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# DAMAGE AND STRAIN LOCALIZATION AROUND A PRESSURIZED SHALLOW-LEVEL MAGMA RESERVOIR

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**Abstract.** Structures developing above long-term growing shallow-level magma reservoirs, such as resurgent domes, may contain information on the reservoir itself. To understand the formation of such tectonic features, we have investigated the deformation process around a shallow pressurized magma reservoir embedded in a damaging elastic volcanic edifice. Our model allows evidencing the effect of the progressive damage in producing the fault pattern associated to tectonic surface deformation. Damage is first isotropic around the cavity and constitutes a damaged zone. Then the free-surface effect appears and an anisotropic shear strain develops from the boundary of the damaged zone; it localizes on reverse faults that propagate upward to the surface. When the surface deformation is sufficient, normal faulting appears. Finally, the complete structure shows an undeformed wedge above the damaged zone. This structure is similar to those found by analogue modelling and from field geologic observations. From this model, we found a relation to estimate the reservoir radius and depth from the graben and dome widths. From limit analysis, we deduced an analytical expression of the magma reservoir pressure which provides a better understanding of the magma pressure build-up during doming. The dip of reverse faults limiting the dome can be

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inferred from the minimal pressure required to rupture the crust around the reservoir. Finally, the magma reservoir overpressure, the dip of the faults, the reservoir depth and the damaged zone radius is inferred from three parameters: the ratio  $\rho_R$  computed from the dome and graben widths, the cohesion and the friction angle.

## 1. Introduction

1 The location of most volcanoes is related to large tectonic structures controlled by  
2 geodynamics. These structures are created by regional horizontal tectonic stresses: for  
3 instance, rifts or cordilleras are created by tensile or compressive stresses, respectively.  
4 At the volcano scale, some remarkable structures like resurgent domes result from the  
5 pressurization of shallow-level magma reservoirs, which provides a vertical component  
6 to the stress field. Retrieving the geometrical and some physical characteristics of the  
7 pressure source from geological and geophysical surface observations, especially taking  
8 into account the faulting structure, is a challenge that requires the development of realistic  
9 models involving strain localization.

10 Resurgent domes are commonly observed in the central part of large active calderas.  
11 They are found, for example, at Yellowstone, Long Valley, Lake Toba, Valles Caldera (e.g.  
12 *Smith and Bailey* [1968]), Siwi Caldera, Campi Flegrei [*Sacchi et al.*, 2014]. Caused by  
13 the long-term uplift of the caldera floor, they are attributed to renewed magma intrusion  
14 after the caldera collapse and subsequent pressurization of a shallow-level magma reservoir.  
15 Most resurgent domes are elongated in shape along the same elongation direction of their  
16 hosting caldera, they are therefore mostly 2D structures.

17 The apex of the domes is generally occupied by one longitudinal graben (in rare cases,  
18 such as in Toba or Long Valley, domes can bear several parallel grabens). Normal faults  
19 forming this graben are well identified at the surface and have been studied at depth  
20 by geothermal drillings in the case of Redondo dome in Valles Caldera (Figure 1) or  
21 seismic imaging at Campi Flegrei [*Sacchi et al.*, 2014]. The nature of dome borders is

22 less well understood, as they are not associated to outcropping faults identified at the  
23 surface. However, they represent a sharp transition between the dome flanks and the flat  
24 caldera moat, strongly suggesting that the dome lateral extension could be limited by  
25 reverse faults. This hypothesis is corroborated by the evidence of fluid circulation along  
26 borders. Multiple geyser basins (including the renown Old Faithful) concentrate along  
27 the border of Mallard Lake resurgent dome in Yellowstone caldera [*Christiansen, 2001*].  
28 Electrical surveys in the Siwi caldera also evidenced probable hydrothermal circulation  
29 along the Yenkahe dome border [*Brothelande et al., 2016a*]. Dome borders also represent  
30 a preferential pathway for magmatic fluids, as late volcanic products seem to commonly  
31 originate from there. Examples include late cinder cones along the Yenkahe (Figure 1),  
32 late rhyolite around Long Valley resurgent dome (moat rhyolite; *Bailey and Dalrymple*  
33 [1976]), Redondo dome in Valles Caldera (ring domes, Figure 1; *Smith and Bailey [1968]*),  
34 and La Pacana resurgent dome [*Gardeweg and Ramirez, 1987; Lindsay et al., 2001*].

35 To understand the structural evolution leading to the building of such resurgent domes,  
36 analogue experiments have been conducted [*Sanford, 1959; Marti et al., 1994; Merle and*  
37 *Vendeville, 1995; Acocella et al., 2000, 2001; Galland et al., 2009; Brothelande and Merle,*  
38 *2015*]. The models revealed the presence of reverse faults limiting the dome, and the  
39 ones using elongated sources could reproduce the development of longitudinal grabens, as  
40 observed in natural domes (Figure S1).

41 The question of the expansion of a cylindrical or spherical cavity has been extensively  
42 studied since the beginning of the twentieth century, first in an infinite or semi-infinite  
43 elastic medium (see, e.g., *Jeffery [1920]*). This has led to an improved understanding of the  
44 structures around magma reservoirs and the development of the first mechanical models

45 of pressurized magmatic structures (*Anderson* [1936]; *Sanford* [1959]; *Robson and Barr*  
46 [1964]; *Phillips* [1974]; see, e.g., *Tibaldi* [2015] for a review). However, modelling faulting  
47 around a pressurized cavity requires taking into account the plastic deformation to localize  
48 strain. In this aim, a considerable amount of work has been produced in mechanics and  
49 geomechanics to model the stresses and displacement around over- or underpressurized  
50 cavities in linear elastic, perfectly plastic materials, with various plastic flow rules (see,  
51 e.g., *Nadai* [1931]; *Hill et al.* [1947]; *Hill* [1950]; *Nadai* [1950]; *Chadwick* [1959]; *Salencon*  
52 [1966]; *Vesic* [1972]; *D'Escatha and Mandel* [1974]; *Carter et al.* [1986]; *Sulem et al.* [1987];  
53 *Vardoulakis et al.* [1988]; *Bigoni and Laudiero* [1989]; *Yu* [2000]; see *Bigoni and Laudiero*  
54 [1989] for a historical introduction of this question).

55 Numerical modelling of volcano-tectonic structures around a pressurized magma reser-  
56 voir in an elasto-plastic semi-infinite medium was first made by *Grosfils* [2007] and *Ger-*  
57 *bault et al.* [2012], and by *Brothelande et al.* [2016b] in the case of resurgent domes. This  
58 later model leads to phenomenological relations between source parameters and surface  
59 observables. However, elasto-plastic modelling is closely linked to mechanical instability  
60 and subject to numerical instabilities that may prevent from modelling the complete fault-  
61 ing geometry, especially when the magma reservoir is overpressurized and close to the free  
62 surface of a semi-infinite medium. In this work we first derive a stable representation of the  
63 faulting geometry by using continuum damage mechanics. Continuum damage mechanics  
64 is a widely used approach in mechanics and geomechanics for modelling strain localization  
65 before rupture through progressive change of mechanical properties (see, e.g., *Kachanov*  
66 [1958]; *Lemaitre* [1994]; *Krajcinovic* [1996]; *Lyakhovskiy et al.* [1997]; *Main* [2000]; *de Borst*  
67 [2002]; *Turcotte and Glasscoe* [2004]; *Amitrano and Helmstetter* [2006]; *Heap et al.* [2010]).

68 It has been used to model the progressive landslide failure [*Eberhardt et al.*, 2004; *Lacroix*  
69 *and Amitrano*, 2013], and more recently to model the dynamics of basaltic volcanoes pre-  
70 eruptive deformation and seismicity [*Carrier et al.*, 2015; *Got et al.*, 2017]. In the present  
71 work our aim was to get a complete representation of the faulting geometry around a  
72 pressurized reservoir by continuum damage mechanics, in order to use it to establish geo-  
73 metrical and physical relations between source parameters (reservoir overpressure, depth  
74 and radius, fault dip), surface observables (graben and dome widths) and rock mass pa-  
75 rameters (friction angle and cohesion), by using limit analysis. Limit analysis is also a  
76 widely used method to provide analytical or numerical relations in stability problems (see,  
77 e.g., *Davis and Selvadurai* [2002] as a simple and good reference textbook on the subject)  
78 and may be applied to volcanological mechanical questions.

## 2. Model

79 We investigated the damage process and its consequence for the dome growth by defin-  
80 ing an initially axisymmetric, homogeneous, and isotropic two-dimensional model which  
81 represents a shallow-level pressurized magma reservoir in a damageable elastic half-space  
82 (Figure 2). Discretizing this model in finite elements, we compute the plane strain de-  
83 formation using roller boundary conditions along the model vertical borders, and fixed  
84 boundary conditions at the bottom. Surface is free. Model was scaled to physical dimen-  
85 sions (Figure 2), and set to a 10 km x 10 km square, with an initial linear elastic medium  
86 containing an initial 0.5 km radius circular central cavity. A uniform pressure is applied  
87 along the boundary of the cavity all along the numerical experiment. Gravity is applied  
88 to each node, using a rock density  $\rho = 2700kg/m^3$ , a value comprised between andesite

89 ( $\rho = 2600\text{kg}/\text{m}^3$ ) and basalt ( $\rho = 2900\text{kg}/\text{m}^3$ ) rock densities in the 0-25 km depth interval  
 90 (see, e.g., *Christensen and Mooney* [1999], Table 4).

91 In this modelling we use the finite element progressive damage approach of *Amitrano*  
 92 *et al.* [1999]; *Amitrano* [2003]; *Amitrano and Helmstetter* [2006], previously used in *Lacroix*  
 93 *and Amitrano* [2013] and more recently in *Riva et al.* [2018]. It allows the reproduction  
 94 of the progressive failure of rock by using an elastic damage model. It follows *Kachanov*  
 95 [1958]’s elastic damage approach in which the Young’s modulus  $E_i$  of each element  $i$   
 96 decreases when a failure criterion is met and a fracture occurs. Damage onset has been  
 97 shown to follow a Mohr-Coulomb criterion [*Heap et al.*, 2009], which has been chosen  
 98 as the instantaneous damage threshold by *Amitrano and Helmstetter* [2006], *Lacroix and*  
 99 *Amitrano* [2013] and in the present model. This Mohr-Coulomb criterion is truncated  
 100 when the normal stress is equal to the tensile strength. Progressive damage induces the  
 101 progressive decrease of the Young’s modulus with the number of damage events; for one  
 102 damage event:

$$E_i(n+1) = (1 - \delta) E_i(n) \quad (1)$$

103 or, for  $n$  damage events:

$$E_i(n) = (1 - \delta)^n E_0 \quad (2)$$

104 where  $\delta$  is the incremental damage occurring during one damage event, and  $E_0$  is the  
 105 initial Young’s modulus. Heterogeneity in the model is introduced by using a random  
 106 perturbation of the Young’s modulus spatial distribution.

107 Stress redistribution occurs around a damaged element and may induce an avalanche  
 108 of damage events. The number of damaged elements is the avalanche size, which may be  
 109 thought as a measure of its magnitude.

