Asymmetric impacts and over-provision of public goods

Louis-Gaëtan Giraudet*[1] and Céline Guivarch[1]

[1]CIRED, Ecole des Ponts-ParisTech, 45 bis avenue de la Belle Gabrielle, Nogent-sur-Marne Cedex 94736,
France

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Abstract

We elicit simple conditions for an old puzzle – over-provision of a public good. An asymmetric public good that benefits some contributors while harming others is subject to both free riding and free driving. Even though aggregate impacts are net positive, it can be over-provided if free drivers face provision costs that are sufficiently lower than free riders'. Asymmetric impacts further impose restrictions on Hicks-Kaldor improvements. We establish these results in a parsimonious model that can easily be applied to a variety of so-called NIMBY problems, for instance new public infrastructures and global warming mitigation.

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*Corresponding author: `giraudet@centre-cired.fr`

1 Introduction

The ordinary prediction of the theory of public goods is that a non-rival, non-excludable commodity will be subject to free riding, hence under-provided in equilibrium (Samuelson, 1954; Musgrave and Musgrave, 1973). This prediction may fail in a variety of empirically-relevant, yet little-studied, contexts. In this note, we propose a simple model that provides a better understanding of the causes and welfare implications of an anomaly – over-provision of a public good, under-provision of a public bad – as well as a handy tool for its empirical identification.

Our model involves two key ingredients: heterogeneity in individual provision costs and, crucially, asymmetry in impacts from the public good. The latter is defining of so-called 'not-in-my-backyard' (NIMBY) problems, in which some contributors benefit from the public good while others are harmed. The problem is typically posed by public infrastructures such as roads, airports, waste treatment plants, which locally generate aesthetic, air and noise pollution. One well-documented example is irrigation dams in India. Duflo and Pande (2007) show that while every farmer in the community may be willing to gain access to irrigation, the dam has side effects on agricultural production – negative upstream and positive downstream. Global warming is another important example. While virtually every individual on the planet enjoys energy services from the combustion of fossil fuels, it is increasingly documented that the ensuing increase in atmospheric temperature may produce benefits in high latitudes and damage in low latitudes, essentially through opposite effects on agriculture (Mendelsohn and Massetti, 2017) or heating and cooling expenditures (Davis and Gertler, 2015). Heterogeneity in provision costs, the second ingredient of our model, is even more evident. Two communities differing in their exposure to impacts may well differ in other respects – wealth, technology endowment, etc. For instance, the contributors most vulnerable to global warming tend to be poorer, both within (Hallegatte et al., 2015) and across countries (Althor et al., 2016).

Building on a parsimonious linear-state, two-player structure, our model produces two results. First, we find that when impacts are asymmetric, one player free rides in equilibrium while the other free drives. If the former player faces higher provision costs than her competitor, free driving can exceed free riding, thereby leading to excessive aggregate provision, even though impacts are net positive. Second, we find that under these conditions, net compensation, if needed to establish the social optimum, can flow from either player to the other. In the domain where aggregate provision is insufficient, in contrast, compensation systematically flows from the positively impacted player to the negatively impacted one.

Our contribution is two-fold. First, we make a connection between an early, somewhat forgotten literature on anomalous provisions of public goods and an emerging literature on free driving in greenhouse gas (GHG) emission abatement. The former was initiated by Buchanan and Kafoglis (1963) through a numerical counter-example involving immunization. Despite subsequent developments,[1] the anomaly-generating conditions remained poorly understood, let alone empirically investigated. More recently, several studies have pointed to geoengineering as a free-driving technology allowing countries to mitigate GHG emissions in excess of what would be privately optimal (Weitzman, 2015; Manoussi and Xepapadeas, 2015; Heutel et al., 2016). Our modeling structure allows us to reconcile both insights by characterizing in a simple manner how the interplay between free riding and free driving can cause anomalous provisions. Second, we propose a model flexible enough to be brought to the laboratory and the field in order to assess the empirical significance of anomalous provisions of public goods.

