrequency in Figure 3, is always westward,
 188 as the phase velocity itself, like in the case of AEW in the observations.

189 The phase portraits of the first and the second unstable modes of Figure 2, with the same
 190 zonal wavenumber $k = 20$, are presented in Figure 4. First, we clearly see in the Figure that
 191 the phase portrait of the most unstable mode is qualitatively similar to that of its counterpart
 192 on the f -plane, cf. Figure [A2](#), although it is subject to deformations due to the beta-effect,
 193 especially in the lower layer. Second, we see that the first unstable mode is varicose, i. e.
 194 having anomalies of the same sign at both sides of the jet at a given longitude, and the
 195 second unstable mode is sinuous in the upper layer, where the jet is located, a general fact
 196 which is well-known for jet instabilities. Note that baroclinic character of the unstable mode is
 197 more pronounced at the southern side of the jet. The instability in question can be, as usual,
 198 interpreted as a resonance between Rossby waves counter-propagating across the meridional
 199 gradients of potential vorticity (cf. [Thorncroft and Hoskins 1994a](#)). An important question
 200 raised in the literature ([Hall et al. 2006](#)) is sensitivity of the results of the stability analysis to
 201 the parameters of the jet. Within our simple model, and with our simplest parameterization of
 202 the jet, a study of sensitivity is straightforward. The sensitivity of the results to the latitudinal
 203 position of the jet is clear from Figure 2: the lower the latitude of the jet center, the larger the

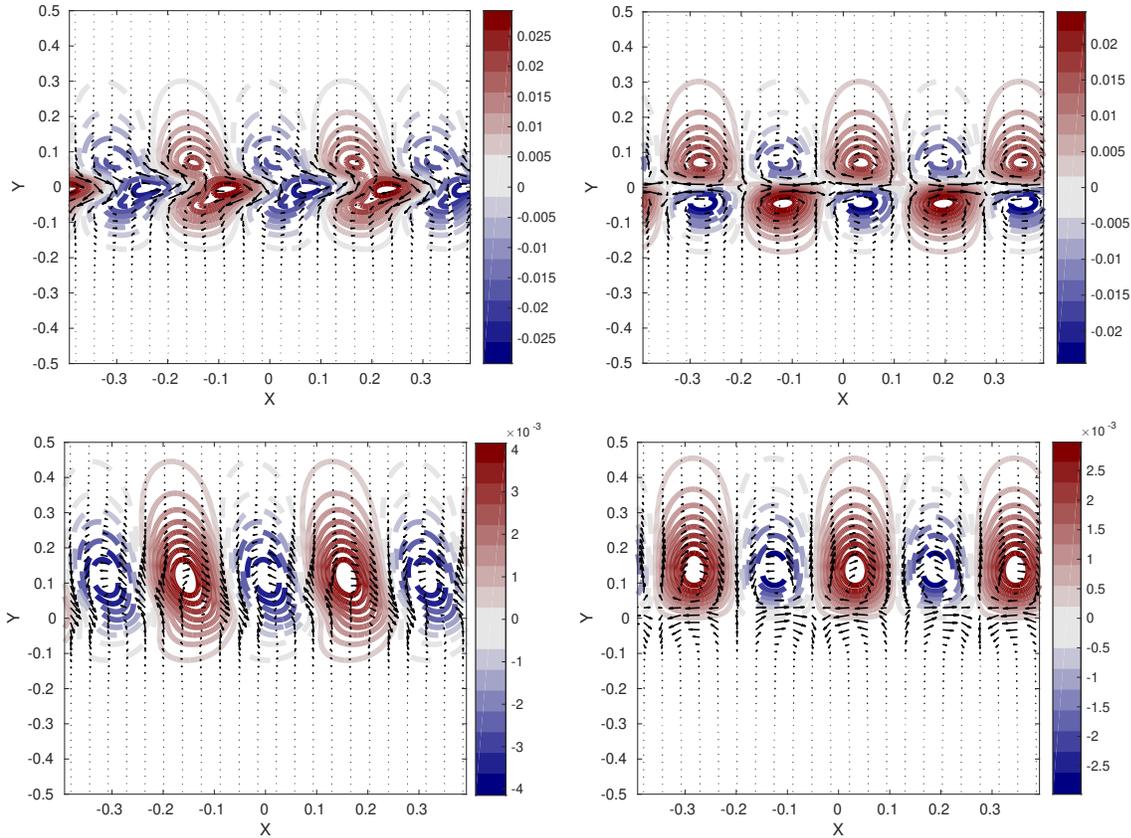


Figure 4. Geostrophic stream-functions of the unstable modes on the β -plane in the upper layer (upper row) [$\psi_2 = \eta_1 + s\eta_2$] and the lower layer (lower row) [$\psi_1 = \eta_1 + \eta_2$] of the first (left column) and the second (right column) unstable modes. Positive (negative) values are represented by solid (dashed) lines, and the velocity field is represented by arrows. $Ro = 0.15, U_0 = 0.045, Bu = 11.35, a = 20, H_{01} = 1/3, H_{02} = 2/3, k = 20, \Phi = 13^\circ \text{ N}$ (Colour online).

204 wavenumber of the most unstable mode. The sensitivity to the value of the Burger number,
 205 i.e. to the width of the jet, can be inferred already from Figure A1. The sensitivity to the
 206 Burger number on the β - plane is presented in Figure 5. As follows from these figures, at lower
 207 Bu , i.e. for wider jets, the wavenumber of the most unstable mode diminishes, and at higher
 208 latitudes the wavenumber of the most unstable mode shifts down, as well.

209 The results of the analysis of sensitivity to the aspect ratio and stratification are presented,
 210 respectively, in Figures 6 and 7, which show that these sensitivities are rather low in the
 211 interval of wavenumbers corresponding to the highest (and, thus, most interesting) growth
 212 rates.

213 We should finally comment on the role of the bottom friction, which is easy to include in
 214 the model, as already mentioned. We, however do not do this, as the lower layer in our basic
 215 state is quiescent, and the amplitude of velocity of the unstable modes in this layer is an
 216 order of magnitude lower than in the upper layer, cf. Figure 4. Yet in the configuration with
 217 lower-layer westerlies, which we neglect, the lower layer damping does influence the structure

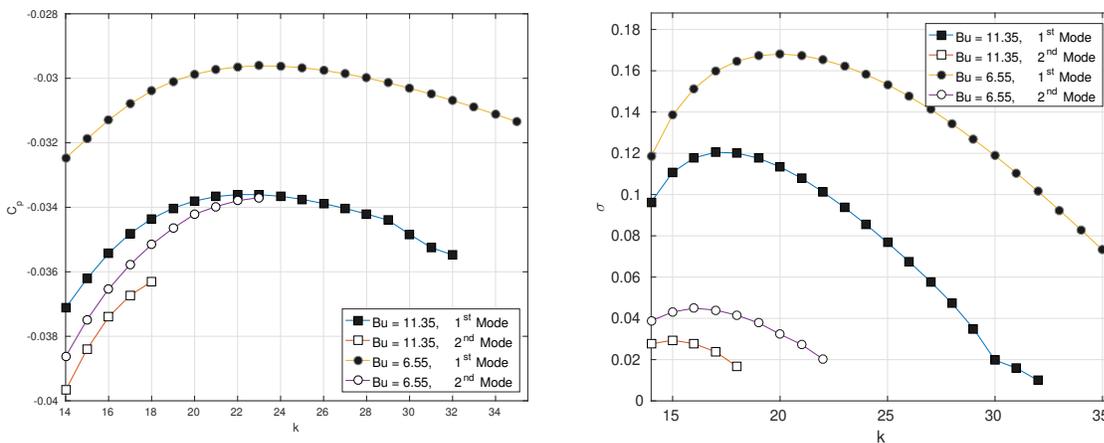


Figure 5. Sensitivity of the stability diagram, in the region of the highest growth rates to the value of Bu . Phase velocity c_p (left panel) and growth rate σ (right panel). $Ro = 0.15$, $U_0 = 0.0447 [f_0 L_d]$, $k = 20$, $s = 1.1$, $H_{01} = 1/3$, $H_{02} = 2/3$.