110 Damage also occurs as a subcritical process, on elements where the applied shear stress  
 111 is lower than the instantaneous, maximal, strength. These elements are located outside  
 112 the zone of maximum damage. In that case, damage is delayed and described by a law  
 113 determined from static fatigue experiments, which expresses the time to failure  $t_f$  as a  
 114 function of the major principal stress  $\sigma_1$  and of the instantaneous compressive or tensile  
 115 strength  $\sigma_0$  [*Wiederhorn and Bolz, 1970; Das and Scholz, 1981*]:

$$t_f = t_0 \exp\left(-b \frac{\sigma_1}{\sigma_0}\right) \quad (3)$$

116 The time  $t_0$  and the constant  $b$  depend on rock properties and ambient conditions [*Scholz,*  
 117 *1972*]. They are scaling parameters for the dynamics of the rupture process. Each element  
 118  $i$  is characterized by its failure time  $t_i$  without interaction between elements (initially  
 119  $t_i = t_f$ ), and the proportion of consumed lifetime [*Amitrano and Helmstetter, 2006*],  
 120 which allows taking into account the stress history for estimating the remaining time  
 121 to failure. Taking into account this subcritical crack growth process and the random  
 122 part in the spatial distribution of Young's modulus perturbations avoids an instantaneous  
 123 localization in a very narrow fault plane. It allows the dissipation of the elastic potential  
 124 energy outside this plane and widens the damaged zone, creating heterogeneity in this  
 125 zone. It contributes to regularize the strain localization process, spreading damage in  
 126 time and space, and controlling its time dynamics. Therefore, in the absence of data  
 127 (deformation, seismicity), the information on physical processes operating during damage  
 128 is contained in the regularization process; it is the physical a priori knowledge of this  
 129 process.

130 As a consequence, in this model, the pressure applied in the reservoir is constant, but  
 131 larger than the long-term strength of the rock mass, so that earthquakes occur around the

132 reservoir and weaken the volcanic edifice. This weakening occurs through the progressive  
 133 decrease of the Young's modulus; it may induce strain localization when stress conditions  
 134 are sufficient and remain unchanged. In this work, we will not perform a parametric  
 135 study that would allow us to find phenomenological relations; we will instead search for  
 136 a faulting geometry and use limit analysis to find physical analytical relations between  
 137 source parameters and observables.

### 3. Results and discussion

#### 3.1. Strain localization and spatial distribution of damage

138 At the beginning of the pressurization process (Figure 3 (a)), a damaged shell appears  
 139 around the pressurized reservoir, and isotropic strain localization develops along more  
 140 or less radial features around the reservoir. These features correspond to the logarithmic  
 141 spirals that are well observed when an infinite isotropic elastic-plastic medium is plastified  
 142 under the pressure applied in a hole or cylindrical cavity (see, e.g., *Hill* [1950]; *D'Escatha*  
 143 *and Mandel* [1974]; *Davis and Selvadurai* [2002]; *Gerbault et al.* [2012]) when no spatial  
 144 regularization is applied. During this initial phase, the free surface is not perturbed  
 145 and the external boundary of the damaged zone is quasi-circular, as it is in the case of  
 146 an infinite medium. *Hill* [1950] gives an expression for the radius of the damaged zone  
 147 around a pressurized reservoir in an infinite medium:

$$\frac{r_{DZ}}{r} = \left( \frac{2E}{3Y} \right)^{\frac{1}{3}} \quad (4)$$

148 where  $r_{DZ}$  is the radius of the damaged zone,  $r$  is the radius of the cavity,  $E$  is the Young's  
 149 modulus and  $Y$  is the yield stress of the rock mass. Taking a value of 5 GPa for  $E$  and

150 10 MPa for  $Y$ , in the order of magnitude of those for an average quality rock mass (see,  
151 e.g., *Hoek and Brown* [1997]), gives  $r_{DZ} \approx 6r$ , which is close to our result.

152 However our model is not infinite. Large superficial deformation develops after isotropic  
153 damage occurs around the reservoir (Figure 3); it appears to be a free-surface effect. It  
154 produces an anisotropic strain component that dominates the isotropic component out-  
155 side the damaged zone. Free surface and damage induce the punching failure of the rock  
156 mass above the pressurized cavity. Damaged elements delimit zones where anisotropic  
157 shear strain progressively localizes, and forms a coherent and complex system of faults  
158 with reverse and normal faulting. Reverse faults  $\Phi_r$  progressively delimit an uplifted com-  
159 partment similar to resurgent domes (see, e.g., *Brothelande et al.* [2016b]). These faults  
160 initiate first at depth, at the external boundary of the damaged zone, at some distance of  
161 its top, and progress towards the surface. Their average dip is about  $60^\circ$ . Normal faults  $\Phi_n$   
162 develop at the surface when the dome progressively builds; they delimit a central graben,  
163 which is a frequently observed feature in such structures. A characteristic undeformed  
164 wedge, delimited by faults  $\Phi_w$  is present above the top of the damaged zone. Though the  
165 pressure source is not located at the surface, this undeformed wedge shares strong sim-  
166 ilarities with the well-known Prandtl's wedge for shallow strip footings in geomechanics  
167 (see, e.g., *Davis and Selvadurai* [2002]). These results show that the final damage spatial  
168 distribution reproduces quite accurately the results of published geological field observa-  
169 tions (see, e.g., *Merle et al.* [2013]) and analogue modelling (*Sanford* [1959], *Davison et al.*  
170 [1993], *Marti et al.* [1994], *Merle and Vendeville* [1995], *Acocella et al.* [2001], *Merle et al.*  
171 [2013] and *Brothelande and Merle* [2015]): resurgent domes are structures with reverse  
172 faulting on their external boundaries and internal normal faulting (see, e.g., *Acocella et al.*

173 [2001], and, for a typical example, the Yenkahe complex [*Merle et al.*, 2013; *Brothelande*  
 174 *et al.*, 2016b]). Field geological observations show that magmatic intrusions associated  
 175 with resurgence are bordered by reverse faults [*Fridrich et al.*, 1991], which can explain  
 176 the abrupt transition between the flat caldera moat and the dipping layers of the dome  
 177 flanks. Elements of this pattern can be found in *Anderson* [1936], *Gerbault et al.* [2012]  
 178 ( $\Phi_w$ ), in *Robson and Barr* [1964], *Phillips* [1974] ( $\Phi_r$ ), where  $\Phi_r$  faults are associated  
 179 with cone sheets, whereas  $\Phi_w$  faults may prefigure ring dykes during reservoir deflation  
 180 stages.

### 3.2. A geometrical analysis of the spatial deformation pattern

181 On the external boundary of the damaged zone, the orthoradial (hoop) stress is not  
 182 constant, due to the free-surface effect. This stress is tensile when the reservoir is over-  
 183 pressurized. The orthoradial stress on the boundary is maximum at the Jeffery's points  
 184 (*Jeffery* [1920], Figure 4). In the case where a damaged zone exists, the faults  $\Phi_r$  and  $\Phi_w$   
 185 initiate from the Jeffery's points located at the external boundary of the damaged zone.  
 186 At these points only the orthoradial or hoop stress is sufficient to rupture the sound rock.

187 These points may be used to determine the radius of the damaged zone. Using the  
 188 distance  $B$  between the Jeffery's points, and their depth  $H$  as model parameters, we find  
 189 two simple linear relations between the dome half-width  $W_D$ , the graben half-width  $W_G$ ,  
 190 which are observables, and these parameters (Figure 4):

$$W_D = W_G + 2B \quad (5)$$

191 so that

$$B = \frac{W_D - W_G}{2} \quad (6)$$

192 and

$$W_D = B + H \cot \beta \quad (7)$$

193 where  $\beta$  is the dip of the reverse fault  $\phi_r$ ; from the latter equation we deduce:

$$H = \frac{W_D + W_G}{2} \tan \beta \quad (8)$$

194 By the Jeffery's points we can draw an infinity of circles, one only being tangent to the  
195 straight lines issued from the point I (Figure 4). Considering that the reservoir center C  
196 is located at the intersection of the normal to these straight lines, we find the depth of  
197 this center:

$$h_c = \left( 1 + \left( \frac{B}{H} \right)^2 \right) H \quad (9)$$

198 and the radius of the damaged zone:

$$r_{DZ} = B \left( 1 + \left( \frac{B}{H} \right)^2 \right)^{\frac{1}{2}} \quad (10)$$

199 If the reservoir section is not circular,  $h_c$  is the depth of the center of curvature of the  
200 damaged zone top, with the curvature taken at Jeffery's points. We notice that the ratio:

$$\lambda = \frac{H}{B} = \frac{W_D + W_G}{W_D - W_G} \tan \beta \quad (11)$$

201 may be known only from field data, so that  $h_c$  may be written:

$$h_c = \left( 1 + \left( \frac{W_D - W_G}{W_D + W_G} \cot \beta \right)^2 \right) \left( \frac{W_D + W_G}{2} \right) \tan \beta \quad (12)$$

202 One can also express the reservoir radius  $r_c$  as a function of the damaged zone radius:

$$r_{DZ} = (1 + \Delta) r_c \quad (13)$$

203 where  $\Delta$  is the relative damage zone thickness (close to 5 in our case), so that  $r_c$  may be  
204 written:

$$r_c = \frac{W_D - W_G}{2} \left( 1 + \left( \frac{W_D - W_G}{W_D + W_G} \cot \beta \right)^2 \right)^{\frac{1}{2}} \quad (14)$$

### 3.3. Magma reservoir pressure derived from Limit Analysis

205 In such a model, when the geometry of damaged zones and the displacement field are  
206 known, limit analysis theorems (see, e.g., *Davis and Selvadurai* [2002], *Souloumiac et al.*  
207 [2009]) give us the possibility to compute an upper bound for the pressure in the magma  
208 reservoir. The aim of this approach is to provide an analytical expression for the magma  
209 reservoir pressure, to understand which are the control parameters of the pressurization  
210 process and what is the relation with the deformation process.

211 The upper bound theorem of Limit Analysis states that failure will occur if, for any  
212 kinematically admissible displacement field, the rate of work of the external forces is  
213 equal to or exceeds the rate of energy dissipation. It allows us to determine the maximum,  
214 ultimate value (upper bound) of the applied external loads that a system can bear without  
215 collapsing, which in our case corresponds to the loss of equilibrium. For collapse modes  
216 of translational type, the upper bound is equivalent to the solution found by the limit  
217 equilibrium method (see, e.g., *Drescher and Detournay* [1993]): the upper bound theorem  
218 expresses the energy balance of the limit load, which is equivalent to the limit equilibrium  
219 of the system on the base of the principle of virtual powers.