2 Model

2.1 Set-up

We consider provisions of private goods $g \geq 0$ by two players, i and $-i$, which together produce a non-rival, non-excludable good G:

$$G = g_i + g_{-i}. \tag{1}$$

Player i derives some linear-quadratic value $V_i(g_i)$ from her provision of the private good:

$$V_i(g_i) \equiv \alpha_i g_i - \frac{\beta_i}{2} {g_i}^2, \tag{2}$$

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[1] Williams (1966) noted that the anomaly required some agents to over-contribute while others under-contribute, without further specifying the determinants of this dichotomy. Vincent (1969) pointed to economies of scale in external effects as a necessary condition for the anomaly, unless externalities have opposite signs – a condition deemed implausible by the author. Diamond and Mirrlees (1973) found sufficient conditions to rule out the anomaly which express own-consumption preferences and normality of demand. Buchholz and Peters (2001) later found these conditions to be too restrictive. Rather, they emphasized skewness in the distribution of incomes as a key determinant of the anomaly. Lastly, Sandmo (1980) noted coincidences between the anomaly and the notion of stability of the long-run aggregate demand function.

with parameter α_i reflecting amenities and $\beta_i > 0$ reflecting provision costs (which can be determined by wealth, technology endowment, etc.). In addition, she is impacted by the public good through an external effect $X_i(G)$:

$$X_i(G) \equiv \gamma_i G, \tag{3}$$

with parameter γ_i, of unspecified sign, reflecting individual (or local) impacts. The aggregate (or global) impact of the public good is $\gamma_i + \gamma_{-i}$. The players' utility functions are common knowledge and defined as:

$$U_i(g_i, G) \equiv V_i(g_i) + X_i(G). \tag{4}$$

The model has a very parsimonious structure, with separability of amenities, provision costs and impacts, and linearity of the latter. Importantly, we allow the sign of impact parameters γ to differ across players – an extreme form of heterogeneity we refer to as *asymmetry.* The problem is one of *pure* public good if g does not generate private amenities ($\forall i \ \alpha_i = 0$) and of *impure* public good otherwise (Cornes and Sandler, 1984; Kotchen, 2006).

In the case of a dam, g could reflect the services privately purchased from the dam (e.g., electricity, irrigation) and G the total capacity of the dam – a proxy of its impacts on agriculture, plausibly negative upstream and positive downstream. In the case of anthropogenic global warming, g could be a fossil fuel input to energy services (e.g., mobility, building weatherization) and G the build-up of greenhouse gases resulting from the combustion of fossil fuels – a proxy of atmospheric temperature, a modest rise of which plausibly has negative impacts in low latitudes and positive ones in high latitudes.

2.2 Taxonomy

Based on the sign and magnitude of impact parameters, we propose the following taxonomy of public goods (broadly speaking):

- $\gamma_i + \gamma_{-i} > 0$: G is a public *good* (strictly speaking).
- $\gamma_i + \gamma_{-i} < 0$: G is a public *bad.*
- $\gamma_i \gamma_{-i} > 0$: G is a *universal* public good or bad;
  - if $\gamma_i = \gamma_{-i}$, it is *homogeneous*,

  - otherwise, it is *heterogeneous.*

- $\gamma_i\gamma_{-i} \leq 0$: G is a *restricted* public good or bad;
  - if $\gamma_i = -\gamma_{-i}$, it is *symmetric*,
  - otherwise, it is *asymmetric.*

While most analyses of impure public goods focus on the universal case, our more general model accommodates empirically relevant asymmetries in impacts.[2] Figure 1 illustrates the different regimes in the (γ_i, γ_{-i}) plane. In the remaining of the note, unless notified, we use 'public good' in its strict sense of a non-rival, non-excludable good generating positive impacts.

3 Predictions

3.1 Provisions

We start by comparing the Nash equilibrium of the game, N, to the social optimum, S. The former emerges through own-utility maximization and the latter through joint-utility maximization. First-order conditions lead to individual provision $g_i^N = (\alpha_i + \gamma_i)/\beta_i$ in equilibrium and $g_i^S = (\alpha_i + \gamma_i + \gamma_{-i})/\beta_i$ at the optimum. These provisions are positive as long as impacts are not too harmful ($\gamma^i > -\alpha_i$ and $\gamma^i + \gamma^{-i} > -\alpha_i$, respectively).