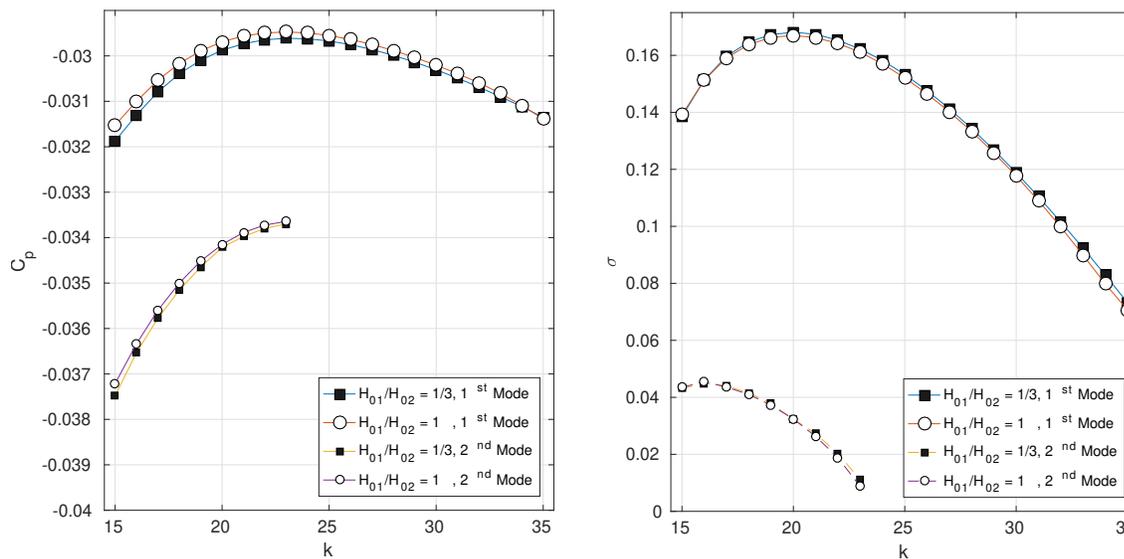


Figure 6. Sensitivity to the ratio of the layers' thicknesses of the stability diagram for phase velocity c_p (left panel) and growth rate σ (right panel). $Ro = 0.15$, $Bu = 11.35$, $U_0 = 0.045 [f_0 L_d]$, $a = 20$, $k = 20$, $s = 1.1$, $H_{01} + H_{02} = 1$.

218 of the normal modes (Hall *et al.* 2006).

219 4. Nonlinear evolution of the unstable disturbances

220 4.1. Parameters and setup

221 To investigate the nonlinear evolution of the instabilities of the jet, we used a high-resolution
 222 finite-volume code by Bouchut and Zeitlin (2010). The simulations were set in the domain
 223 with the zonal dimension $L_x = n \lambda$, $n = 5$, in units of L_d , where λ is the wavelength of an

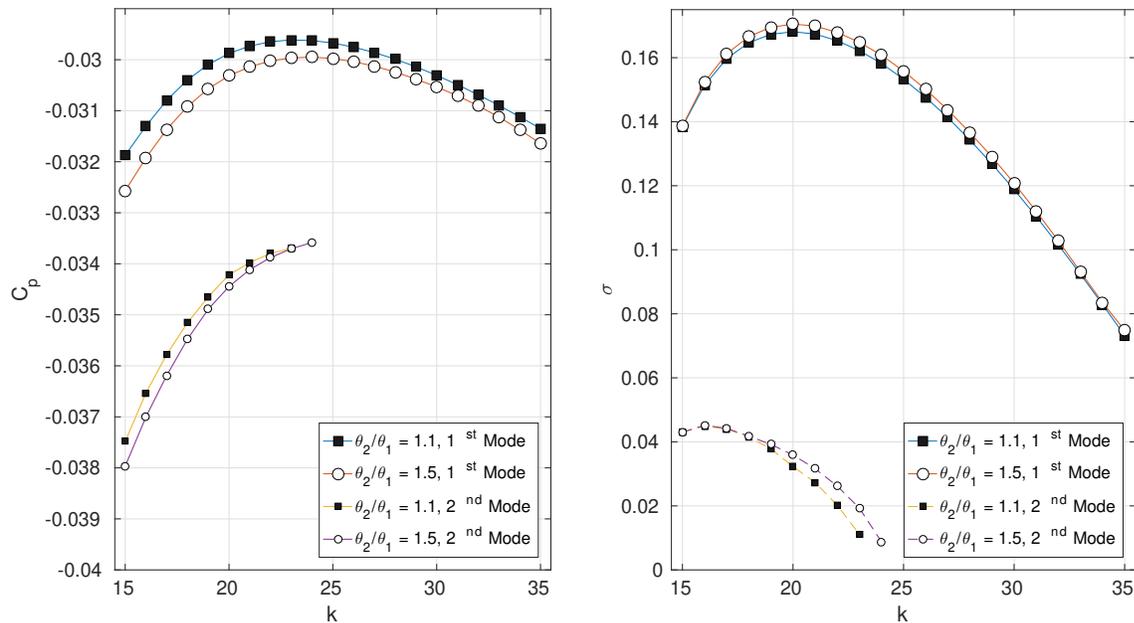


Figure 7. Sensitivity to stratification of the stability diagram for phase velocity c_p (left panel) and growth rate σ (right panel). $Ro = 0.15$, $Bu = 11.35$, $U_0 = 0.045 [f_0 L_d]$, $a = 20$, $k = 20$, $s = 1.1$, $H_{01} = 1/3$, $H_{02} = 2/3$.

224 unstable mode used in the simulation, and meridional dimension $L_y = 3.8L_d$. A “continent”
 225 in the simulations with topography was represented by a meridional band of width $0.7L_d$,
 226 of uniform height $b = 0.05H_0$. (The results presented below do not qualitatively vary with
 227 changes of this value, within the same order of magnitude). Periodic and sponge boundary
 228 conditions were implemented, respectively, in zonal (along-jet), and meridional (cross-flow)
 229 directions. Notice that the periodic boundary conditions imply that the background jet has
 230 uniform intensity across the domain, which is, obviously, an idealization of the observed AEJ.
 231 We should also say that, concerning the land-sea boundary, the model, by construction, cannot
 232 capture the associated thermal contrast. To do this, a variant of the model, which was recently
 233 proposed in Liu *et al.* (2020), and allows for horizontal temperature gradients, is necessary.
 234 This development is postponed to a future work.