220 Results of the modelling allow us to identify the blocks that are sliding and to deduce  
221 their relative velocities (Figure 4). This type of collapse mode is of translational type and  
222 therefore the limit equilibrium method can be used as well. However, the upper bound  
223 theorem is easier to apply given the geometric complexity of the system. Applying the  
224 upper bound theorem requires computing the rate of the work of external forces (weights  
225 and pressure force) and the rate of the dissipation. In that aim we need to compute the  
226 relative velocities of the sliding blocks; they are found by constructing the hodograph

227 (see Appendix 1, Figure S2), taking into account the angle of dilatancy  $\psi$ . We can then  
 228 express the directions of the velocities, and find the modules by projecting the Chasles's  
 229 relation on each coordinates axis (see Appendix 1 for detailed calculations). Given that  
 230 there is no horizontal displacement of the vertical boundaries, the horizontal forces do not  
 231 work. The rate of work may therefore be found by computing the weights of the various  
 232 sliding blocks (see Appendix 1):

$$W_{ext} = -(2\rho gHBv_{01} + w_2v_{02}) + 2PBv_{01} \quad (15)$$

233 Applying the upper bound theorem, that is, writing that the rate of external work of  
 234 the external forces is equal to the rate of internal energy dissipation  $D$  leads to find the  
 235 pressure:

$$P = \rho gH + \frac{w_2v_{02}}{2Bv_{01}} + \frac{D}{2Bv_{01}} \quad (16)$$

236 that may be written explicitly as (see Appendix 1 for the detailed computation of  $w_2v_{02}$   
 237 and  $D$ ):

$$P = \rho gH + \frac{1}{4}\rho gB \tan \beta \left( \frac{\lambda}{\tan \beta} - 1 \right)^2 \left( 1 - \frac{\tan(\beta - \psi)}{\tan(\beta + \psi)} \right) + 2C \cos \phi \frac{\lambda \cos^2 \beta \cos \psi + \sin^2 \beta \sin \psi}{\sin 2\beta \sin(\beta + \psi) \cos(\beta - \psi)} \quad (17)$$

238 where  $\lambda = H/B$  and  $C$  is the cohesion. In the general case where a deviatoric stress  
 239 exists at the depth  $H$ , the initial lithostatic pressure may be expressed as a fraction of  
 240 the vertical stress:

$$P_0 = k\rho gH \quad (18)$$

241 where  $0 \leq k \leq 1$ . This can lead to derive an estimator of the overpressure:

$$\frac{P - k\rho gH}{\rho gB} = (1 - k)\lambda + \frac{1}{4} \tan \beta \left( \frac{\lambda}{\tan \beta} - 1 \right)^2 \left( 1 - \frac{\tan(\beta - \psi)}{\tan(\beta + \psi)} \right) + 2\kappa \cos \phi \frac{\lambda \cos^2 \beta \cos \psi + \sin^2 \beta \sin \psi}{\sin 2\beta \sin(\beta + \psi) \cos(\beta - \psi)} \quad (19)$$

242 where  $\kappa = \frac{C}{\rho gB}$ .

243 In the common case where rock friction is described by associated plasticity, the angle  
244 of dilatancy  $\psi$  is taken equal to the friction angle  $\phi$ , which means that the surface rugosity  
245 alone explains the friction along fault planes. In that case, the normalized overpressure  
246  $\Pi$  may be expressed as:

$$\Pi = \frac{P - k\rho gH}{\rho gB} = (1 - k)\lambda + \frac{1}{4} \tan \beta \left( \frac{\lambda}{\tan \beta} - 1 \right)^2 \left( 1 - \frac{\tan(\beta - \phi)}{\tan(\beta + \phi)} \right) + 2\kappa \cos \phi \frac{\lambda \cos^2 \beta \cos \phi + \sin^2 \beta \sin \phi}{\sin 2\beta \sin(\beta + \phi) \cos(\beta - \phi)} \quad (20)$$

247 Equations (16) and (20) show that overpressure may be decomposed in three contribu-  
248 tions and allows an analysis of the pressurization and deformation process:

249 - a term  $\Pi_0 = (1 - k)\lambda$  controlled by the pre-existing lithostatic pressure. This term is  
250 the amount of pressure necessary to reach, from the initial lithostatic pressure, the vertical  
251 equilibrium of the rock column of height H above the reservoir considering this one isolated  
252 (ruptured) from the host rock. It causes the reservoir volume increase and isotropic  
253 damage that precede the strain localization (formation of fault planes). It corresponds to  
254 the initial pressurization phase; during this phase there is no surface deformation, no free  
255 surface effect and the state of stress around the reservoir is compressive;

256 - a frictional term  $\Pi_f = \frac{1}{4} \tan \beta \left( \frac{\lambda}{\tan \beta} - 1 \right)^2 \left( 1 - \frac{\tan(\beta - \phi)}{\tan(\beta + \phi)} \right)$  which is a budget measured  
257 by the displacement of the block number 2; it corresponds to the overpressure necessary  
258 for the vertical, anisotropic, displacement and deformation along the fault plane structure

259 that appears after strain localization. This term is controlled by the friction angle  $\phi$   
 260 and it is zero when  $\phi$  is null. It is also null for  $\tan \beta = \tan \beta_c = \lambda$ . It results from  
 261 the friction along the fault plane structure after its creation, after the weakening of the  
 262 volcanic edifice and the progressive opening of the eruptive/intrusive system; this term  
 263 is dominant during large (vertical) deformation processes, especially immediately before,  
 264 or during, eruptions or intrusions; it tends to be dominant during the final pressurization  
 265 phase, and constitutes a minimum value for the pressure;

266 - a dissipative term  $\Pi_D = 2\kappa \cos \phi \frac{\lambda \cos^2 \beta \cos \phi + \sin^2 \beta \sin \phi}{\sin 2\beta \sin(\beta + \phi) \cos(\beta - \phi)}$  that corresponds to the (short-  
 267 term) irreversible part of the deformation. It is controlled by the  $C \cos \phi$  term, that is,  
 268 mostly by the cohesion  $C$  since  $\phi$  keeps values in a relatively narrow interval for rocks,  
 269 even at the scale of the rock mass. If  $C$  is null, no effort is spent in rock decohesion  
 270 and this term vanishes. In this case minimal pressure corresponds to  $\beta = \beta_c$ , that is  
 271  $W_G = 0$  (no graben). Decohesion is associated to microseismicity and damage during  
 272 the pre-eruptive/intrusive process; it leads progressively to strain localization when the  
 273 free-surface effect appears. It precedes the phase of large displacement along the fault  
 274 planes that are finally created. This term corresponds to the pressure necessary to create  
 275 the fault plane structure, and to open the eruptive/intrusive system; it works during the  
 276 intermediate phase between the initial and final pressurization phases described above.  
 277 This is a transient contribution that decreases with cohesion and time during the pre-  
 278 eruptive/intrusive process. In this work cohesion is used to describe the unconfined rock  
 279 strength, but cohesion may also be easily related to tensile strength, which is more often  
 280 used to quantify rock strength around magma reservoirs. Cohesion may eventually be

low, at the scale of the rock mass, in fractured media. This term is generally considered alone to quantify the overpressure necessary for the rupture of magma reservoirs.

Therefore in this simple expression we retrieve the main phases of the deformation process, and their contribution to the magma reservoir pressure. It helps to understand how overpressure builds and may be used to describe the time evolution of this pressure, eventually from geophysical observables: seismicity and surface deformation. It highlights the multiple causes of overpressure, which may be the origin of contradictory interpretations [Gudmundsson, 2012; Grosfils, 2007].

Equation 20 and Figure 5 show that the overpressure  $\Pi(\lambda)$  is a parabola with a first order term in  $\lambda$  dependent on  $\kappa$ . For values of  $\kappa$  larger than 0.5,  $\Pi$  is close to a linear function of  $\lambda$  in the vicinity of  $\tan \beta$  (close to 2 in Figure 5), that is, in the practical interval of interest for  $\lambda$ . The overpressure  $\Pi$  is linearly dependent on  $\kappa$  (equation 20). These results may be compared to those obtained by Haug *et al.* [2018], who found that overpressure was linearly dependent on the cohesion, and decreasing as a function of  $B/H$  (inverse of  $\lambda$ ) with a power law.

### 3.4. Inferring the faulting structure from $\lambda = \frac{H}{B}$ ratio, cohesion and friction angle

Expression (20) also contains information on the structure itself, and on its control parameters.

For a given reservoir position and size, and given rock mechanical properties, the fault plane structure is created in such a way that minimal effort, that is minimal pressure, is required. The only parameter of the structure that controls the pressure is the angle  $\beta$ . Minimization of the pressure with  $\beta$  allows us to find the optimal angle for which

the structure is created (Figure 6). As an example, we find that, for realistic rock mass  
 mechanical properties (Figure 6), and  $\lambda$  close to 2, the pressure reaches a minimum for  
 the optimal value  $\beta = 61$  degrees, close to the value found from the finite element model  
 (Figure 3).

From equation (11), we find

$$\begin{aligned}\lambda &= \rho_R \tan\beta \\ &= \frac{1 + \alpha}{1 - \alpha} \tan\beta\end{aligned}\tag{21}$$

where  $\rho_R = \frac{W_D + W_G}{W_D - W_G}$  and  $\alpha = \frac{W_G}{W_D}$ . As a consequence,  $\alpha$  may be written:

$$\alpha = \frac{\lambda - \tan\beta}{\lambda + \tan\beta}\tag{22}$$

The ratio  $\alpha$  is between 0 and 1. From equation (21) we can write

$$\frac{\tan\beta}{\lambda} = \frac{\tan\beta}{\tan\beta_c} = \chi = \frac{1 - \alpha}{1 + \alpha}\tag{23}$$

where  $\tan\beta_c = \frac{H}{B} = \lambda$ .

From equations (22, 23) we infer that  $\alpha$  is non-zero and the central graben exists if  
 $\tan\beta < \lambda = \tan\beta_c$ , that is, if the dissipation effect is non-zero. Therefore  $\alpha$  and  $\beta$  depend  
 on  $\lambda$  and  $\kappa$  (Figures S4 and S5). From these results we can infer that the graben exists  
 ( $\alpha > 0$ ) for sufficiently high values of  $\lambda$  and  $\kappa$ .

Figure 7 shows the variation of  $\beta$  as a function of  $\lambda$ , for various values of  $\kappa$  and  $\phi$ . It  
 shows that the family of  $\beta = f(\lambda)$  curves has a characteristic point  $(\lambda_0, \beta_0)$  independent  
 of  $\kappa$ ; the family is comprised between two extreme curves corresponding to  $\kappa = 0$  (black  
 line and crosses) and  $\kappa \rightarrow \infty$  (red circles). The geometry, showing one central graben,  
 studied by using limit analysis only exists for  $\beta$  curves located below the black line, which  
 corresponds to the limit case  $\beta_c = \arctan(\lambda)$ . This result shows that an increased cohesion  
 induces a decrease in  $\beta$  (see also Figure S5); it also shows that an increase in  $\phi$  from 10 to

321  $40^\circ$  induces a decrease of  $\beta$  of at most  $10^\circ$ , which is reached for high cohesions and large  
 322  $\lambda$  (Figure S6). The characteristic values  $(\lambda_0, \beta_0)$  correspond to the minimal values of  $\lambda$   
 323 (and  $\beta$ ) for which the central graben exists; it is independent of the material cohesion.  
 324 Expressing the relation between  $\beta_0$  and  $\phi$  shows that  $\beta_0 = \frac{3\pi}{8} - \frac{\phi}{2}$  for  $15^\circ < \phi < 75^\circ$   
 325 degrees (Figure S7).