The players exert reciprocal externalities γ_{-i} on one another (but internalize own-externalities γ_i). Player i is said to *free ride*, or under-contribute, if facing a positive externality, and *free drive*, or over-contribute, if facing a negative one ($\gamma_{-i} > 0 \Leftrightarrow g_i^N < g_i^S$). The provision of the aggregate G will be determined by impact and cost parameters:

Proposition 1 *Provisions of the public good in the Nash equilibrium (N) and the social optimum (S) compare as follows:*

$$G^N \geq G^S \Leftrightarrow \beta_i\gamma_i + \beta_{-i}\gamma_{-i} \geq 0. \tag{5}$$

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[2] Dekel et al. (2017), to whom we borrow the term 'universal,' use 'Potential Pareto Public Good' to designate what we refer to as 'asymmetric public good.' Our terminology is meant to avoid confusion between Pareto and Hicks-Kaldor improvements, which, as we show in Proposition 2, can both occur in asymmetric cases.

Remark 1 *Let us call the equilibrium provision 'ordinary' if* $G^S - G^N$ *is of the same sign as* $\gamma_i + \gamma_{-i}$*, and 'anomalous' otherwise. The provision is ordinary whenever impacts are universal* ($\gamma_i\gamma_{-i} > 0$)*. Impacts need to be asymmetric* ($\gamma_i\gamma_{-i} < 0$) *and provision costs heterogeneous* ($\beta_i \neq \beta_{-i}$) *for the anomaly to occur. These predictions do not depend on amenity parameters* α_i *and hence equally apply to pure and impure public goods.*

When impacts are asymmetric, one player free rides while the other free drives. If provision costs β are identical, which effect dominates is directly given by the aggregate impact $\gamma_i+\gamma_{-i}$. Equilibrium provision is therefore ordinary. This implication no longer holds if relatively high impacts (in absolute value) are paired with relatively low provision costs. Assuming for instance $\gamma_i > 0$, an asymmetric public good will be over-supplied if $-(\beta_i/\beta_{-i})^{-1} \leq \gamma_i/\gamma_{-i} \leq \min(-1, -(\beta_i/\beta_{-i})^{-1})$ and an asymmetric public bad will be under-supplied if $-1 \leq \gamma_i/\gamma_{-i} \leq \max(-1, -(\beta_i/\beta_{-i})^{-1})$. This is illustrated in Figure 2.

In the case of a dam, the upstream community will be willing to contribute too little to the construction of the dam while the downstream community will be willing to contribute too much. If, in addition, provision costs are lower downstream than upstream (perhaps because the downstream community is wealthier, or equipped with better technology), the dam will be over-sized; it will be under-sized with the reversed cost distribution. Likewise, global warming can be seen as an asymmetric public bad generating, at least in the short term, important damage in low latitudes and modest benefits in high latitudes. At odds with the ordinary prediction, our theory predicts that global warming could be under-supplied provided that emission reduction costs are lower in low latitudes than in high latitudes – a condition that however lacks empirical support.[3]

3.2 Welfare

We now examine the welfare implications of moving from the Nash equilibrium to the social optimum. The reciprocal externalities can be internalized by a pair of prices $(p_i, p_{-i}) = (-\gamma_{-i}, -\gamma_i)$ each imposed on every unit of the associated player's provision.[4] In a Pigovian

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[3] Emission reduction cost estimates are inherently difficult to obtain and therefore vary widely. Take India, a major emitter expected to severely suffer from global warming. According to some estimate, its emission reduction costs are among the lowest (Nordhaus, 2015) while another suggests it is among the highest (Aldy et al., 2017).

[4] Player i's utility is then lessened by $p_i g_i$. Matching the ensuing equilibrium provision, $(\alpha_i - p^i + \gamma_i)/\beta_i$, with the socially optimal one yields $p_i = -\gamma_{-i}$. Unless the players face identical impacts ($\gamma_i = \gamma_{-i}$), prices

perspective, prices are implemented as domestic taxes and subsidies by a central authority. In a Coasian perspective, implicit prices are self-imposed by cooperating players. Either way, prices may be accompanied by inter-player compensations, depending on conditions elicited below.

