Design and construction of a shell-nexorade hybrid timber structure
Cyril Douthe, Romain Mesnil, Olivier Baverel, Tristan Gobin, Xavier Tellier, Nicolas Ducoulombier, Nicolas Montagne

To cite this version:
Cyril Douthe, Romain Mesnil, Olivier Baverel, Tristan Gobin, Xavier Tellier, et al.. Design and construction of a shell-nexorade hybrid timber structure. IASS Annual Symposium 2018 Creativity in Structural Design (IASS 2018), Jul 2018, Boston, MA, United States. hal-01852280
Design and construction of a shell-nexorade hybrid timber structure

Cyril DOUTHEa, Romain MESNILa,b,*, Olivier BAVERELa,c, Tristan GOBINa,d, Xavier TELLIERa, Nicolas DUCOULOMBIERa, Nicolas MONTAGNEa

*a Laboratoire Navier, UMR 8205, Ecole des Ponts, IFSTTAR, CNRS, UPE, France
romain.mesnil@enpc.fr
b Ecole des Ponts ParisTech, France
c Laboratoire GSA, ENS Architecture Paris Malaquais, France
d HAL Robotics Ltd

Abstract
This paper presents the design and fabrication of an innovative structure, spanning 6.5m with a structural thickness of 12cm. The structural system derives from nexorades, also called reciprocal frames by some authors, obtained by a novel form-finding method based upon translation of the edges. It also allows for the covering with planar facets if the geometry before transformation is covered with planar facets. This property is used for the pavilion, covered with planar timber panels used as bracing elements. The benefit of this structural system, called shell-nexorade hybrid is discussed through a comparative study between the braced and un-braced nexorade (the stiffness is multiplied by ten adding only 30% mass). Finally, the fabrication process, achieved through robotized milling of the beams, is shown, illustrating the potential of pluri-disciplinary approach for conceptual design.

Keywords: Nexorade; reciprocal frame; PQMesh; form finding; robotised manufacturing; timber pavilion; 3Dphotogrammetry

1. About the design of nexorades
1.1 Existing form-finding methods
Nexorades, also called reciprocal frame or reciprocal structures in the literature, are constituted of load bearing members which support each other in a cyclic manner along their side rather than at their extremities [1,2]. This specific arrangement of members’ end simplifies connections because, contrary to most structural systems, members are here always connected by pairs. This simplification of connections goes with an increased complexity of the structure geometry because it often induces eccentricities between member neutral axes… To cope with this geometric complexity, a form-finding step in generally introduced in the design of nexorade to determine a configuration that will fit a desire form and constructional and topological constraints induced by this specific structural morphology. A wide variety of methods have been used: genetic algorithm by Baverel et al. [2], dynamic relaxation by Douthe and Bavere[3], analytic and semi-analytic methods [8], or even iterative least-square optimisation [5,6]. Each method is a response to a specific problem, generally adapted to the treated geometry and to the chosen connection detail. According to most authors interested in the form-finding of such structures, their algorithm performs well, so that creativity is not limited by the method but rather by the allowed design space which is determined by the technological solution envisioned at the beginning of the whole design process. Varying the connection detail seems thus a promising track for renewing the design of nexorade through a renewing of their design space.

1.2 Main aspects of the structural behaviour
Geometric complexity is however not the only issue in the design of nexorade. Indeed, it has been shown that, from a structural point of view, the arrangement of members in a cyclic manner increases bending
moment in the structure and by there the local rotation of nodes [2,3,9]. Furthermore, Brocato [7,8] demonstrated that, when employed in doubly curved structures, nexorades should map shapes that are optimised by minimising the axial forces, because membrane actions are carried by axial forces in the members that causes strong bending moments in the weak axis due to their local arrangements. The design of double curved nexorades is therefore counter-intuitive. The control of nodes or fans local rotation is thus a key issue to the structural design of nexorades. Adding bracing within the plane of the structure is a natural idea, but from a practical point of view, the geometric complexity, especially eccentricities induced by the member arrangement, generally makes the uses of covering panels for bracing very hard (except for very regular geometries like revolution surfaces).

1.3 Proposed hybrid timber structure
To tackle this issue of introducing planar quads for bracing, the authors took advantages of recent developments in the field of architectural geometry to derive a new structural system, that they called shell-nexorade hybrid, from a surface initially meshed by planar quad. This new structural system takes also advantage of the possibilities offered by robotic manufacturing which allows for a renewing of connection technologies (here tailor made T-joints). The purpose of the present paper is hence to show how creativity in structural design is the result of theoretical setting of a design problem (here the parametrisation of the transformation into a nexorade via translation presented in section 2) and technological choices made at the beginning of the process (section 3 for the structural design and 4 for the robotic manufacturing). The whole process is illustrated by a 50m² wooden pavilion shown in section 5 and figure 1.

