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Determining Bishop’s parameter $\chi$ based on pore size distribution

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Abstract

Extension of the effective stress concept to unsaturated soils has been a major concern for decades. Recent studies significantly contributed to the understanding of the fundamentals behind Bishop’s parameter $\chi$ which is generally used to define the effective stress for unsaturated soils. Examination of the recently proposed methods showed that the contribution of suction to effective stress was often overestimated, especially in high suction range. In this study, considering that soil pores with different sizes contribute differently to the overall hydro-mechanical behaviour, a new method to determine Bishop’s parameter $\chi$ is proposed. The key variable used in this method is the ratio of the change in water volume to the change in macro-pore void volume due to loading at constant suction. Shear strength data for a weakly expansive clay were used for validation. A good agreement was obtained between prediction and measurement, indicating the validity of the proposed method.

Keywords:

constitutive relations; fabric/structure of soils; shear strength; partial saturation; suction
NOTATION

$e_{w2}$  water ratio caused by change of strain at constant suction

$e_{w}^{m}$  microscopic water ratio

$e^{m}$  microscopic void ratio

$S_{r}^{e}$  effective degree of saturation

$S_{r}^{m}$  microscopic degree of saturation

$S_{r}^{cap}$  capillary degree of saturation

$S_{r}^{ads}$  adsorbed degree of saturation

$s_{m}$  suction corresponding to the median pore diameter

$V_{w1}$  water volume governed by suction

$\alpha$  ratio of the change in incremental water volume caused by the change of strain under constant suction to total pore volume increment

$\lambda$  material parameter defining the effective degree of saturation

$\bar{\xi}$  standard deviation of the log-transformed pore radius

$\phi$  fitting parameter related to the theoretical degree of saturation due to adsorption at 1 kPa suction

$\chi$  Bishop’s effective stress parameter
INTRODUCTION

The definition of stress variables is essential in developing constitutive models for unsaturated soils. There are two approaches for modeling their mechanical behaviour: the effective stress approach (Bishop, 1959) and the two independent state variables approach (Coleman 1962, Fredlund & Morgenstern, 1977). Currently, using two independent stress variables to describe the behavior of unsaturated soils is widely accepted (Cui and Delage, 1996; Wheeler et al., 2003; Liu et al., 2018). Bishop stress is often considered as one variable in the two independent stresses approach, because it represents a smooth extension of Terzaghi’s effective stress (Sheng et al., 2004; Pereira et al., 2005; Zhou et al., 2018). In the Bishop’s effective stress concept, the key parameter is \( \chi \), as shown in Eq. 1 (Bishop, 1959):

\[
\sigma' = \sigma - \left[ \chi u_w + (1 - \chi) u_a \right]
\]

(1)

where \( \sigma' \) is unsaturated effective stress; \( \sigma \) is total stress; \( \chi \) is the effective stress parameter (Bishop’s parameter); \( u_w \) and \( u_a \) are water and air pressures, respectively.

In the past few decades, various criteria were reported for determining \( \chi \). In the early contributions (Bishop & Blight, 1963), \( \chi \) was set equal to degree of saturation \( S_r \):

\[
\chi = S_r
\]

(2)

which is the simplest, and most used assumption to date. But several studies showed that this choice significantly overestimates the contribution of suction to effective stress, especially in the high suction range (Sheng et al., 2011; Zhou et al., 2016). In a later effort, the overestimation was reduced by using an effective degree of saturation (Vanapalli et al., 1996). To define this latter, attempts have been made to
directly relate $\chi$ to the total degree of saturation, such as (Vanapalli et al., 1996):

$$\chi = f(S_r) = (S_r)^\lambda$$  \hspace{1cm} (3)

where $\lambda \geq 1$ is a material parameter. For coarse-grained soils, parameter $\lambda$ is almost equal to unity, while for fine-grained soils $\lambda$ is larger than unity. Garven & Vanapalli (2006) found that $\lambda$ is correlated with plasticity index, $I_p$.

Soil water consists in capillary water and adsorbed one. Since adsorbed water is strongly bonded to soil particles (Lu et al., 2010; Konrad & Lebeau, 2015; Zhou et al., 2016; Gao et al., 2018), it does not contribute to the contact stress and thus to the effective stress. Thereby, Zhou et al. (2016) related the Bishop’s parameter $\chi$ to the capillary degree of saturation $S^\text{cap}_r$ which is the difference between the total degree of saturation and the adsorbed degree of saturation $S^\text{ads}_r$:

$$\chi = S^\text{cap}_r = \frac{C(s) - \varphi C(s) A(s)}{1 - \varphi C(s) A(s)}$$ \hspace{1cm} (4)

where $C(s) = \frac{1}{2} \text{erfc} \left[ \frac{\ln(s/s_m)}{\sqrt{2} \xi} \right]$, $A(s) = 1 - \frac{\ln(s)}{\ln(s_d)}$