326 The Figure S8 shows that  $\tan \beta - \tan \beta_0$  is a linear function of  $\lambda$  depending on  $\kappa$  and  
 327  $\phi$ . A first-order approximation of this function leads to find the analytical expression (see  
 328 Appendix 2 for details of the calculations):

$$\tan \beta - \tan \beta_0 = R(\kappa, \phi) (\lambda - \lambda_0) \quad (24)$$

329 for  $\lambda > \lambda_0$ , where

$$R(\kappa, \phi) = 1 - \frac{a_0 \kappa}{c_0 \kappa + d_0} \left( 1 + \frac{a_1 \kappa + b_1}{c_1 \kappa + d_1} \phi \right) \quad (25)$$

330  $\lambda_0 = \tan \beta_0 = \tan \left( \frac{3\pi}{8} - \frac{\phi}{2} \right)$ , and  $a_0, c_0, d_0, a_1, b_1, c_1, d_1$  are constants. Equation (24) may  
 331 be considered as the first order Taylor's series expansion in  $\phi$  for large values of  $\phi$ , of the  
 332 solution found by optimization in  $\beta$  of the equation (20). The first-order term  $\frac{a_1 \kappa + b_1}{c_1 \kappa + d_1} \phi$   
 333 takes values in the  $[-0.25, 0.15]$  interval so that the dependency of  $\beta$  on  $\phi$  is relatively  
 334 moderate in equation (24), and mostly due to the  $\lambda_0$  term. However neglecting the first-  
 335 order term leads to significant changes for the highest values of  $\kappa$ . Increasing the order of  
 336 the expansion enlarges the interval wherein the approximation is valid, at the expense of  
 337 a larger number of parameters. The optimal order is found to be the first order, using a  
 338 compromise between the parameter number and the goodness of fit. Results (Figure S9)  
 339 show that  $\beta$  is more strongly decreasing with the cohesive term  $\kappa$ . Using the values issued  
 340 from our computations, we can infer that, when the central graben structure exists, for  
 341 realistic rock mass characteristics ( $\phi \approx 30^\circ$ ),  $\beta$  grossly varies from about  $50^\circ$  for  $\lambda \approx 1$ , to

342 about  $60^\circ$  for  $\lambda \approx 2$  and  $70^\circ$  for  $\lambda \approx 6$  (Figure 7). These results are coherent with those  
 343 obtained from analogue models, where values of 50 degrees ( $\lambda = 0.8$ , *Acocella et al.* [2001];  
 344 *Brothelande et al.* [2016b]) and 60 degrees ( $\lambda = 2$ , *Acocella et al.* [2001]) were found for  $\beta$ .  
 345 The variation of  $\beta$  with  $\lambda$  is also coherent with the results obtained by *Haug et al.* [2018].

346 From  $\tan\beta_c = \lambda = \rho_R \tan\beta$  and equation (24), we find that

$$\begin{aligned} \tan\beta &= \tan\beta_0 + R(\kappa, \phi) (\lambda - \lambda_0) \\ &= (1 - R(\kappa, \phi)) \tan\beta_0 + R(\kappa, \phi) \rho_R \tan\beta \end{aligned} \quad (26)$$

347 where  $\rho_R = \frac{W_D + W_G}{W_D - W_G}$  and  $\lambda_0 = \tan\beta_0 = \tan\left(\frac{3\pi}{8} - \frac{\phi}{2}\right)$ ; we infer:

$$\tan\beta = \frac{1 - R(\kappa, \phi)}{1 - \rho_R R(\kappa, \phi)} \tan\beta_0 \quad (27)$$

348 Therefore  $\rho_R = 1$ , that is  $W_G = 0$ , corresponds to  $\beta = \beta_0$  when  $\kappa \neq 0$  (and  $\beta = \beta_c =$   
 349  $\tan^{-1}(\lambda)$  when  $\kappa = 0$ ).

350 Equation (27) has to verify the condition  $\tan\beta_c \geq \tan\beta \geq \tan\beta_0$  for  $\rho_R \geq 1$  (that is  
 351  $W_G \geq 0$ ). The condition  $\tan\beta \geq \tan\beta_0$  is true when  $\rho_R \geq 1$  and  $R(\kappa, \phi) \geq 0$ , whatever  
 352 can be  $\lambda, \kappa, \phi$ . The condition  $\tan\beta \leq \tan\beta_c$  implies that  $R(\kappa, \phi) \leq R(0, \phi) = \frac{\lambda - \lambda_0}{\rho_R \lambda - \lambda_0}$ , so  
 353 that equation (27) is valid when  $0 \leq R(\kappa, \phi) \leq \frac{\lambda - \lambda_0}{\rho_R \lambda - \lambda_0}$ .  $R(\kappa, \phi)$  is always lower than  $\frac{1}{\rho_R}$   
 354 when  $\lambda$  tends to infinity. As  $\rho_R$  is always larger than 1, we can infer that  $0 \leq R(\kappa, \phi) \leq 1$ .

355 In volcanic environments, the angle  $\beta$  is generally not directly found from field geo-  
 356 logical measurements, as inverse faults that reach the surface are more inferred from the  
 357 deformation and topography rather than directly evidenced. Equation (27) therefore al-  
 358 lows the estimation of  $\beta$  from values of  $\rho_R$ , that is  $W_D, W_G$  measured on the field, and  
 359 estimates of  $\kappa$  and  $\phi$  determined at the scale of the rock mass. This estimation is used to  
 360 compute  $h_c$  and  $r_c$ , and the normalized overpressure terms  $\Pi_f + \Pi_D$ , since  $\lambda = \rho_R \tan\beta$ .  
 361 We used our results to compute estimates of  $\beta, h_c, r_{DZ}$  and  $\Pi_f + \Pi_D$  in the case of the

362 resurgent dome of Yenkahe (Vanuatu). From field and map measurements, we know that  
 363 (see, e.g., *Brothelande et al.* [2016b])  $W_D = 2.3$  km,  $W_G = 0.6$  km so that  $\rho_R = 1.7$ ; the  
 364 corresponding interval of validity for the equation (27) is  $0 \leq R(\kappa, \phi) \leq 0.59$ . For reason-  
 365 able values of the friction angle  $\phi$  comprised between  $25^\circ$  and  $45^\circ$ , we find (Figure 8) that  
 366  $60^\circ \leq \beta \leq 75^\circ$ ,  $h_c$  comprised between 2.5 km and 4 km, and  $r_{DZ} \approx 1$  km. The radius of  
 367 the magma reservoir is not directly known, as it depends on the damaged zone relative  
 368 thickness  $\Delta$ , which is poorly known and likely to be comprised between 1 and 5. Using a  
 369 value of 2 to 3 for  $\Delta$  allows us to find a magma reservoir radius of about 300 meters and a  
 370 magma reservoir depth of about 2 to 3.5 km, the depth being compatible with the numer-  
 371 ical results of *Brothelande et al.* [2016b]. Variations of  $h_c$  and  $r_{DZ}$  with  $W_G$  and  $W_D$  are  
 372 presented Figure S17; they show that  $h_c$  is sensitive to variations in  $W_G$  more than in  $W_D$ .  
 373 Computation of the overpressure terms for Yenkahe within the intervals  $25^\circ \leq \phi \leq 40^\circ$   
 374 and the corresponding  $75^\circ \geq \beta \geq 60^\circ$  yields  $0.06 \leq \Pi_f \leq 0.21$  and  $3.26\kappa \geq \Pi_D \geq 1.98\kappa$ ;  
 375 for  $\phi = 30^\circ$  and  $\beta = 65^\circ$ ,  $\Pi_f = 0.069$  and  $\Pi_D = 2.38\kappa$ . This result helps to quantify the  
 376 fact that when the eruptive/intrusive system is closed or sealed (what happens after a  
 377 long period of quiescence), the normalized overpressure required for the edifice rupture is  
 378  $10\kappa$  to  $50\kappa$  ( $0.4 \leq \kappa \leq 2$ ) larger than the normalized overpressure needed when the edifice  
 379 is already ruptured and deforms only by frictional sliding. From the former results, the  
 380 overpressure involved in the crustal anisotropic deformation by the magma reservoir is  
 381 close to  $\rho g B \Pi_D$ , that is approximately  $2C$ . For cohesion values between 0.1 and 10 MPa,  
 382 this part of the overpressure varies between 0.2 and 20 MPa. Computation of the isotropic  
 383  $\rho g B \Pi_0 = (1 - k)\rho g H$  term shows that it scales with the depth  $H$  of the magma reservoir,  
 384 and reaches a value of 50 MPa in the case of the Yenkahe dome. This order of magnitude

385 is compatible with those computed by *Grosfils* [2007]. This isotropic overpressure term  
386 induces the isotropic damage around the magma reservoir. However in real cases where  
387 inflation/deflation cycles exist and correspond to time variation of the overpressure with  
388 irreversible strain in contraction, after the first cycle isotropic damage may pre-exist be-  
389 fore the re-pressurization of the magma reservoir. In those cases it can be questioned if  
390 any depth-dependent overpressure is needed to localize the shear strain at the boundary  
391 of the damaged zone.

392 Values of reservoir radius and depth have been computed for some other resurgent domes  
393 (Table 1 and Figures S13-17) and may be compared to direct geophysical estimations.  
394 Campi Flegrei is the most well-investigated caldera. At this site, dome and graben half-  
395 widths are taken from *Sacchi et al.* [2014], though it is difficult to know which graben  
396 structure is actually active nowadays; variations of the graben structure may indicate that  
397 depth and radius vary with time. At Campi Flegrei, most of these studies have estimated  
398 the source location to be beneath the Pozzuoli area at a depth around 3 km below sea level  
399 (e.g. *Trasatti et al.* [2008] and references therein; *Bonafede et al.* [1986]; *Berrino et al.*  
400 [1984]). The order of magnitude for the depth is corroborated by the seismicity recorded  
401 during the 1982-84 uplift that clustered between 1 and 4 km below sea level [*Ferrucci*  
402 *et al.*, 1992]. Furthermore, seismic attenuation images provided by *De Siena et al.* [2010]  
403 indicate the presence of a small melt pocket beneath Pozzuoli located between 3 and 3.5  
404 km depth. All these estimations match well with ours.

405 At Yellowstone, several tomography surveys have imaged a very large low-velocity zone,  
406 interpreted as a giant magma reservoir. *Miller and Smith* [1999] identified probable partial

407 melting zones at about 8 km beneath the domes surface, a depth later confirmed by *Husen*  
408 *et al.* [2004] whereas *Farrell et al.* [2014] found a depth of about 6 km.

409 At Valles caldera, resurgence is no longer active and probably finished more than 1  
410 Myrs ago [*Phillips et al.*, 2007]. The only direct geophysical constraint on the depth  
411 of the magma reservoir associated to resurgence is that it is greater than 3.25 km, the  
412 maximum depth of the geothermal drilling campaign that did not penetrate any intrusive  
413 rock [*Nielson and Hulen*, 1984].

#### 4. Conclusion

414 In this work we have investigated the deformation process around a pressurized magma  
415 reservoir embedded in a damaging elastic volcanic edifice. It has allowed us to evidence  
416 the action of the progressive damage process, and the structure created by the damage  
417 distribution. Damage is first isotropic around the cavity and constitutes a damaged zone.  
418 Then the free-surface effect appears and an anisotropic shear strain develops from the  
419 boundary of the damaged zone; it localizes on reverse faults that propagate upward to  
420 the surface. When the vertical surface deformation is sufficient, normal faulting appears.  
421 Finally, the complete structure shows an undeformed wedge above the top of the damaged  
422 zone, which strongly recalls the Prandtl's wedge. This structure is very similar to what  
423 is found by analogue modelling and from field geologic observations. From this model,  
424 we found a relation for reservoir radius and depth as a function of dome and graben  
425 widths. Magma reservoir pressure is deduced from limit analysis, which allows a better  
426 understanding of the magma pressure build-up. The dip of the reverse faults is inferred  
427 from the minimization of the pressure needed to rupture the crust around the magma  
428 reservoir. From that analysis, the magma reservoir overpressure, the dip of the faults, the

429 magma reservoir depth and the damaged zone radius may be inferred analytically from  
430 three parameters only: the ratio  $\rho_R$  computed from the dome and graben widths, the  
431 cohesion, and the friction angle.

