



**HAL**  
open science

# Multiphase continuum models for fiber-reinforced media

Jeremy Bleyer

► **To cite this version:**

Jeremy Bleyer. Multiphase continuum models for fiber-reinforced media. CMCS 2017 - Computational Modeling of Complex Materials across the Scales, Nov 2017, Paris, France. hal-02434342

**HAL Id: hal-02434342**

**<https://enpc.hal.science/hal-02434342>**

Submitted on 9 Jan 2020

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

## MULTIPHASE CONTINUUM MODELS FOR FIBER-REINFORCED MEDIA

JEREMY BLEYER\*

\* Laboratoire Navier, Ecole des Ponts ParisTech-IFSTTAR-CNRS UMR 8205,  
Université Paris-Est  
6-8 av. Blaise Pascal, Cité Descartes, 74555 Champs-sur-Marne, FRANCE  
e-mail: jeremy.bleyer@enpc.fr

**Key words:** fiber-reinforced materials, multiphase continua, homogenization, scale effects

**Abstract.** Fiber-reinforced materials are nowadays very common in various industrial applications, ranging from composite laminates in aerospace industry to fiber-reinforced concrete or piled-raft foundations in civil engineering. Homogenization theory is an extremely attractive approach aiming at replacing such highly heterogeneous media by an equivalent homogeneous material, reproducing the macroscopic behaviour of the heterogeneous composite when the microscopic and macroscopic scales are well separated. However, many situations lead to non-separated scales such as a large dimension of the heterogeneity compared to the structure scale or important contrast in terms of material properties. Unfortunately, both situations are very typical for fiber-reinforced material in which the inclusions are usually thin and very stiff compared to the surrounding matrix [1].

Multiphase models have been proposed as a generalized continuum capable of accounting for such effects in media reinforced by linear inclusions [2]. In such models, the reinforced medium is represented as the superposition of two mutually interacting continuous media, each of them possessing its own kinematics (Figure 1). In the specific case where both phases have the same kinematics, one recovers standard monophase continua. The generalized internal forces of the multiphase model can be interpreted as the partial stress of the matrix and the reinforcement phase as well as the interaction force between the two interacting phases, this latter quantity being work-conjugate to the relative displacement between both phases.

Initially proposed as a phenomenological approach and strongly related to the theory of mixtures for fluid/solid interacting media, such models require the identification of many constitutive parameters, which have been related to the constituents mechanical properties only in simple situations [3]. A more systematic way of performing the up-scaling procedure from heterogeneous Cauchy continua towards multiphase continua is

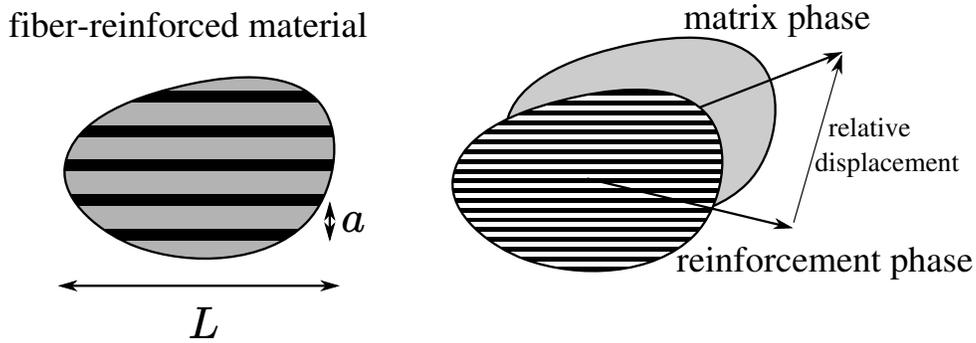


Figure 1: Principle of the multiphase model: the reinforced medium is modeled as two homogeneous phases with separate kinematics and in interaction with each other.

therefore required in order to use such models in more complex situations, in particular with non-linear constitutive models.

In this work, we build a rigorous homogenization framework in which the generalized behavior of the multiphase model is obtained from the solution of a specific auxiliary problem formulated on the unit cell of the periodically heterogeneous material. In particular, we identify the equivalent stiffness of each of the phases as well as the interaction stiffness describing the effect of a relative displacement between both phases. Extensions to non-linear constitutive behaviors, in particular regarding the overall yield strength of such media, will also be discussed. It is shown that such models encompass classical homogenization approaches in the limit of well separated scales and are able to capture accurately the previously mentioned scale effects.

## REFERENCES

- [1] Boutin, C. and Soubestre, J., Generalized inner bending continua for linear fiber reinforced materials, *International Journal of Solids and Structures*, (2011), **48**, 517–534
- [2] de Buhan, P. and Sudret, B., A two-phase elastoplastic model for unidirectionally-reinforced materials, *European Journal of Mechanics-A/Solids*, (1999) **18**(6), 995–1012
- [3] Bedford, A. and Stern, M., A multi-continuum theory for composite elastic materials, *Acta Mechanica*, (1972), **14**(2), 85–102