with $s$ is matric suction; $\varphi$ and $\xi$ are fitting parameters determined by soil water retention curve (SWRC); $s_m$ is the suction which corresponds to the median pore diameter; $s_d$ is the suction at extreme dry state, assumed equal to $10^6$ kPa. However, the lastest investigation showed that using capillary degree of saturation underestimates the contribution of suction to Bishop’s effective stress for most soils (Gao et al., 2020). It has been well documented that for compacted soils, the microstructure is characterised by two pore populations: i) macro-pores where capillary effects dominate, and ii) micro-pores where the hygroscopic effect dominates (Romero & Vaunat, 2000; Alonso et al., 2010). It has been admitted that
the contribution of suction to Bishop’s effective stress is only related to the water trapped in macro- or inter-aggregate pores (Alonso et al., 2010; Zhai et al., 2019).

This was confirmed by some experimental results which showed that loading and drying paths predominantly influenced the macropores, while micropores remain almost undisturbed (Monroy et al., 2010; Mašín, 2013). Therefore, Bishop’s parameter $\chi$ can be considered as the effective degree of saturation of macro-pores $S^e$ (Alonso et al., 2010):

$$\chi = S^e = \frac{S_e - S^m_e}{1 - S^m_e} \quad (5)$$

where $S^m_e$ the degree of saturation of micro-pores. This method can improve the prediction of effective stress, but still overestimates the contribution of suction in some cases (Sheng et al., 2011; Zhai et al., 2019).

In this study, a new method for determining $\chi$ using the macro-PSD is proposed, which satisfies Houlsby’s power equation.

**DETERMINING BISHOP’S PARAMETER $\chi$ BASED ON MACROPOROSITY**

Fig. 1 shows a conceptual sketch of water retention curves at two different void ratios (Vaunat & Casini, 2017). The incremental change in water ratio $\delta e_w$ (path OB) caused either by suction change or mechanical loading can be split into $\delta e_{w1}$ (path OA) and $\delta e_{w2}$ (path AB). The change of water content along path OA is caused by the increment of suction at constant volume, while the incremental component of path AB is caused by the change of void ratio under constant suction (net stress effect). This latter can be expressed as a proportion of total volume change (Vaunat & Casini, 2017). The total incremental work input per unit volume in unsaturated soils $\delta w$ (Houlsby, 1997)
can be expressed as:

\[
\delta w = -\left[\sigma - \alpha u_w - (1 - \alpha)u_a\right] \frac{\delta V}{V} - (u_a - u_w) \frac{\delta V_{w1}}{V}
\]

\[
= -\left[\sigma - \alpha u_w - (1 - \alpha)u_a\right] \delta e_V - (u_a - u_w) \frac{\delta V_{w1}}{V}
\]

where \(\sigma\) is the total mean stress; \(V\) is the total soil volume; \(V_v\) is the volume of voids;
\(\delta V_{w1}\) is the change of volume occupied by water due to suction; \(\alpha\) is the ratio of the increment of volume occupied by water caused by the incremental volumetric strain under constant suction to the pore volume increment. As the first term between brackets in Eq. (6) expresses Bishop’s effective stress, Vaunat & Casini (2017) proposed:

\[
\chi = \alpha = \frac{\delta V_{w2}}{\delta V} = \frac{\delta e_{w2}}{\delta e}
\]

where \(\delta e_{w2}\) is the increment of water ratio caused by the change of volumetric strain under constant suction; \(\delta e\) is the increment of void ratio.

Fig. 2 shows the SWRCs under different void ratios for several soils (Li et al., 2007; Salager et al., 2013; Seiphoori et al., 2014; Gao & Sun, 2017). It appears that in low suction range, the water retention curve is dependent on void ratio. By contrast, in higher suction range, the SWRCs are almost the same (Fig. 3). The separating point corresponds to the delimitation between the range of capillary water in macro-pores and the range of adsorbed water in micro-pores (Romero, 1999). Mašín (2013) also discussed the structure evolution with mechanical loading and assumed that loading (compaction) influenced predominantly the macro-pores, while micro-pores remained untouched (Fig. 3). Thus, as the volume change is mainly due to macro-pores changes, Bishop’s parameter \(\chi\) is solely related to the low suction range (Fig. 3). Thereby, Eq.
(7) is modified as:
\[ \chi = \frac{\delta e_{w2} - \delta e_{w1}}{\delta e - \delta e_m} \]  
(8)
where \( \delta e_{w} \) is the incremental microscopic water ratio; \( \delta e_m \) is the incremental microscopic void ratio. In Fig. 3, two SWRCs at different void ratios should be provided and the void ratio of every SWRC should be kept constant in the whole suction range. However, in a SWRC test, the void ratio changes with suction changes.