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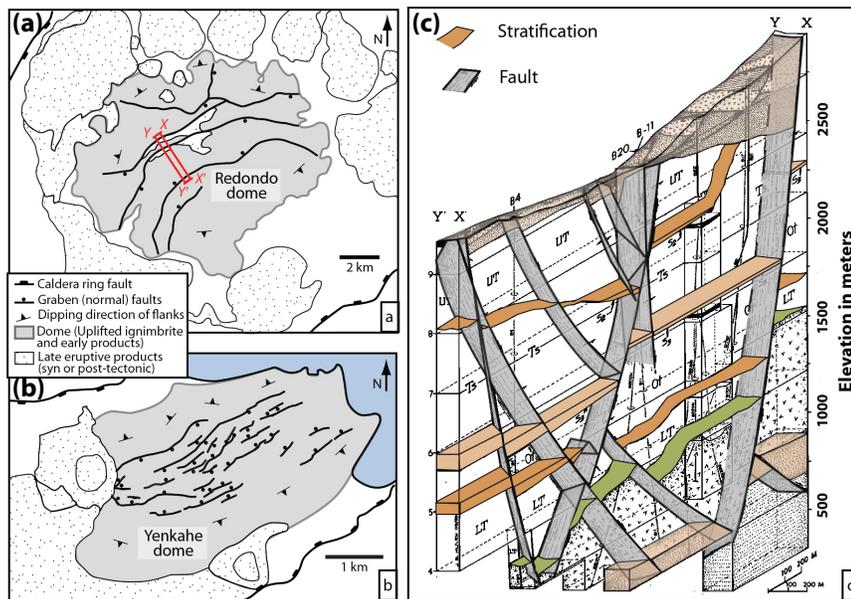
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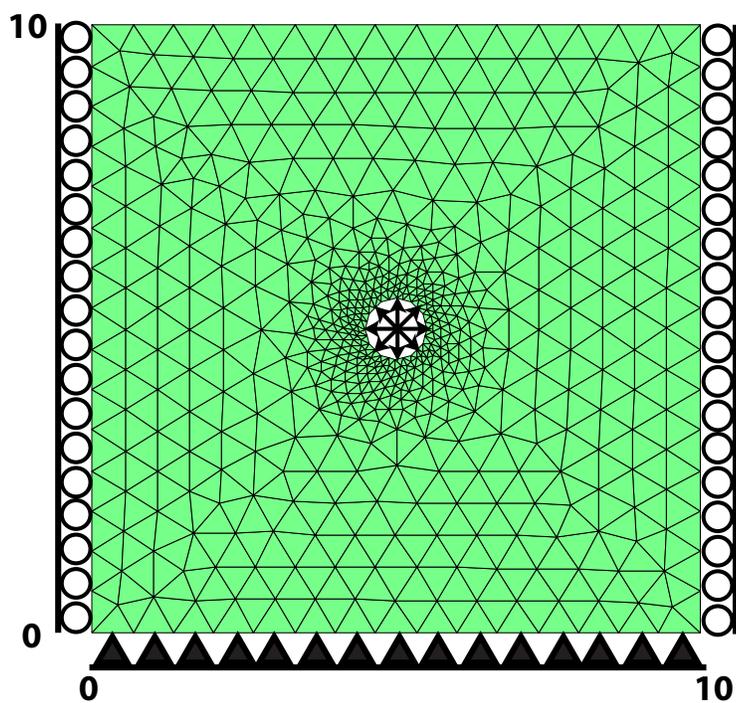
Caldera	Dome	$W_D$ (km)	$W_G$ (km)	$\rho_R$	$\beta(^{\circ})$	$h_c$ (km)	$r_{DZ}$ (km)
Siwi	Yenkahe	1.5-2.2	0.4-0.6	1.5-2.3	60-70	2-4	0.8-1
Valles	Redondo Dome	4.5-4.9	1.6-1.8	2-2.3	65-75	9-12	1.7
Yellowstone	Mallard Lake	4.5-5	1.3-1.7	1.7-2.2	65-75	9-12	1.9-2
Creede	Snowshoe Mountain	5.3-5.7	0.9-1	1.4-1.5	60-65	6.5-7.5	2.6-2.7
La Pacana	Purifican-Bola	5-5.5	0.4-0.6	1.15-1.3	50-55	5.5-6	2.9-3.2
Campi Flegrei	Pozzuoli Bay	2.2-2.6	0.5-0.7	1.5-1.9	62-70	2.5-3.5	0.8

**Table 1.** Results for the ratio  $\rho_R = \frac{W_D+W_G}{W_D-W_G}$ , the fault dip angle  $\beta$ , the reservoir

depth  $h_c$  and the damaged zone radius  $r_{DZ}$  for various resurgent domes. Half-widths of the dome  $W_D$  and the graben  $W_G$  are inferred from *Smith and Bailey* [1968] (Valles caldera), *Smith et al.* [2009] (Yellowstone), *Steven and Lipman* [1976] (Creede), *Gardeweg and Ramirez* [1987] (La Pacana), *Sacchi et al.* [2014] (Campi Flegrei, Figure 13).



**Figure 1.** Simplified structural maps of (a) Redondo dome in Valles caldera (after *Smith and Bailey* [1968]) and (b) Yenkahe dome in Siwi caldera (after *Brothelande et al.* [2016a]). (c) Reconstructed 3D structure of the graben along  $XX'YY'$  inferred from geothermal drilling (modified after *Nielson and Hulen* [1984]).