235 The following values of parameters characterizing the moist processes were used in the
 236 simulations: $\gamma = 0.3$, $\gamma^* = 0.1\gamma$, $\alpha_e = 0.25$, $\epsilon = 0.3$, $Q^s = 0.9$, $W_{cr} = 0.01$. The simulations
 237 in the moist-convective environment were initialized with a uniform, humidity in the lower
 238 layer, close to saturation, according to the hypotheses of the model: $Q_1 = Q^s - 0.01$ over
 239 the ocean, and far from the saturation $0.2Q^s$ over the land. The humidity in the upper layer
 240 was taken to be far from the saturation ($Q_2 = Q^s - 0.2$) The liquid water content was set to
 241 be zero everywhere at the initial moment. The difference between the evaporation properties

242 between the land and the sea was accounted for in the simplest way: the evaporation was
 243 switched off over topography. The condensation and precipitation relaxation times were taken
 244 to be $\tau_c = 120 \delta t \approx 2 h$, $\tau_p = 30 \delta t$, respectively, where $\delta t = 0.002$ is the non-dimensional
 245 time-step of the code. Increasing the relaxation time of condensation within the same order
 246 of magnitude does not change the results qualitatively. The non-dimensional vaporization
 247 coefficient was chosen as $\alpha_v = (10 \cdot Q^s \tau_c)^{-1}$. We also compared simulations without and with
 248 radiative cooling, with the relaxation time 40 days in the latter case.

249 4.2. “Dry” vs moist-convective simulations without topography initialized with the 250 most unstable mode

251 We start with the analysis of nonlinear evolution of the most unstable mode in the absence
 252 of topography in the “dry” configuration, by initializing the simulations with the background
 253 jet with superimposed small (\approx one tenth of percent, compared to the values in the jet
 254 itself) perturbation of all fields of an unstable mode. The corresponding evolution of relative
 255 vorticity is presented in Figure 8, and reveals no characteristic comma-shaped pattern of
 256 a breaking unstable wave, which is typical for midlatitude westerly jets (Lambaerts *et al.*
 257 2011a). Nevertheless, a steepening of the streamlines, which was called “breaking AEW” in
 258 the analysis of observations (Cornforth *et al.* 2017, Lafore *et al.* 2017), does take place. After
 259 a period of initial growth a nonlinear saturation at finite amplitude takes place. At the same
 260 time, lateral spreading of vorticity anomalies in the lower layer is observed. What is important
 261 in the following, this spreading is meridionally asymmetric, extending farther at the northern
 262 than at the southern flank of the jet. As is easy to check by using the values of the phase
 263 velocity and frequency from Figures 2, 3, neither of them matches their counterparts for
 264 free Rossby waves estimated from the quasi-geostrophic expression $\omega = -\bar{\beta}k/(k^2 + l^2 + L_d^{-2})$,
 265 where l is meridional wavenumber of the free wave, and L_d is deformation radius (barotropic or
 266 baroclinic). Analysis of the evolution of divergence (not shown) reveals no significant emission
 267 of inertia-gravity waves either. Hence, the origin of the saturation of the instability is not a loss
 268 of energy by radiation, and is essentially nonlinear. It is not a breaking of the unstable mode,
 269 like in westerly mid-latitude jets, either, but a repartition of energy through the spectrum of
 270 unstable modes. This is confirmed by the Fourier analysis of the dynamical variables, which
 271 reveals the appearance and growth of modes with lower values of k during the process of
 272 saturation, not shown. (Notice that subharmonic modes were excluded in Lambaerts *et al.*
 273 (2012) by the choice of boundary conditions, unlike the present case).

274 As compared to the “dry” simulation of Figure 8, the evolution of the instability in the moist-

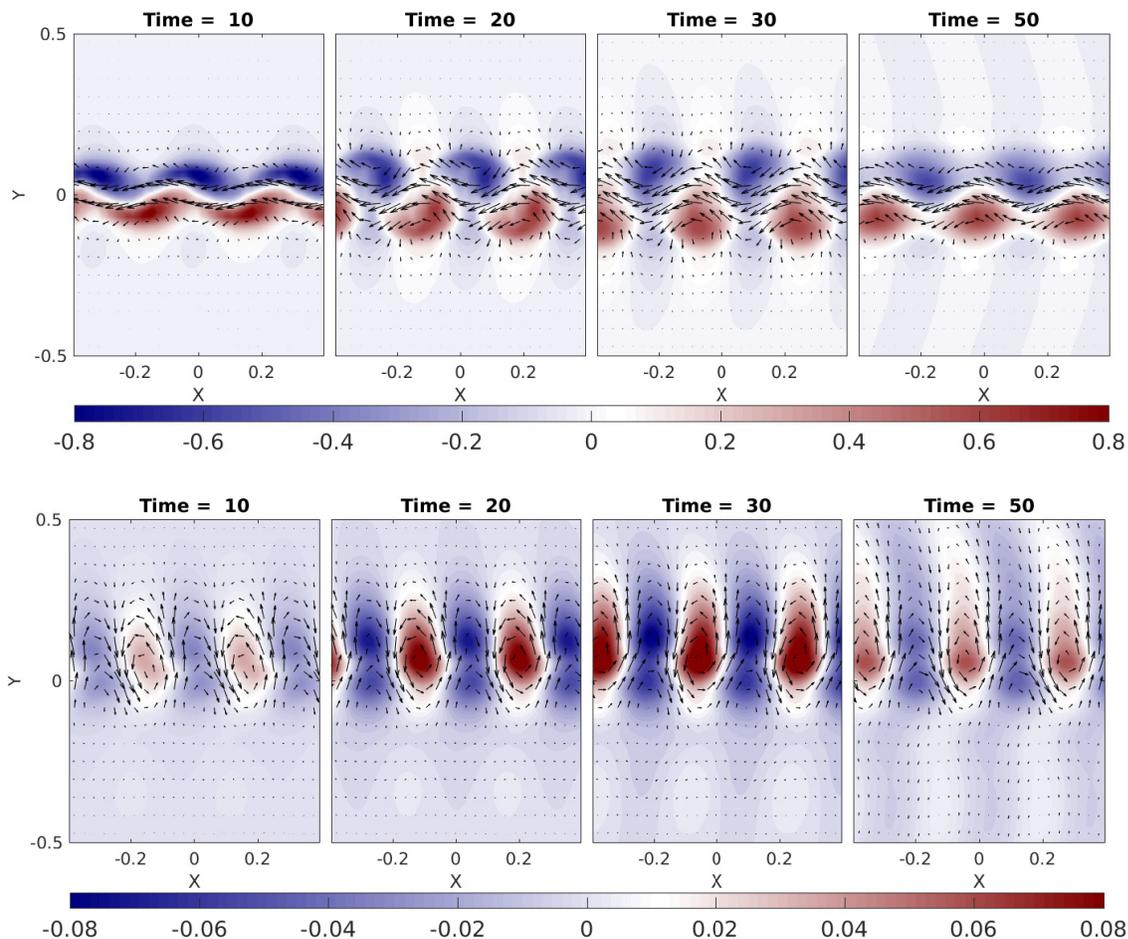


Figure 8. Evolution of vorticity and velocity in the upper (upper row) and lower (lower row) layers in “dry” environment in a simulation initialized with the full periodic unstable mode of small non-dimensional amplitude (0.005), without radiative cooling. The global configuration is the same as in Figure 4. The length unit at the axes here and below is L_d (Colour online).