The variation of utility to player i is $U_i^S - U_i^N = -{\gamma_{-i}}^2/(2\beta_i) + {\gamma_i}^2/\beta_{-i}$. By construction, $U_i^S + U_{-i}^S \geq U_i^N + U_{-i}^N$. Cooperation is said *Pareto-improving* if both $U_i^S \geq U_i^N$ and $U_{-i}^S \geq U_{-i}^N$. It is said *Hicks-Kaldor-improving* if only one of the latter two conditions holds, thus requiring the player whom cooperation makes better-off, say i, to transfer a positive amount τ to the worse-off player, say $-i$, such that $U_{-i}^N - U_{-i}^S < \tau < U_i^S - U_i^N$. Working out the different cases,

Proposition 2 *Moving from equilibrium to the optimum is*

- *Pareto-improving if* $\frac{1}{\sqrt{2\beta_i/\beta_{-i}}} \leq \mid \gamma_i/\gamma_{-i} \mid \leq \frac{2}{\sqrt{2\beta_i/\beta_{-i}}}$,
- *Hicks-Kaldor-improving otherwise. Specifically,*
  - *Player $-i$ should compensate Player i if the left inequality is violated;*
  - *Player i should compensate Player $-i$ if the right inequality is violated.*

Remark 2 *In the domain where the equilibrium provision of an asymmetric public good is ordinary (see Proposition 1), net compensation, if needed, systematically flows from the positively impacted contributor to the negatively impacted one (and vice versa with an asymmetric public bad). In all other regimes, net transfers can go either way.*

To grasp the essence of the proposition and the associated remark, let us start by considering a public good provided by fully identical players, characterized by $\beta_i = \beta_{-i}$ and $\gamma_i = \gamma_{-i} > 0$. Then, cooperation is Pareto-improving: it is in both player's interest to overcome the free-riding problem. This holds as long as cost and impact ratios are close to unity. If, now, Player i's marginal provision cost is less than half that of $-i$'s, she free rides substantially less than her competitor, to a point where cooperation requires the former to compensate the latter. But if, with that same cost distribution, player i's impact is twice

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p_i and p_{-i} will differ.

that of $-i$'s, the former is now the biggest free rider, and the direction of transfers is reversed. More generally, for any cost ratio, if transfers are needed, they can go either way, depending on the impact ratio; reciprocally, for any impact ratio, transfers can go either way, depending on the cost ratio. This is illustrated in Figure 3.

In asymmetric regimes, this two-way possibility is restricted to the domain of anomalous provisions. In the domain of ordinary provisions, in contrast, transfers only occur in one direction. Recall that for an asymmetric public good to be under-supplied, free riding must exceed free driving. Superimposing Figures 2 and 3, we see that the domain where this is the case intersects with the domain in which cooperation either is Pareto-improving or requires i to compensate $-i$, but not with that commanding $-i$ to compensate i. In contrast, in the domain where the public good is over-supplied, for free driving exceeds free riding, both directions of transfers are possible.

That compensation flows from those contributors exposed to positive impacts to those exposed to negative impacts might sound intuitive. Yet our analysis suggests this orientation should not be systematic. The disconnect between intuition and our prediction might stem from a natural tendency to characterize winners and losers in the light of external effects, instead of considering broader utility functions as economic reasoning would command.

4 Discussion

Our model offers a simple framework to better understand the causes and welfare implications of an economic anomaly – over-provision of a public good (and under-provision of a public bad). Its results are robust to pure and impure public-good regimes. It however rests on a parsimonious structure, of which we now question the generality and applicability.

First, our model involves two players (or groups thereof) with marked differences in provision costs and impacts. In practice, the two-player setting seems relevant to the empirical analysis of NIMBY problems, which generally oppose two communities (e.g., upstream versus downstream, or central versus peripheral). It is less relevant to global warming, which involves a multitude of contributors that differ from one another in many other respects than their vulnerability to climate-change impacts. Still, our two-player model can be useful to study bilateral agreements between large GHG emitters, such as that between China and the United States,[5] which played an important role in the international negotiations leading

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[5] `https://obamawhitehouse.archives.gov/the-press-office/2014/11/11/us-china-joint-`

to the Paris Agreement.

Second, the model features separable terms and linear impacts. Such a flexible structure conveniently lends itself to both empirical and experimental tests. Laboratory experiments testing the theory of public goods generally involve linear payoff functions and symmetric players. Dekel et al. (2017) have recently conducted novel experiments involving the kind of heterogeneity we consider in this note. Yet the authors do not examine the possibility of anomalous provision. Bringing our simple model to the lab could fill this gap. The model could also enrich empirical analyses with welfare assessment, for instance by eliciting whether, given the distribution of wealth upstream and downstream, dams such as those studied by Duflo and Pande (2007) are under- or over-sized. While those approaches seem fit for a variety of NIMBY problems, the model should be confined to the analysis of short-term, moderate temperature variations when it comes to global warming, which is known to have non-linear impacts on economic productivity (Burke et al., 2015).