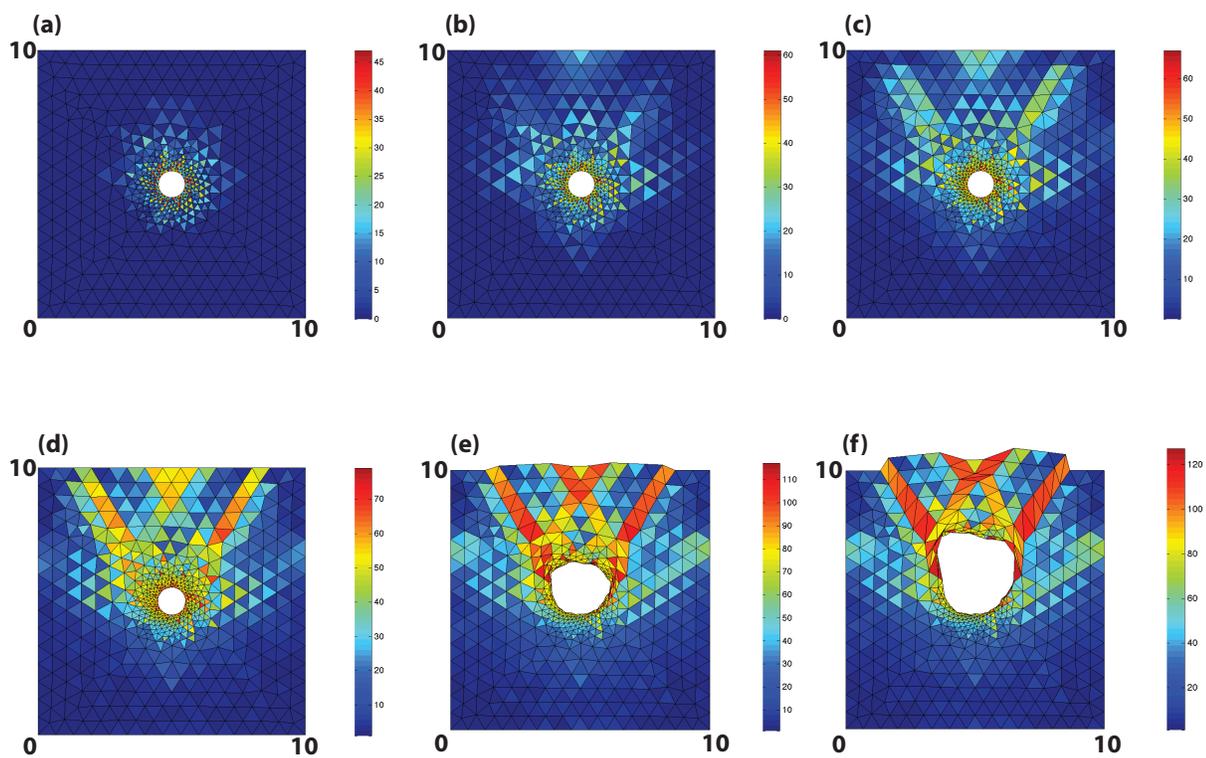


**Figure 2.** Model geometry and boundary conditions.

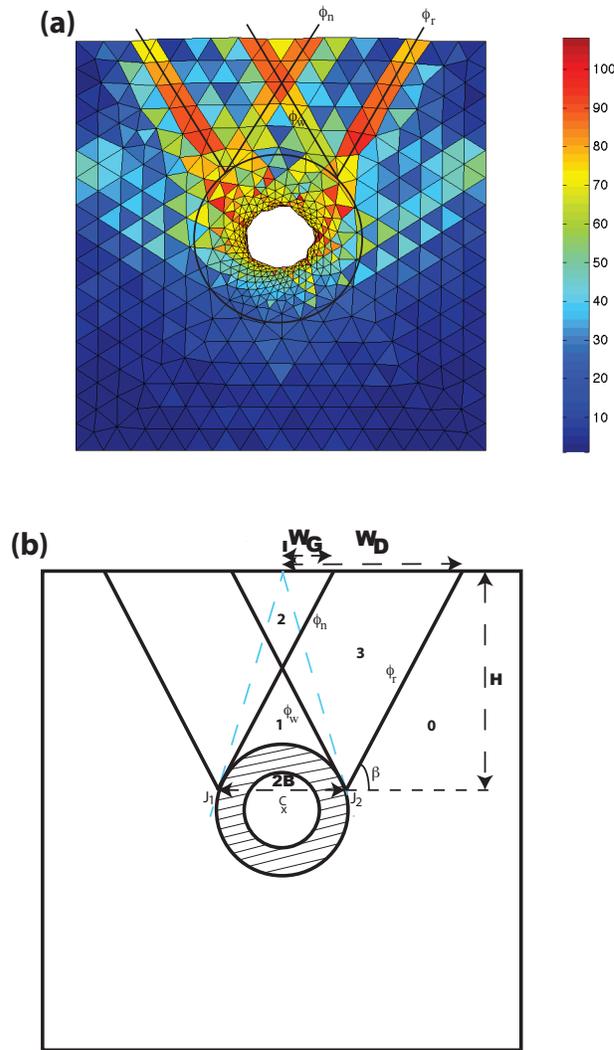
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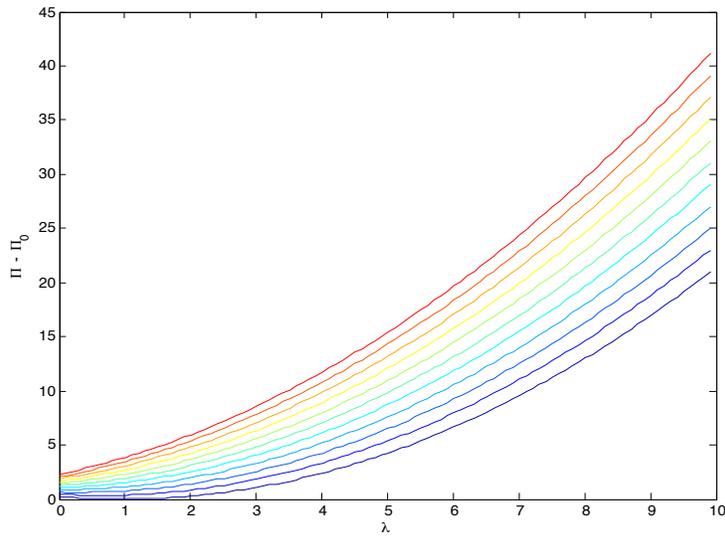
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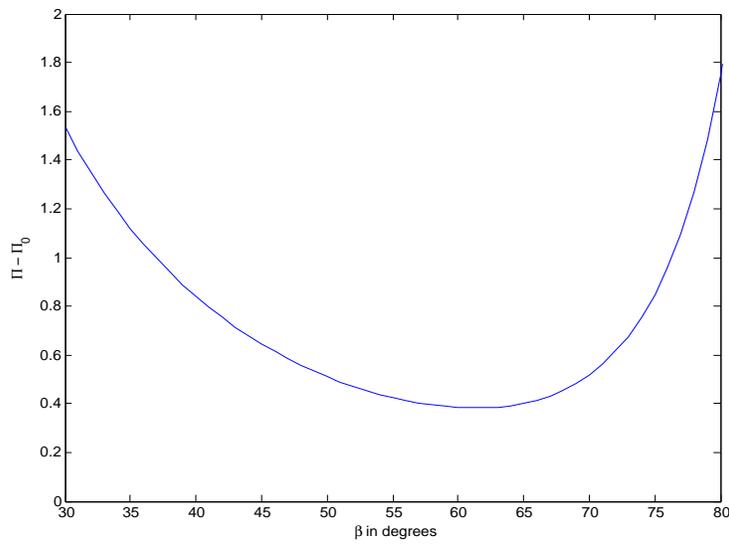
**Figure 3.** Number of damage events as a function of their position in the model at six stages of the strain localization process, showing the free-surface effect.



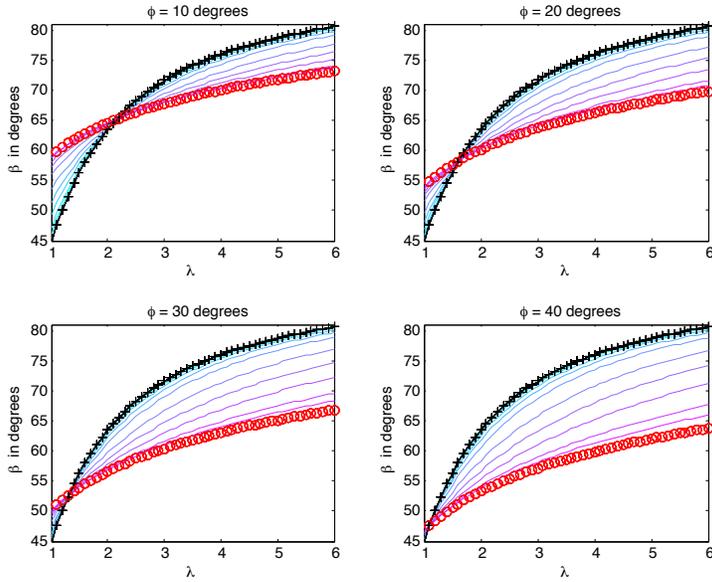
**Figure 4.** (a) Damage distribution around the pressurized magma reservoir, showing the main structural elements: damaged zone (inside the black circle), reverse faults ( $\phi_r$ ), normal faults ( $\phi_n$ ), undeformed Prandtl's wedge limited by the  $\phi_w$  faults. (b) Model of the structure used for the limit analysis study, showing the damaged zone (hatched area), normal and reverse faults ( $\phi_n$ ,  $\phi_r$ ,  $\phi_w$ ), the blocks they delimit (numbers), and the graben and dome extent with their half-widths  $W_G$  and  $W_D$ . Block number 0 corresponds to the fixed, stable, structure; block number 1 corresponds to the Prandtl's wedge, above the magma reservoir; block number 2 corresponds to the graben; block number 3 corresponds to the part of the dome delimited by the normal ( $\phi_n$ ) and reverse ( $\phi_r$ ) faults. Blue dashed lines are tangent to the circle and used to find the Jeffery's points  $J_1, J_2$ ;  $J_1 J_2 = 2B$ . The point I is at the vertical of the center C of the reservoir.



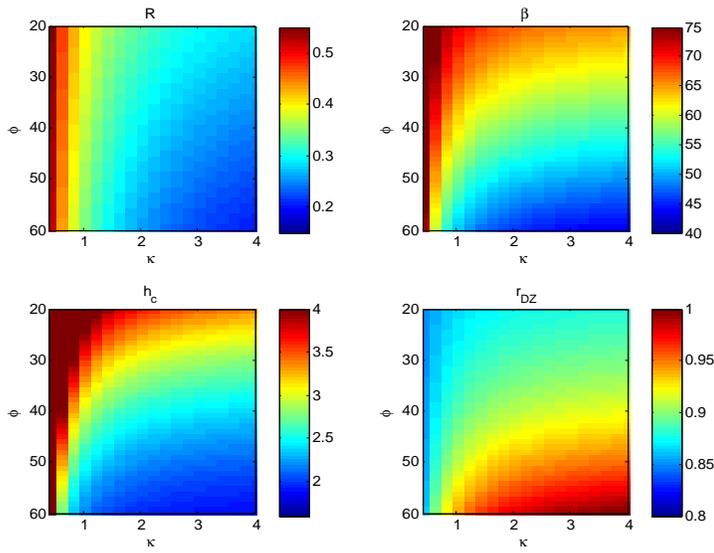
**Figure 5.** Overpressure  $\Pi - \Pi_0 = \Pi_f + \Pi_D$  (see text for details) in the cavity as a function of  $\lambda$ , for values of  $\kappa$  varying between 0 (dark blue) and 2 (dark red) by step of 0.2, with  $\phi = 30$  degrees, and  $\beta = 61$  degrees.



**Figure 6.** Overpressure  $\Pi - \Pi_0 = \Pi_f + \Pi_D$  (see text for details) in the cavity as a function of the angle  $\beta$ , for  $\kappa = 0.2$ ,  $\lambda = 2$ ,  $\phi = 30$  degrees.



**Figure 7.** Angle  $\beta$  as a function of  $\lambda$ , for various values of  $\kappa$  (0.01, 0.02, 0.05, 0.1, 0.2, 0.5, 1, 2, 5, 10, from light blue to magenta), and various values of  $\phi$ ; black crosses correspond to a purely frictional material ( $\kappa = 0$ ), and red circles to a cohesive-frictional material with very high cohesion ( $\kappa = 100$ ); black line (superposed to black crosses) corresponds to  $\beta = \beta_c = \arctan(\lambda)$ . (a):  $\phi = 10^\circ$ ; (b):  $\phi = 20^\circ$ ; (c):  $\phi = 30^\circ$ ; (d):  $\phi = 40^\circ$ . The characteristic point, intersection of all curves, corresponds to  $\beta_0 = \frac{3\pi}{8} - \frac{\phi}{2}$  (see text and Figure S7).



**Figure 8.** Results obtained for the Yenkahe caldera. (a): Function  $R(\kappa, \phi)$  (Equation (25)); (b): Angle  $\beta$  (Equation (27)) in degrees; (c) Depth  $h_c$  (Equation (12)) in km; (d) Radius of the damaged zone  $r_{DZ}$  (Equations (13-14)) in km, as a function of  $\kappa$  and  $\phi$  (in degrees). Values of  $\beta$ ,  $h_c$  and  $r_{DZ}$  in the text corresponds to  $25^\circ \leq \phi \leq 45^\circ$  and  $\kappa \approx 1$ .

**Figure1.**

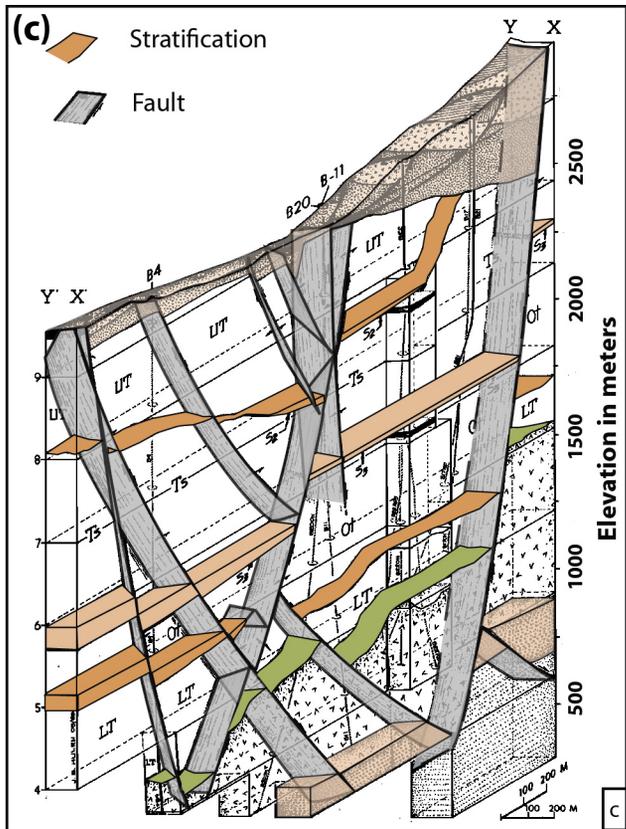
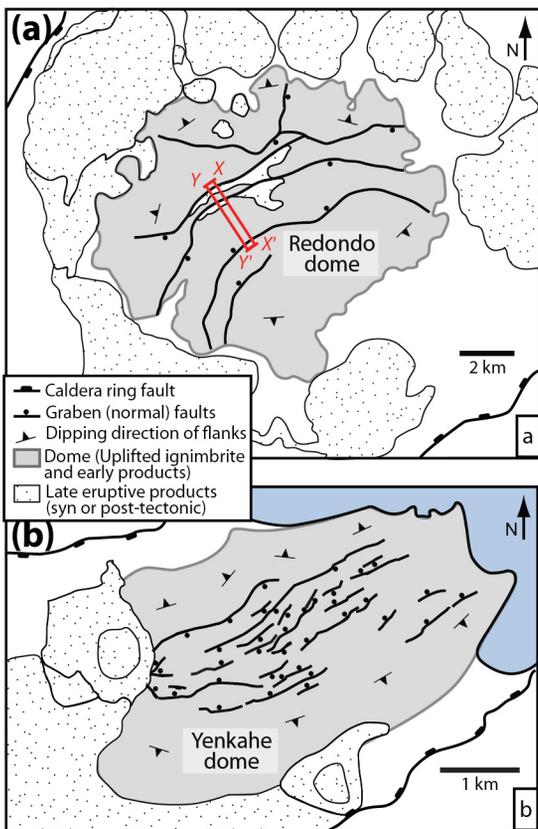


Figure2.