275 convective (MC) environment is totally different, as follows from Figure 9. The meridionally
 276 asymmetric periodic zones of cyclonic vorticity developing from their counterparts in the
 277 initial perturbation, like in the dry case, undergo a different evolution under the influence of
 278 the moist convection. At earlier stages they are intensified due to the moist convection. The
 279 mechanism of this intensification in the present model is very simple, and consists in growth
 280 of cyclonic vorticity due to the convective mass sink, as explained in Lambaerts *et al.* (2011a).
 281 As follows from Figure 9, the condensation is correlated with the enhanced convergence zone
 282 inside of the vorticity anomaly, which is shifted north-east. Convergence is a proxy for ascent
 283 in shallow-water models, so what is observed in the Figure is consistent with the discussion
 284 of the position of maximum ascent in Thorncroft and Hoskins (1994a,b). At later stages a
 285 secondary lower-layer jet, which is *retrograde* with respect to the primary one, is formed
 286 close to the axis of the original upper-layer jet. Interestingly, retrograde jets in the lower

287 troposphere are typically accompanying the AEJ in the observations (Cook 1999), so here
288 we see a dynamical mechanism of their generation. As follows from the Figure, this jet is
289 formed by merging cyclonic anomalies of relative vorticity, which are induced by the unstable
290 mode, are growing due to the instability, and are enhanced by the condensation. On the other
291 hand, localized zones of enhanced cyclonic vorticity are formed at the northern flank of the
292 jet. No such cyclogenesis is observed at the southern flank, due to meridional asymmetry
293 of the unstable mode. As follows from Figure 10, where we present the late stages of the
294 simulation of Figure 9, the cyclones that form at the northern flank of the jet are further
295 intensified due to the upward convective fluxes, as explained in Lambaerts *et al.* (2011a), and
296 have a tendency to detach from the jet. Transformation of positive vorticity into detached
297 cyclones is not fast and takes few weeks to occur. This suggests an importance of non-modal
298 growth, and corresponding initializations of non-linear simulations, see below. Such properties
299 of cyclogenesis by AEJ, and asymmetry between the northern and southern flanks of the AEJ,
300 have been reported in data analyses (e.g. Pytharoulis and Thorncroft 1999, Chen *et al.* 2008).
301 The cloud cover presented in the bottom-right panel of the Figure is in qualitative agreement
302 with observations, in a sense that condensation is within and eastward of the trough, but more
303 complicated cloud structures were also detected in the observations, e.g. Cornforth *et al.* (2017)
304 and references therein.

305 The left and the right panels of Figure 11 display Hovmöller diagrams of vorticity corre-
306 sponding to the above-described “dry” and moist-convective experiments, and clearly show
307 the westward propagation of disturbances, and their intensification in the moist-convective
308 case.

309 We also repeated the same simulation with radiative relaxation. Qualitatively, the scenario
310 of the evolution does not change, but the intensity of the vortex and jet structures generated
311 by the developing instability decreases at later stages, not shown.

312 4.3. *Evolution of localized wave-packets of unstable modes, and the role of the* 313 *land-ocean contrast and topography*

314 In order to render our simulations more realistic, we also initialized them with a localized
315 wave-packets of the unstable modes. An example of such initial condition is given in Figure
316 12. It is obtained by taking a Gaussian envelope in the zonal direction of all fields correspond-
317 ing to the most unstable mode, with a width roughly corresponding to two wavelengths of
318 the latter. (It is possible, by solving a “dry” adjoint problem, to identify the most rapidly
319 growing non-modal perturbation, following e.g. Parker (2008) in the present context, but we