As the pore size distribution (PSD) obtained by Mercury Intrusion Porosimetry (MIP) test can be used to determine the SWRC at a constant void ratio (Zhang et al., 2018), the parameters in Eq. (8) can be obtained using two different PSDs, as illustrated in Fig. 4.

**EVALUATION OF THE PROPOSED METHOD THROUGH SHEAR STRENGTH**

Bishop & Blight (1963) described the shear strength of unsaturated soils by:
\[ \tau_f = c' + [(\sigma_n - u_a) + \chi(u_a - u_w)] \tan \theta' \]  
(9)
where \( \tau_f \) is the shear strength; \( c' \) is the effective cohesion at saturated state; \( \sigma_n \) is the normal stress; \( \theta' \) is the internal friction angle. Using Eq. 9 and considering triaxial conditions, parameter \( \chi \) can be back-calculated, as follows:
\[ \chi = \frac{q_f - c't \frac{\cos \theta_r}{3 - \sin \theta'}}{M} \rho_s \]  
(10)
where \( q_f \) is the deviator stress at failure; \( M = 6 \sin \phi'/(3-\sin \phi') \) is the slope of critical state line; \( \rho_s = [\sigma_1 + 2\sigma_3]/3 - u_a \) is the net mean stress.

The results from the tests on Nanyang expansive clay were used to validate the
proposed method for Bishop’s parameter $\chi$ determination (Fig. 8). Table 1 summarises the physical property indexes and triaxial shear parameters for this clay. Triaxial shear tests were conducted using an unsaturated triaxial testing apparatus supplied by GDS company. In order to obtain the molded samples only in drying process, all triaxial specimens were prepared by static compaction at initial water content of about 0.215 (suction is about 170 kPa) and initial dry density around 1.25 Mg/m$^3$. Table 2 gives a summary of the test conditions and stress paths. For tests No.1-3, suction was applied using axis-translation technique in triaxial cell, while for tests No.4 and No.5 the suctions (0.8 and 2.5 MPa, respectively) were applied through controlling the water contents ($w=14.8\%$ and $w=12.55\%$, respectively) by referring to the SWRC in drying process. For higher suctions, i.e. tests No.6-8, the vapor equilibrium method was employed. When the samples of tests No.4-8 reached the target respective suctions, they were put into triaxial cell for further consolidation and shear tests following the stress and suction paths shown in Figs. 5 (a) and (b). The net confining pressure was 100 kPa, and the suctions were 0.05, 0.2, 0.4, 0.8, 2.5, 3.29, 38 and 368 MPa, respectively. In the triaxial shear tests, $q_t$ and $\bar{p}$ were measured for every suction state and given in Table 3. As illustrated in Fig. 2, when the suction is higher than a specific value, the water retention curve is independent of void ratio. In addition, the modulus of soils with high suction are normally very large and the changes of void during shear tests are thus small. Thereby, shear tests under constant water content are often considered as under constant suction in high suction range (Gao et al., 2019; Zhang et al., 2020). Substitution of $q_t$, $\bar{p}$, $c'$, $\theta'$ and $s$ (see Tables 1, 2 and 3) into Eq. (10)
allows the calculation of Bishop’s parameter $\chi$ for every suction (or degree of saturation), as illustrated in Fig. 8.

As the void ratio of this sample with 3.29 MPa in shear test is 0.99, the PSD curves of two samples compacted at different void ratios (0.81 and 0.92) at the same constant suction of 3.29 MPa are considered. The results are shown in Fig. 6. According to Lloret et al. (2003), the macro- and micro-pores are separated by the point where the PSD becomes independent of loading. For bimodal structure, the diameter at the valley bottom between the two pore size families can be considered as the delimiting diameter between micro- and macro-pores. It appears from Fig. 6 that the PSD curves are not affected by the compaction load when the pore diameter is smaller than 4900 nm. Thereby, this diameter was taken as the boundary between macro- and micro-pores. Eq. (8) can then be applied to determine Bishop’s parameter $\chi$ using the macro-PSDs. Fig. 7 shows the obtained results. For comparison, the parameters in Table 1 are substituted into Eq. (10), and several other methods are applied including Bishop’s method with $\chi = S_r$ (Eq. 2), Vanapalli et al.’s method (Eq. 3), Zhou et al.’s method (Eq. 4), Alonso et al.’s method (Eq. 5), and Vaunat & Casini’s method (Eq. 7). Results are included in Fig. 8. It is observed that when the global degree of saturation (Eq. 2) is considered as Bishop’s parameter $\chi$, significant overestimation of suction contribution is obtained, in agreement with the observation of Alonso et al. (2010) and Zhou et al. (2016). In Zhou et al.’s method, the capillary water is separated from the adsorbed water. Then the capillary degree of saturation (Eq. 4) is used to determine Bishop’s parameter $\chi$. Fig. 7 shows the fitted SWRCs by
Zhou et al.’ SWRC method (Zhou et al., 2016) and the corresponding parameters. The relationship between capillary degree of saturation and degree of saturation is plotted in Fig. 8. It appears that this method underestimates the suction contribution to Bishop’s effective stress over a wide suction range. In Alonso et al.’s method, $S'_m$ is the degree of saturation of the micro-pores with diameter smaller than 4900 nm. It was found to be about 20% by calculation based on the PSD in Fig. 6. The effective degree of saturation (Eq. 5) is used to determine $\chi$. It appears that the prediction is improved, but the suction contribution is overestimated. Similarly, Vaunat & Casini’s method can improve the prediction in the low suction range as Alonso et al.’s method, but remains unsatisfactory in the higher suction range. In particular, a small bump on the curve obtained by Vaunat & Casini’s method appears in the range of degree of saturation from 5% to 40%, owing to the contribution of micro-pores to the calculation of $\chi$. In Vanapalli et al.’s method, a value of 1.6 was fitted for exponent parameter $\lambda$. It is observed that the prediction matches the test data well in the low suction range, but gives higher $\chi$ values in higher suction range. Interestingly, the proposed method (Eq. 8) shows a good agreement with the experimental results in the full suction range.