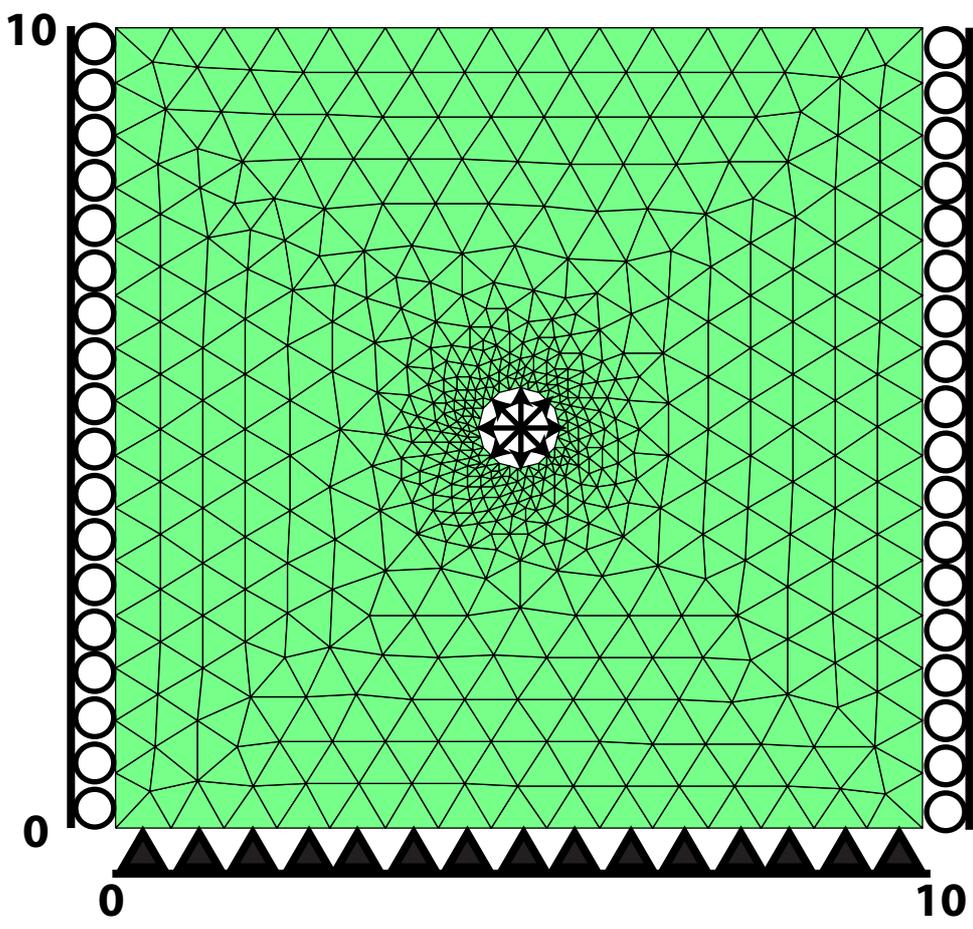


Figure3.

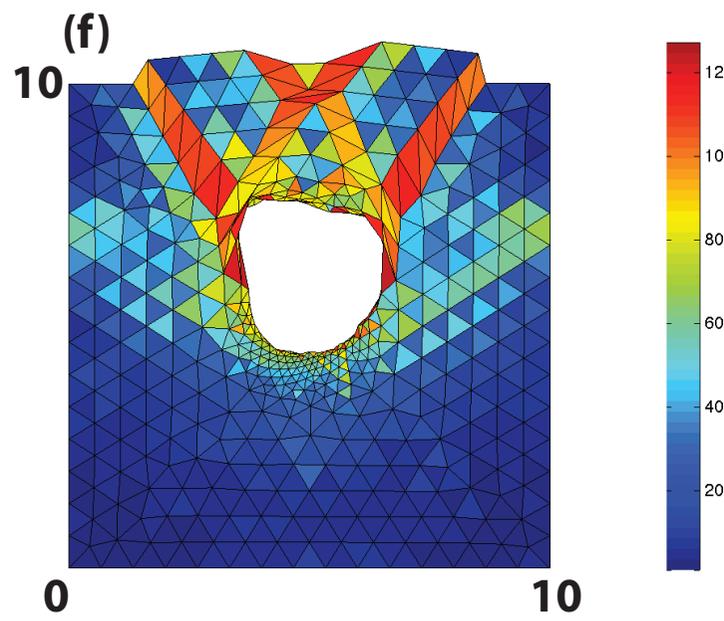
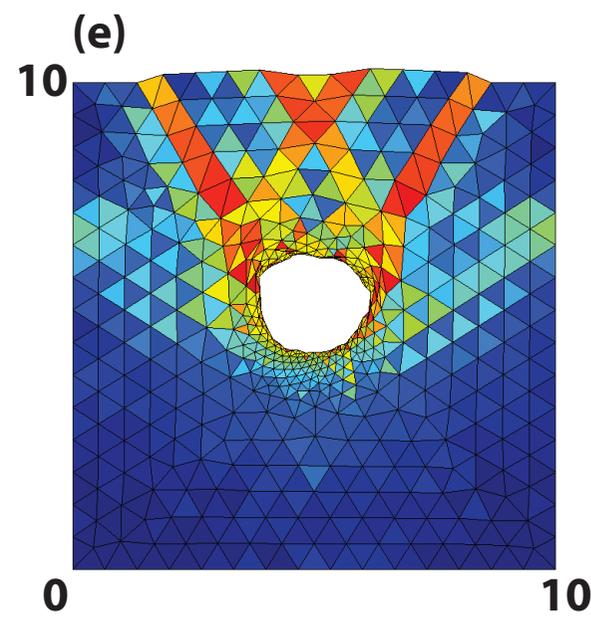
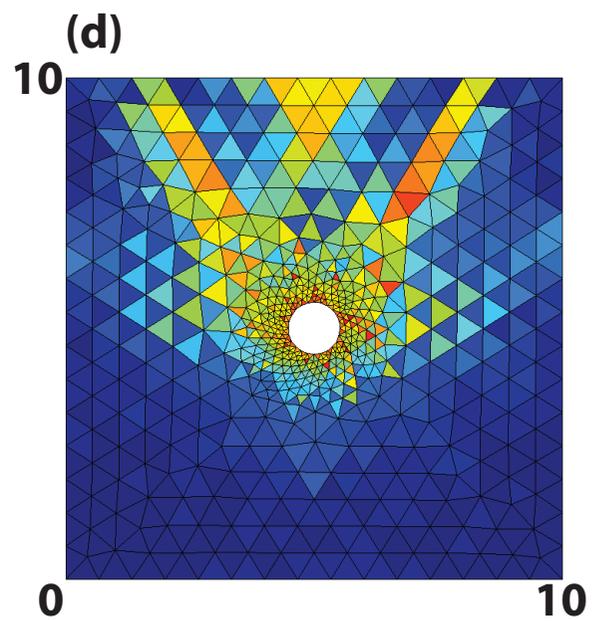
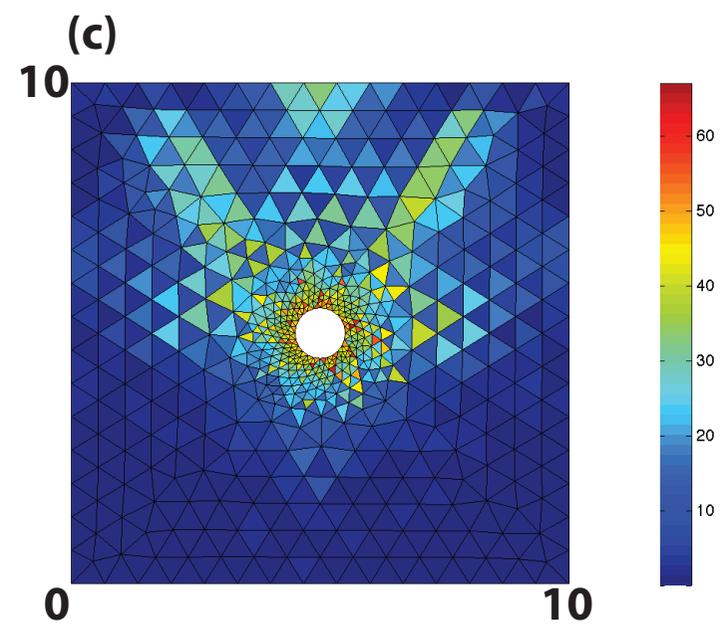
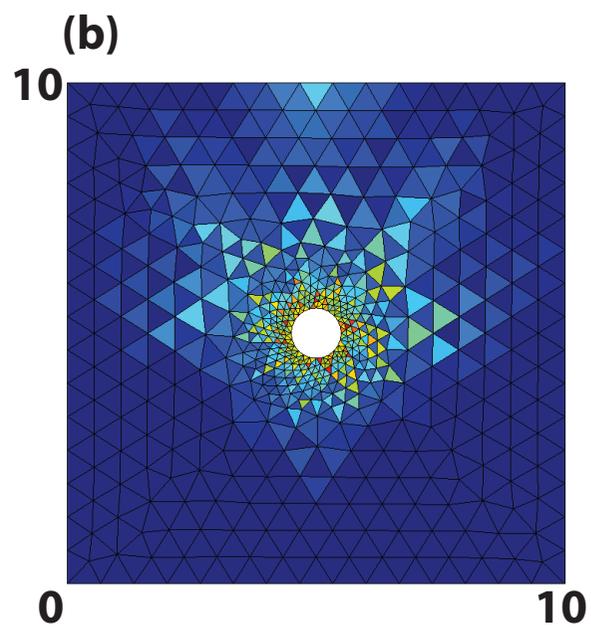
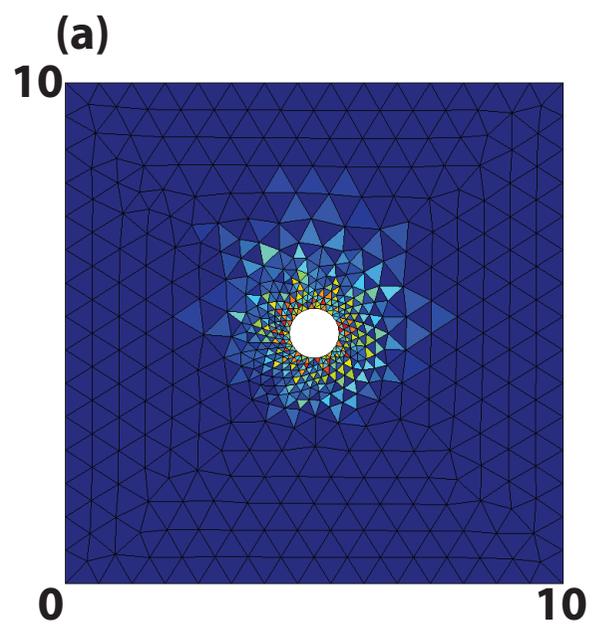


Figure4.



Figure 5.