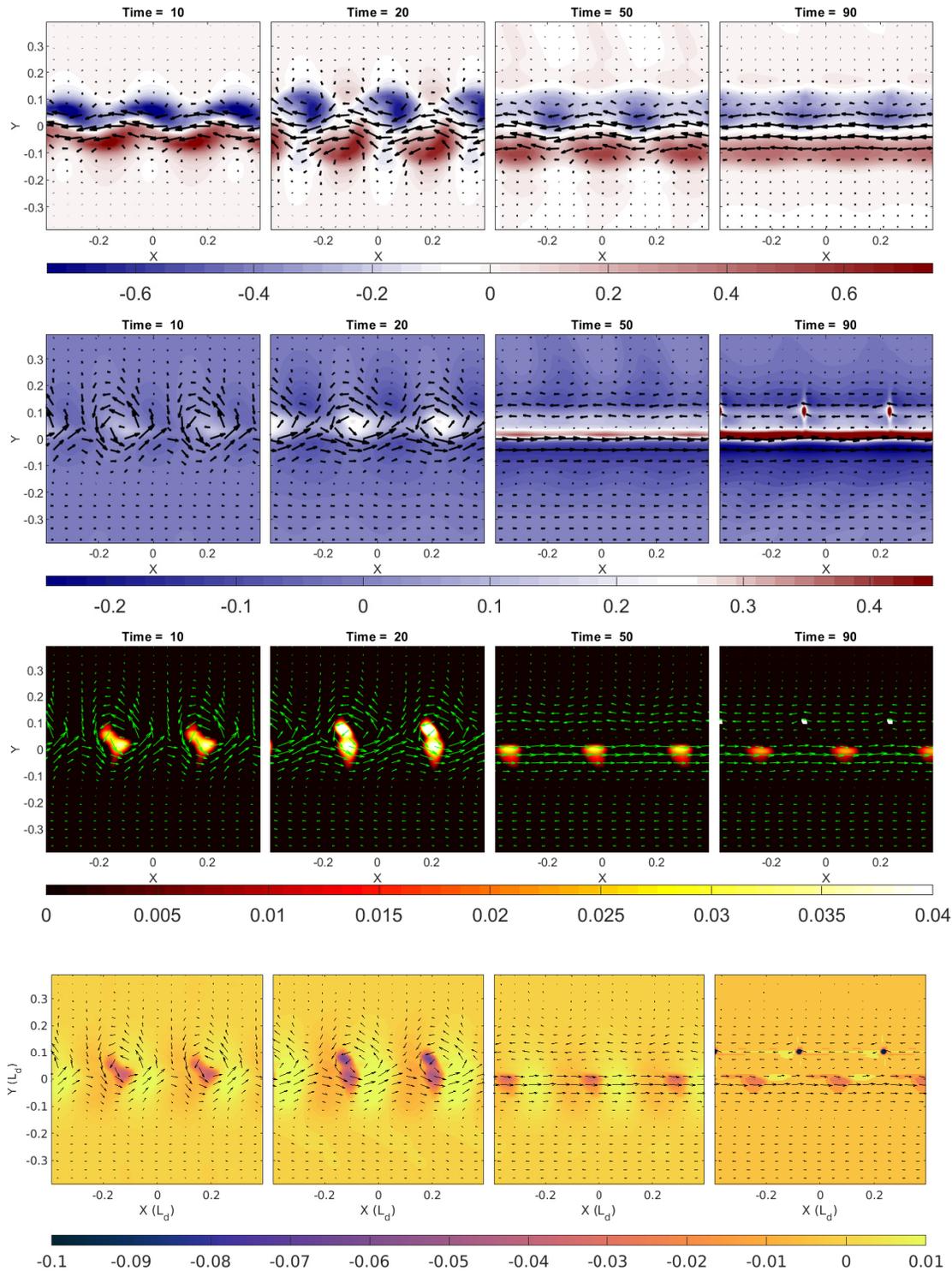


Figure 9. Evolution of the most unstable mode in moist-convective environment, as seen in the vorticity and velocity fields in the upper (upper row) and lower (second row) layer. Corresponding evolution of condensation (third row) and divergence (fourth row) in the lower layer. Same simulation as in Figure 8, but with condensation and evaporation switched on (Colour online).

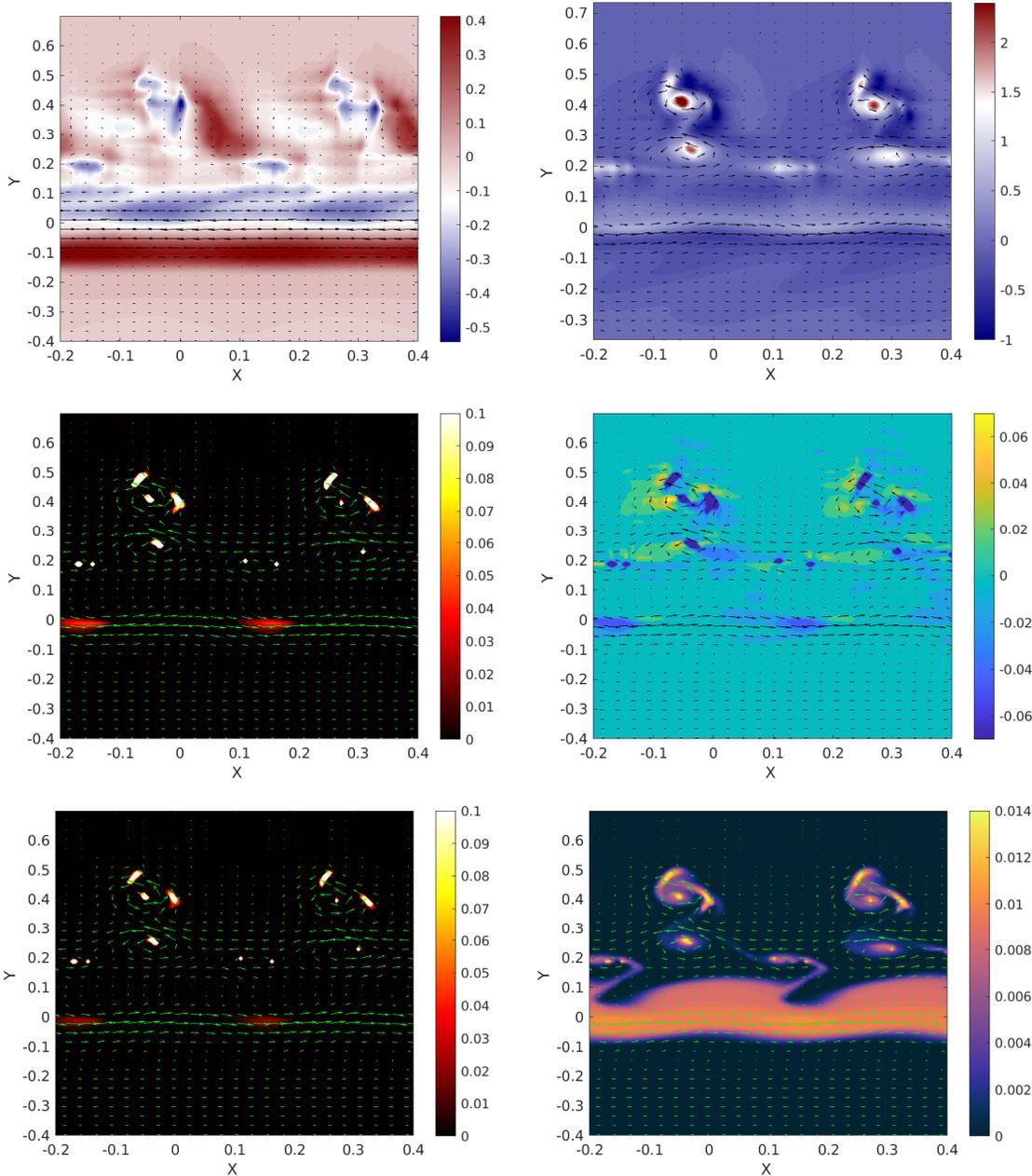


Figure 10. *First row*: Late stages of the evolution of vorticity and velocity in the upper (left panel) and lower (right panel) layers illustrating cyclogenesis in the lower layer due to the moist-convection. *Second row*: Corresponding condensation (left panel) and divergence (right panel) fields in the lower layer. *Third row*: Corresponding precipitation (left panel) and clouds (right panel). To distinguish various details, we made a zoom on two adjacent detaching vortices (Colour online).

320 adopt here a more straightforward approach). We analyze the evolution of the jet with super-
 321 imposed perturbation of Figure 12 in “dry” (not shown) and moist-convective environments,
 322 and determine the role in this process of topography and the land-ocean boundary, which are
 323 roughly mimicking the West-African plateau, The height of the plateau is taken to be constant
 324 $b = 0.05H_0$, and have zonal extension $[-0.35 \ 0.35]$ in non-dimensional terms. The nonlinear

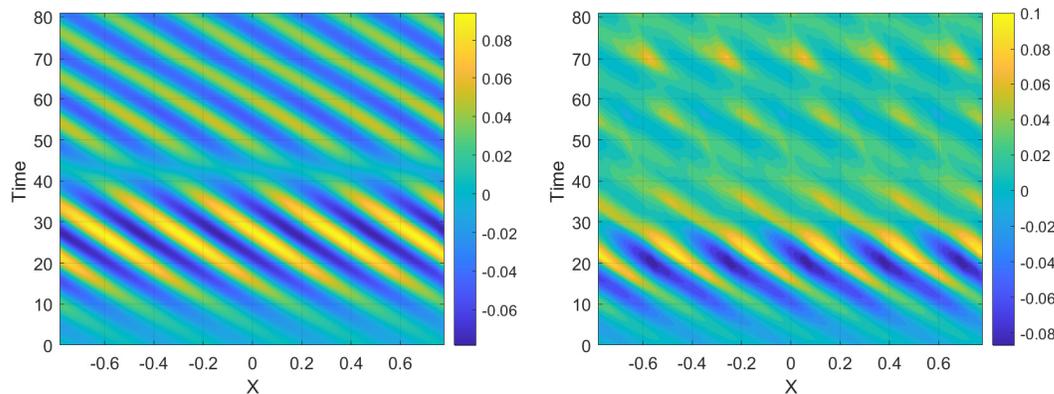


Figure 11. Hovmöller diagrams of vorticity in the lower layer at $y = 0.1$ in “dry” (left panel) and moist-convective (right panel) environments corresponding to Figs. 8 and 9, respectively. Note that colorbars are not the same in the panels, for better visibility (Colour online).