CONCLUSIONS

The hydro-mechanical behavior of unsaturated soils is strongly related to the water distribution in pores. As loading and drying predominantly affect the macro-pores, with the micro-pores remaining almost undisturbed, the pore size below which the PSD curves become independent of loading or drying processes can be considered as the delimiting diameter between macro-and micro-pores. This also implies that the
contribution of suction to Bishop’s effective stress is only related to the water in macro-pores.

Based on a method proposed by Vaunat & Casini (2017) for Bishop’s parameter $\chi$ determination, a modified method was proposed considering the contribution of capillary water in macro-pores. Bishop’s parameter $\chi$ was defined as the ratio of the change in water volume to the change in macro-pores during a loading process at constant suction. When the degree of saturation of macro-pores is zero, $\chi$ becomes zero too. The proposed method was evaluated using experimental shear strength data and compared with different methods reported in literature. It appeared that the proposed method allows good agreement between the prediction and measurement over a wide suction range, as opposed to other methods which either overestimate or underestimate the suction contribution to the effective stress. It is however worth noting that more test results are needed to further validate the proposed method.

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Table captions.

Table 1 Physical property indexes and shear strength parameters of Nanyang weakly expansive soil

Table 2 Relevant state variables in triaxial tests

Table 3 Stress state at triaxial shear failure

Figure captions.

Fig. 1 Partition of total water volume change into components due to suction and deformation only for a path going from the water retention curve at \( e \) to \( e+\delta e \) (after Vaunat & Casini, 2017)

Fig. 2. SWRCs under different void ratios over a wide suction range for soils: (a) a clayey silty sand (data from after Salager et al., 2013); (b) MX-80 granular bentonite (data from Seiphoori et al., 2014); (c) Maryland clay (data from Li et al., 2007); (d) Pearl clay (data from Gao & Sun, 2017)

Fig. 3 Sketches of soil-water retention behavior of specimens with different densities over a wide suction range

Fig. 4 PSDs at two different void ratios (compacted at different void ratios at a constant suction)

Fig. 5 Stress and suction paths for tests: (a) tests in the lower suction range; (b) tests in the higher suction range

Fig. 6 The PSD of Nanyang weakly expansive soil and the criterion adopted to discriminate macro- and micro-pores

Fig. 7 Measured SWRC and fitted curves from Zhou et al.’ SWRC method (2016) for Nanyang weakly expansive soil

Fig. 8 Comparison of measured Bishop’s parameter \( \chi \) with the predictions obtained by various methods for Nanyang weakly expansive soil
Table 1 Physical property indexes and shear strength parameters of Nanyang weakly expansive soil

<table>
<thead>
<tr>
<th>Specific gravity</th>
<th>Liquid limit (%)</th>
<th>Plastic limit (%)</th>
<th>Plasticity index</th>
<th>Shrinkage limit (%)</th>
<th>Maximum dry density (g/cm³)</th>
<th>Optimum water content (%)</th>
<th>Free swelling ratio (%)</th>
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Table 2 Relevant state variables in triaxial tests

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<th>Test No.</th>
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<th>Suction (MPa)</th>
<th>Control suction method</th>
<th>Net cell pressure (kPa)</th>
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<td>S₀(%)</td>
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Table 3 Stress state at triaxial shear failure

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<th>Test No.</th>
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<th>Net cell pressure (kPa)</th>
<th>Deviator stress (q_t) (kPa)</th>
<th>Net mean stress (\bar{p}) (kPa)</th>
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<td>Vapor equilibrium technique</td>
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<td>1680</td>
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