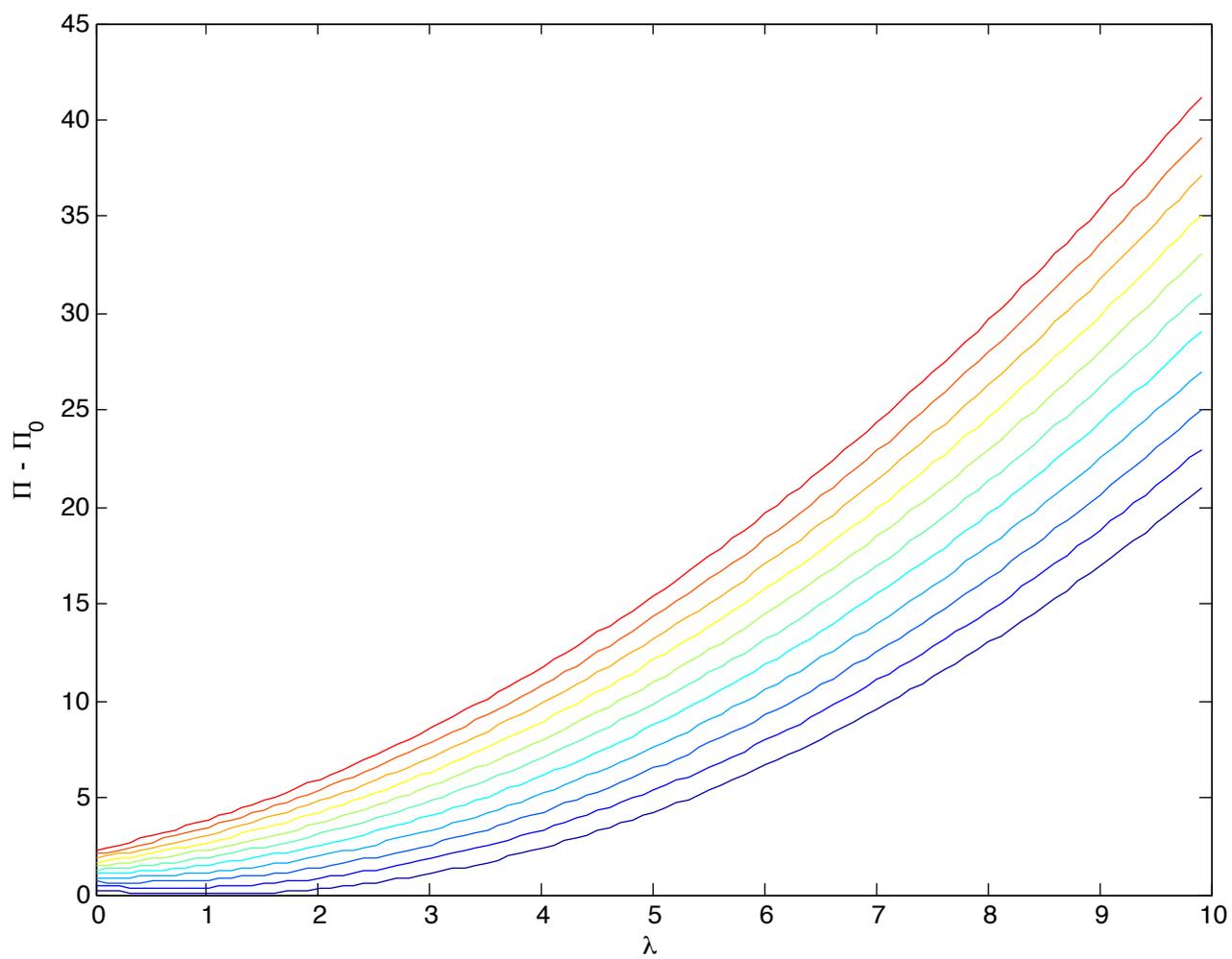


Figure6.

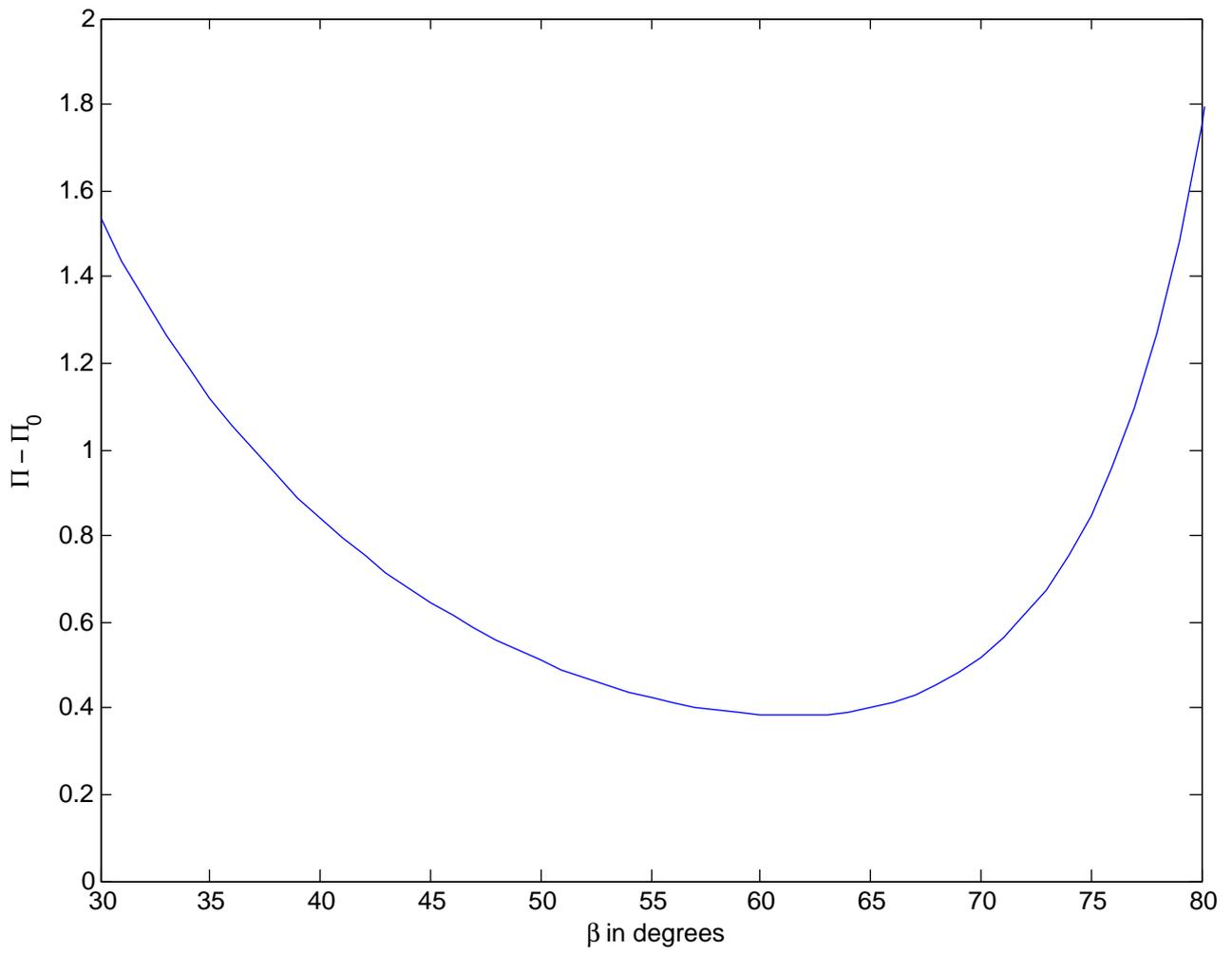
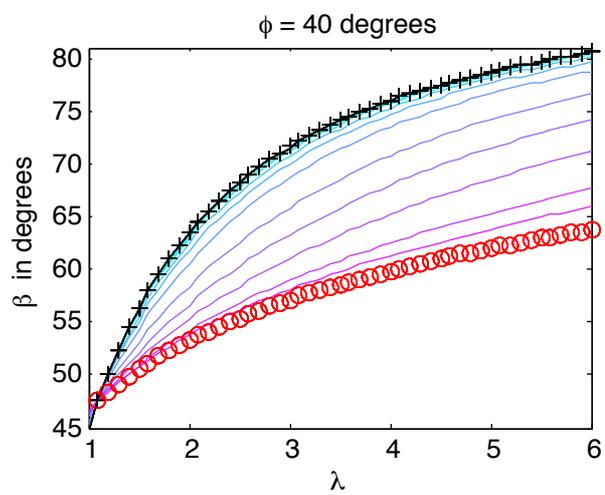
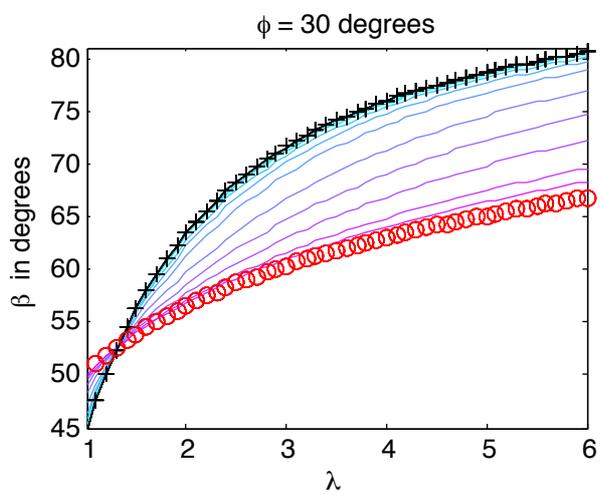
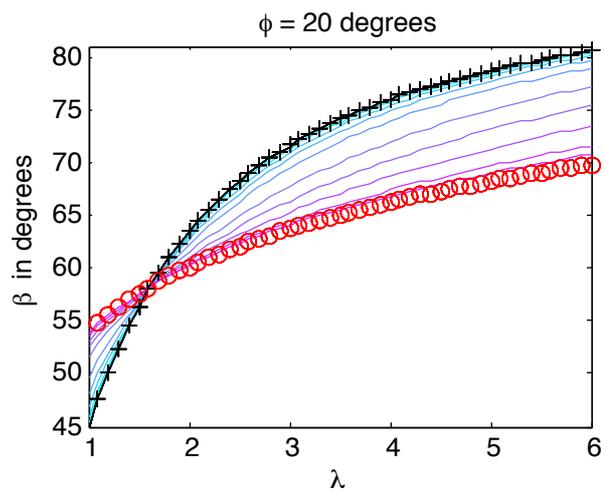
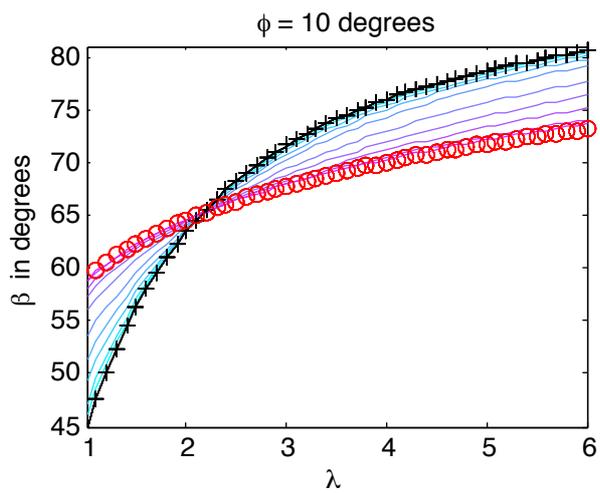


Figure 7.



**Figure8.**