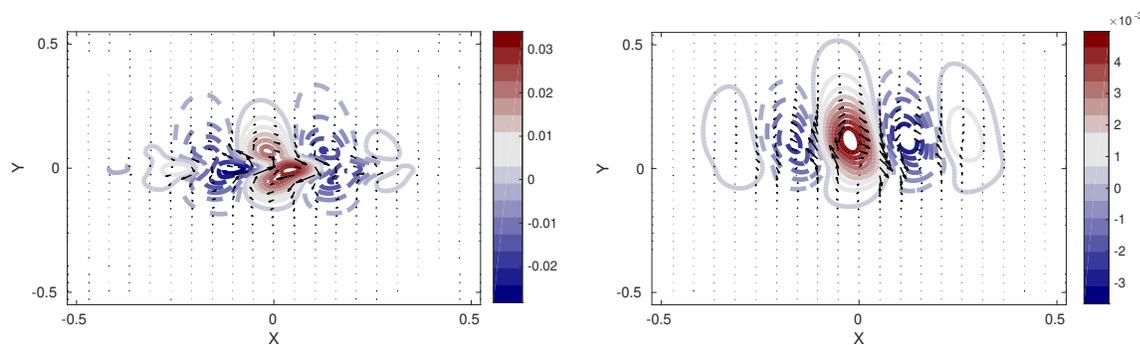


Figure 12. Initial condition as a wave packet constructed with the most unstable mode. Left (right) panel: vorticity and velocity in the upper (lower) layer. Global configuration is the same as in Figure 4. The maximal non-dimensional amplitude of the perturbation is 0.005 (Colour online).

325 simulations are initialized with wave-packets of unstable modes over the land, as this is the
 326 case in the observations of AEW. The evolution of the wave-packet in the moist-convective en-
 327 vironment in such configuration is presented in Figure 14. The corresponding development of
 328 clouds and precipitation is shown in Figure 15. As follows from the Figures, the wave-packet,
 329 as expected, is subject to dispersion, accompanied by wave emission. At the same time, weak
 330 lower-layer vorticity anomalies induced by the initial condition are growing in intensity, due
 331 to the instability of the carrier mode, cf. Figure 9. There is practically no difference between
 332 the evolution with and without topographic elevation in the dry case, except for a weak to-
 333 pographic Kelvin wave propagating along the eastern coast (the corresponding signal, which
 334 is enhanced in the presence of moist convection, can be distinguished in the right-bottom
 335 panel of Figure 14), as the “continental plateau” is low - as already explained, we consider
 336 a configuration which is similar to the real one, within the precision of the model. However,
 337 the land-sea contrast in the moist-convective case is important. Cyclonic vorticity anomalies
 338 due to initial perturbation move over the “continent” practically without condensation, but

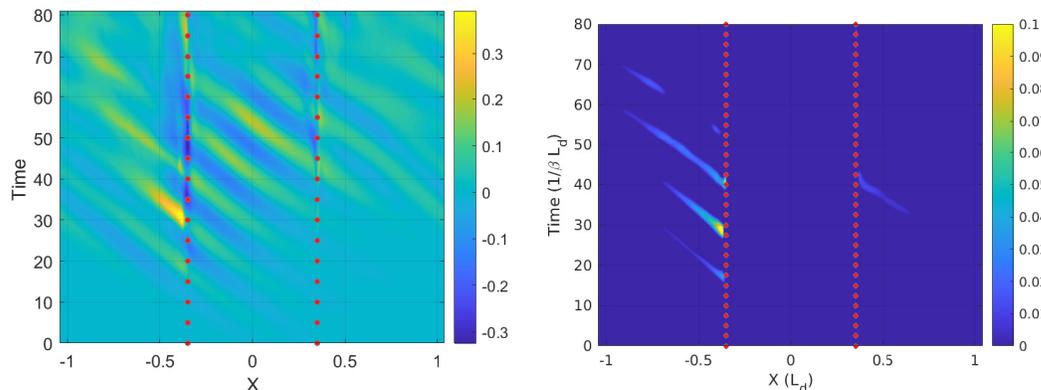


Figure 13. Hovmöller diagrams of vorticity (left panel) and condensation (right panel) in the lower layer corresponding to vorticity anomalies crossing the land-sea boundary (Colour online).

339 when they cross over the ocean the condensation due to evaporation switches on and leads
 340 to their rapid intensification. The result of this process is an appearance of a series of intense
 341 vortices, detaching from the continent and moving westward. The Hovmöller diagrams of vor-
 342 ticity and condensation presented in Figure 13 illustrate this process. Due to periodicity in
 343 zonal direction, these intense vortices eventually hit the continent at its eastern side. In the
 344 idealized setup of the present experiments, with the easterly jet of uniform intensity extending
 345 through the whole zonal extension of the domain, the encounter with the eastern continental
 346 boundary happens before the vortex has time to clearly detach from the jet, as in Figure 10.
 347 In real life, the intensity of the jet decreases across the Atlantic, and the detached vortices
 348 propagate North-West on the β -plane. The evolution of the hurricane-like vortices of different
 349 intensities was recently studied within the mcRSW model (Rostami and Zeitlin 2020), so we
 350 refer to this paper for the details of the behavior of the detached cyclones at later stages.

351 We should emphasize that, in the context of AEJ, the value of the group velocity of the
 352 perturbation is important, as it defines the speed of propagation of the wave-packet. We
 353 repeated the simulations of Figure 14 with a wave-packet built with the mode $k = 13$ with
 354 the fastest of all unstable modes group velocity, cf. Figure 3. The results of this, and other
 355 experiments, where we combined different unstable modes to construct initial wavepackets are
 356 qualitative and quantitatively similar (not shown).