Third, and relatedly, our results are established within a static framework. Again, this might be relevant to a variety of NIMBY problems but less so to global warming, an intrinsically dynamic problem. As long as the structure of the problem remains separable and linear, however, the predictions established in the static framework carry over to a dynamic framework (Giraudet and Guivarch, 2016). Introducing non-linearities would nevertheless raise new questions inherent in dynamic games.

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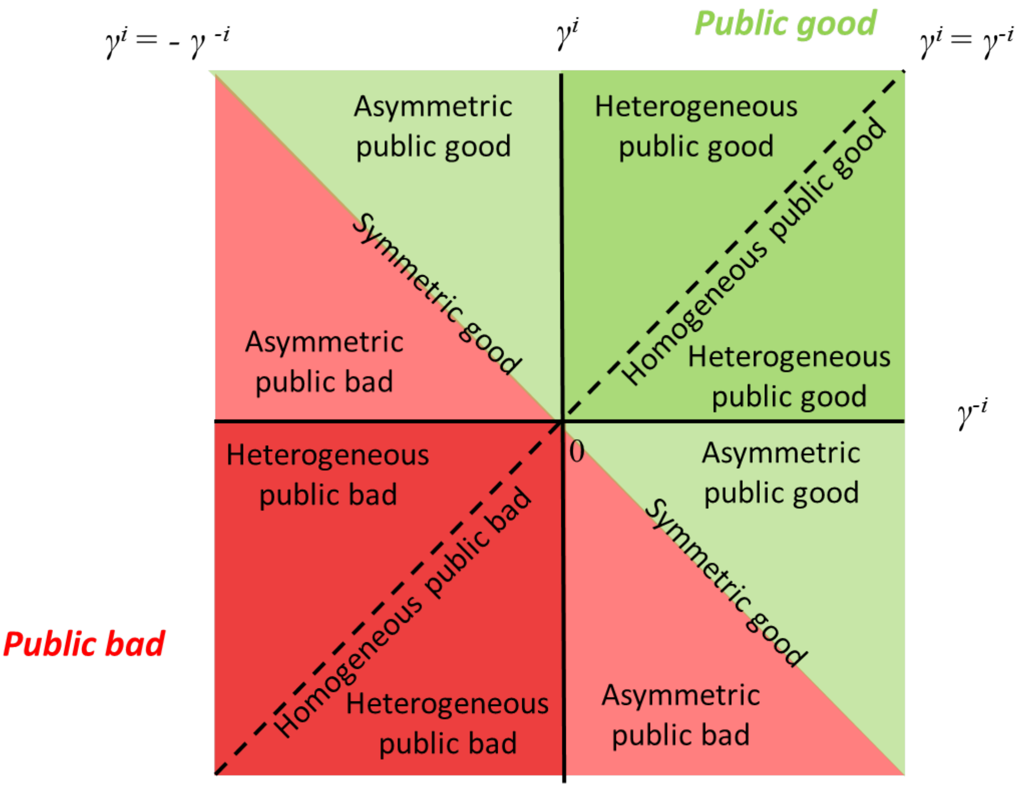

Figure 1: Taxonomy

Notes: The origin corresponds to a pure private good.