357 5. Summary and Discussion

358 We investigated development of instabilities of an easterly jet configuration in the presence of
 359 moist convection and land-sea contrast in a simple atmospheric model, which allows for low-
 360 cost high-resolution linear and nonlinear numerical analyses. We established linear stability

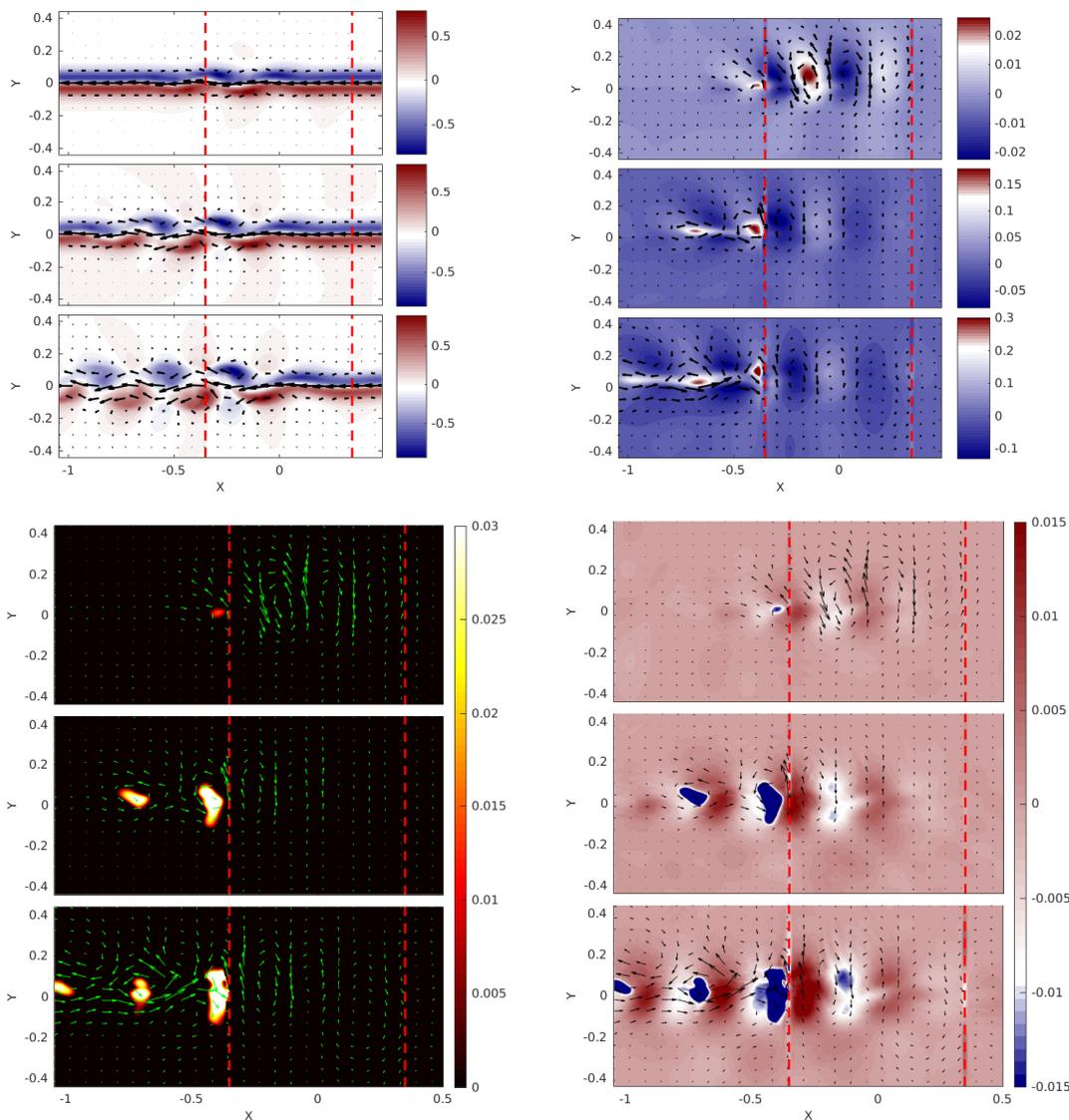


Figure 14. *Upper row*: Evolution of vorticity and velocity in the presence of “continental plateau” of non-dimensional height $b = 0.05$, and of moist convection in the upper layer (left panel) and lower layer (right panel). *Lower row*: The corresponding evolution of condensation (left panel) and divergence fields (right panel). Simulations were initialized with the wave packet of Figure 12, no radiative relaxation. Notice a difference in the colorbar variation of vorticity field. Vertical dashed lines represent the borders of the “continent”. Time= 10, 20, 30 from up to down (Colour online).

361 diagrams for different parameters of the jet, and identified the unstable modes. We showed
 362 that the mechanisms of nonlinear saturation of the instability of the jet with respect to the
 363 perturbation in a form of the most unstable mode are significantly different in adiabatic
 364 and moist-convective environments. We demonstrated that saturation of the instability in the
 365 “dry” case is due to subharmonic interactions and, thus is sensitive to boundary conditions. In
 366 the moist-convective case we demonstrated a robust generation of a secondary westerly jet and
 367 of localized zones of intense cyclonic vorticity in the lower layer at the northern flank of the jet,
 368 similarly to observations. It is the structure of the most unstable mode of the jet in the lower

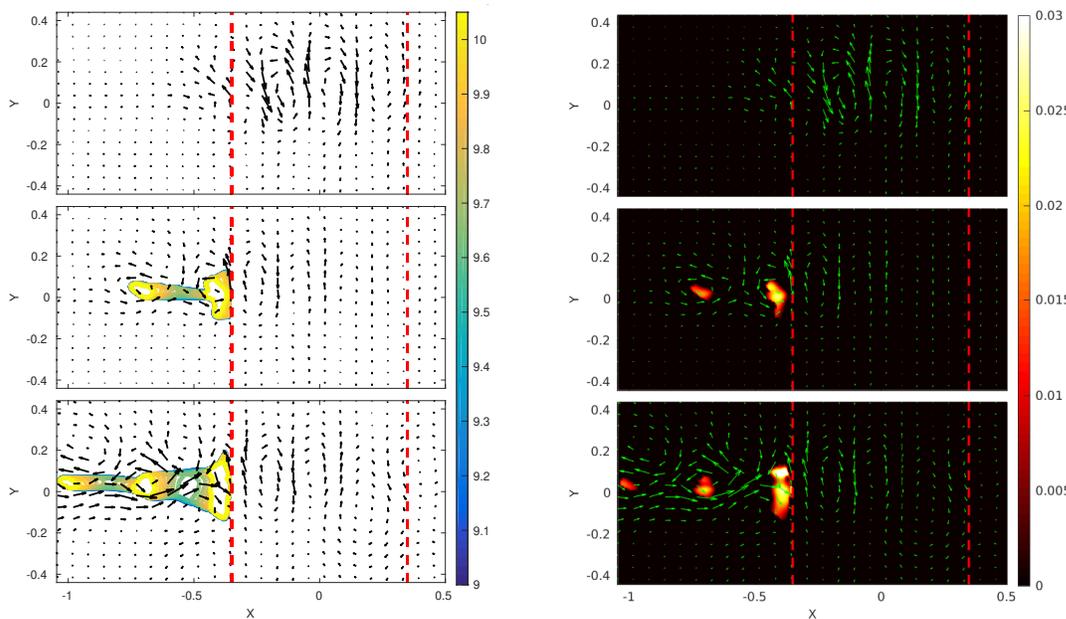


Figure 15. Evolution of high levels of precipitable water in the lower layer (*left panel*), with “continental plateau” of non-dimensional height $b = 0.05$ delimited by dashed lines, and the corresponding evolution of precipitation (*right panel*) (Colour online).

369 layer, with its meridional dissymmetry, combined with the effect of enhancement of cyclonic
 370 vorticity by moist convection which are at the origin of the cyclogenesis process. In a more
 371 realistic case of initialization of numerical experiments with localized wave-packets of unstable
 372 modes over topography mimicking a West-African plateau, we demonstrated that intense
 373 localized vortices are formed in the moist-convective environment over the ocean, to the west
 374 of the “continent” and move westward. The low flat topography does not significantly influence
 375 the scenarios of evolution of the perturbations of the jet, except for a weak topographic
 376 Kelvin-wave signal. On the contrary, the changes of humidity and evaporation at the land-sea
 377 boundary are important for enhancement of the perturbations.

378 Thus, in spite of a highly idealized setup the simple two-layer moist-convective rotating
 379 shallow water model does capture essential features of the development of African Easterly
 380 Waves, and allows to get insights on the fundamental dynamical mechanisms underlying the
 381 observed structures.

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384 **Declaration of Interests** The authors report no conflict of interest.

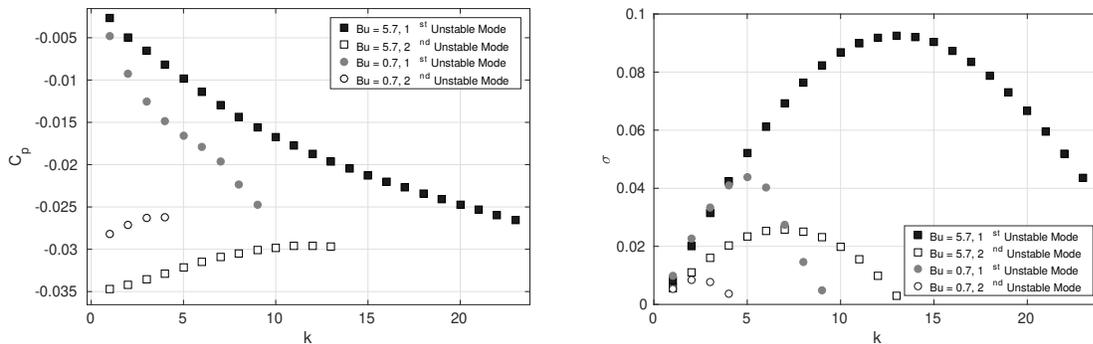


Figure A1. Stability diagram of the upper-layer easterly Bickley jet on the f -plane with $Bu = 5.7$ and $Bu = 0.7$ in terms of phase velocity c_p (left panel) and growth rate σ (right panel) as functions of the zonal wavenumber k . The first and the second unstable modes are represented by black and white squares for $Bu = 5.7$ and by gray and white circles for $Bu = 0.7$, respectively. $Ro = 0.0467$, $U_0 = 0.03$, $a = 12$, $H_{01} = H_{02} = 0.5$, $\beta = 0.5$.

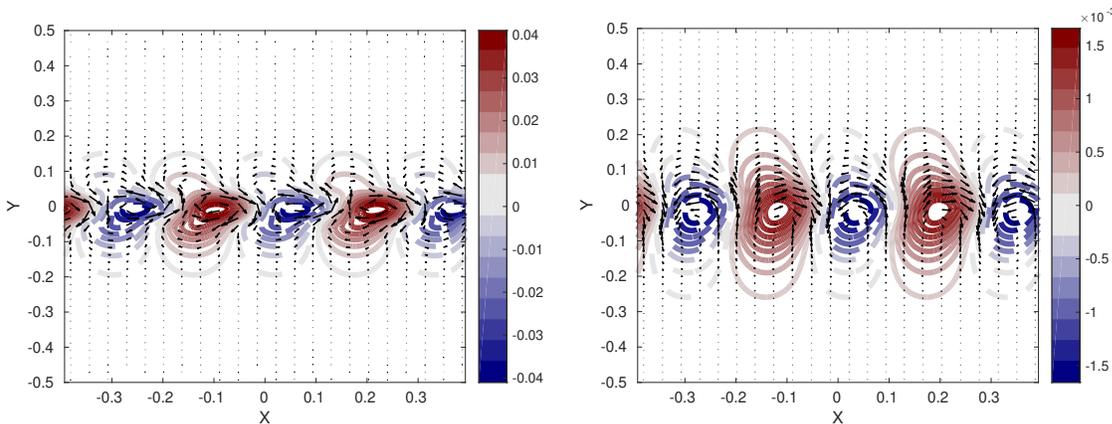


Figure A2. Geostrophic stream-functions of the most unstable mode on the f -plane in the upper layer (left panel) [$\psi_2 = \eta_1 + s\eta_2$] and the lower layer (right panel) [$\psi_1 = \eta_1 + \eta_2$]. Positive (negative) values are represented by solid (dashed) lines, and the velocity field is represented by arrows. $k = 20$, $Ro = 0.15$, $U_0 = 0.045$, $a = 20$, $H_{01} = 1/3$, $H_{02} = 2/3$, $\Phi = 13^\circ \text{N}$, $\beta = 0$ (Colour online).

385 Appendix A: Stability of the easterly jet on the f - plane

386 In this Appendix we present the results of the auxiliary investigation of the stability of the
 387 easterly upper-layer jet on the f - plane. The stability diagram, and the phase-portrait and
 388 meridional cross-section of the most unstable mode are displayed, respectively in Figures A1,
 389 A2, and A3. We present in Figure A1 the results both for large and small Burger numbers of
 390 the jet. The former case is close to that of Lambaerts *et al.* (2012), with the Rossby number
 391 being close, too, and the results quantitatively and qualitatively similar, which is, thus, a
 392 successful benchmark. The latter case highlights the dependence of the results on Bu , which
 393 was not analyzed in Lambaerts *et al.* (2012).

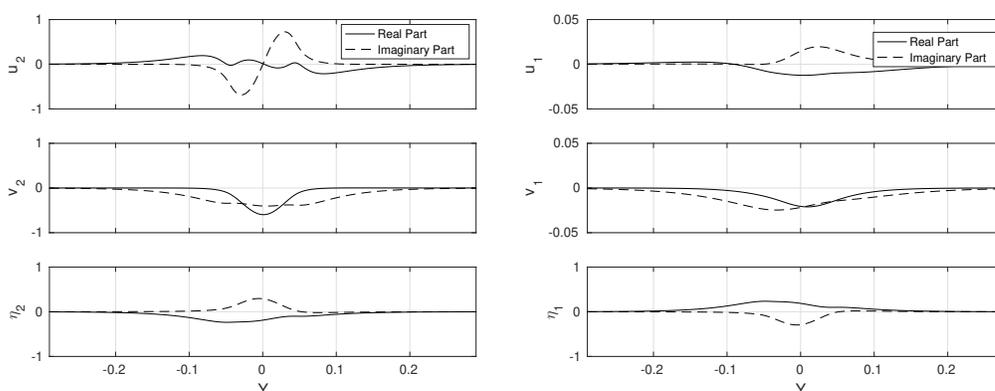


Figure A3. Meridional cross-sections of u_i, v_i, η_i of the most unstable mode on the f -plane shown in the Figure A2, in the upper layer (left panel) and the lower layer (right panel) (Colour online).

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