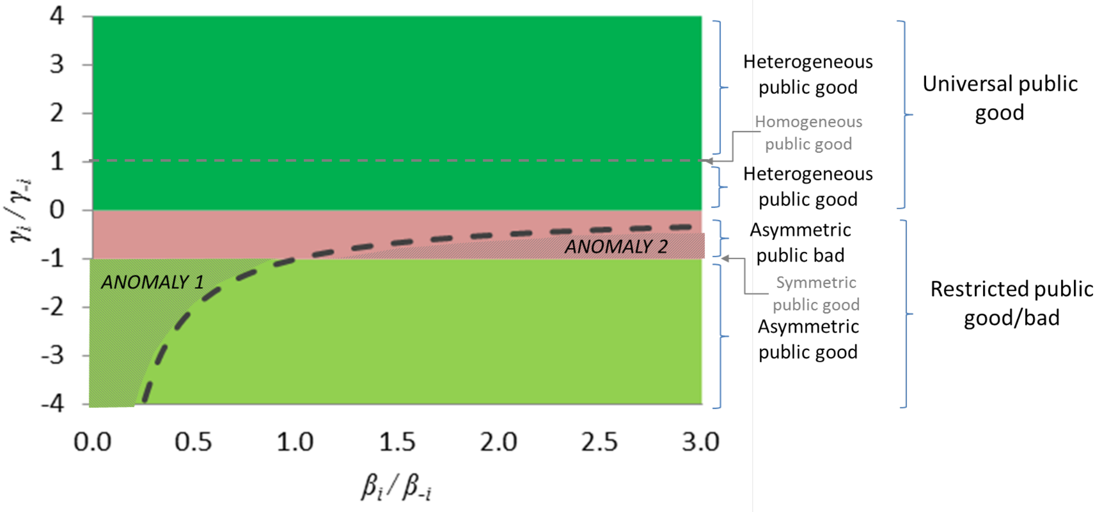

Figure 2: Equilibrium provisions

Notes: Assumption: $\gamma_i > 0$. With $\gamma_i < 0$, the picture would have a heterogeneous public bad on top, an asymmetric public good in the middle and an asymmetric public bad at the bottom. ANOMALY 1 corresponds to over-provision of a public good. ANOMALY 2 corresponds to under-provision of a public bad.

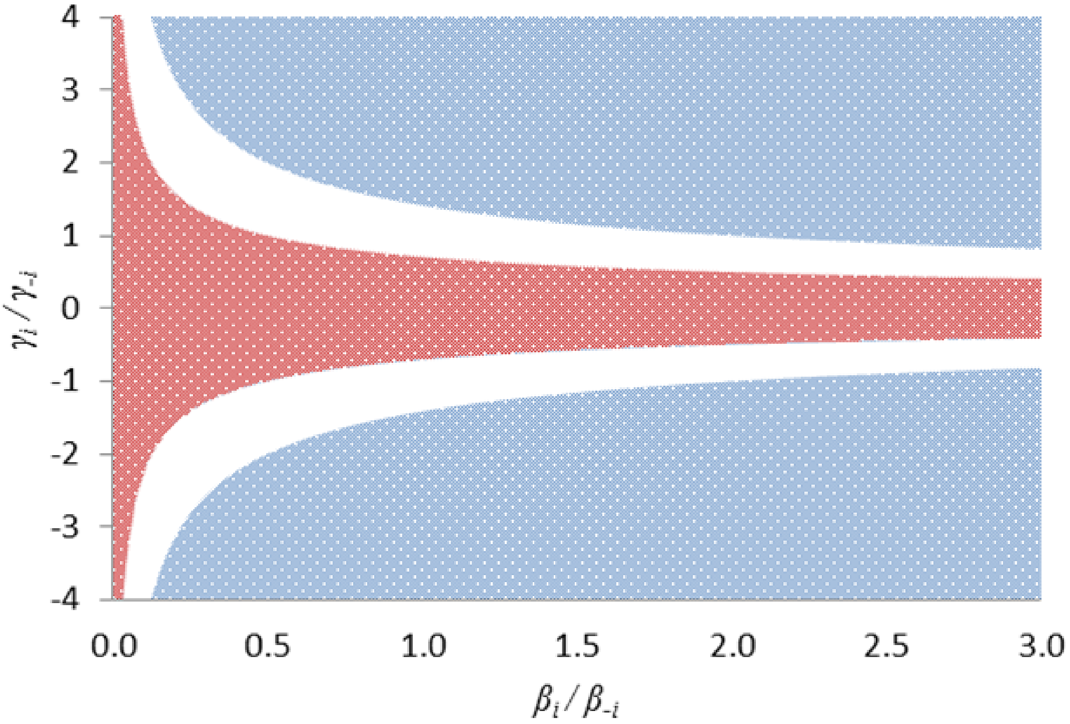

Figure 3: Distributional impacts of cooperation

Notes: The white zone corresponds to Pareto improvements. The sense of the arrows indicate the sense of transfers.