![Figure 1: Build’in pavilion 2017, prototype of wooden shell-nexorade hybrid.](image)

2. Form finding
The form finding of nexorades is here based on the method of translations, which starts from a given mesh and then translates and extends all members to create fans [0]. The goal of the form-finding is to identify all individual translations and scale factors that will lead the desired engagement lengths and eccentricities. It is shown in [10] that the eccentricities and engagement lengths depend linearly on the transformation parameters. The proposed shape-generation method is thus based on the creation of a linear subspace, a well-known topic in computer graphics, which has been applied to the modelling of polyhedral meshes with planar quads recently by the authors [11]. Formulating the form-finding of nexorade in this way eases hence the design exploration and local fitting problem (like surface fitting, boundary fitting, target engagement lengths or target eccentricities) which are expressed with linear least square methods and solved instantly.

The proposed design space is related to parallel transformations, which preserve facet planarity [10]. Applying our method to a mesh with planar facets yields a nexorade with nearly planar panels. Such a mesh can be obtained by solving a fitting problem with the marionette technique [11]. This is shown on the left of figure 2. The reference geometry of the pavilion (shown in light grey) is an iso-parametric quad mesh obtained from a NURBS surface made of 6 patches, figuring a kind of trinoid, a tunnel with three equal entrances. The final mesh with planar quads is shown in black. One can see that it is very close to the initial mesh, which indeed was chosen on purpose to approximately follow curvature lines.
After reciprocation (see middle image of figure 2), some eccentricities are necessarily introduced. The right of figure 2 shows a close-up of a fan: the thick black lines correspond to the beam neutral axes, while the grey lines correspond to planar panels. One has thus to deal with the relative eccentricities between members (white dots), but also with the eccentricities between beams and panels (orange thick dots).

Figure 2. PQ-mesh found with the marionette technique (left), reciprocal frame obtained by translation of the edges (middle): the opposite edges of the quads remain co-planar.

In practice, structural designers aim at controlling two geometrical quantities:
- engagement lengths which should be large enough to ease assembly but not too large to limit bending moments induced by the lever-arm of normal forces in the members and
- eccentricities between members axes whose target length will strongly depend on the technology used for connections but who will be sought constant in most cases. We will see in next session, that the T-joint of the pavilion allows for a target null eccentricity whereas many previous applications used a standard connector and had therefore a target eccentricity equal to the size of the connector [2,3].

Engagement lengths and eccentricities are generally competing objectives, so that one has to balance the relative weight of the quadratic functional linked with each objective before running the optimisation program. In the case of the pavilion, current engagement lengths are sought around 0.2 m while those of the central 6-valent node are sought around 0.25 m (because of sharper angle between members). The eccentricities are simply minimised: the lower, the better because the higher the effective height of the connection (see next section).

3. Structural analysis
3.1. Design of the structure

The new form-finding method presented in previous section is thus aimed at designing nexorades covered by planar quads used for bracing. It had been shown in [3] that alternating the styles of fans (leftwards or rightwards) leads higher bending moments in the members. It was also shown in [11] that this alternation leads a better faithfulness to the initial form (i.e. to the mesh before transformation into nexorade) and higher regularity of fans. Yet, if the nexorade is going to be braced by the panels and if the connection is effective, the bending deformation of the members in the tangent plane of the surface will be blocked, so that the inconvenient of the style alternation can be mitigated. It is thus of primary importance that the panels are connected regularly along the member edge (in the case of the pavilion a 4 x 40 mm screw will be set every 100 mm). This being said, an alternate style of fan was thus chosen for the pavilion.

The efficiency of the structure was then investigated within the framework of Karamba3d. The size of the engagement length was first tested (ranging from 200 mm in average to 500 mm), then the influence of member connection stiffness and finally the influence of the panels on the global behaviour. Prior to the study with the panels, a convergence study is also conducted and lead to an acceptable accuracy for approximately 200 triangular elements by panel. All parametric studies were conducted for three kinds of loading: self-weight, snow load (asymmetric) and wind load (only horizontal forces). Non-linear
static analysis and linear buckling analysis were only considered for the design and verification of the final design of the pavilion.

Typical results of the analyses are presented in figure 3. The amplitude and direction of reaction forces and connection forces were studied, as well as displacements. It was shown that globally before the installation of the panel (in the provisory state), structures with larger engagement length were softer (about 2 times) but also that this difference reduces to almost nothing once the panels are installed, no matter the distribution of the loading. This demonstrates clearly that the panels can be considered as effective bracing elements and confer a shell like behaviour to the structure. The parametric study on the stiffness of the connectors showed then that there is a threshold above which the connection forces can be considered constant and the connectors as efficient. Practical application to the design of the pavilion under Eurocode 5 and fabricator technical agreement showed then that this threshold can be reached easily with reasonable size and density of screws.

Figure 3: (left) Reactions under wind load, the circle radii are proportional to the amplitude of the reaction forces, white circle indicate uplift; (right) Influence of connection stiffness between beams on connection forces

3.2. Design of connections
The T-joints considered for the pavilion consist of end-grain assembly along the side of a member (see figure 4), inspired by [12]. All assembly share the same technological characteristics based on the same construction principles: a 5 mm tenon/mortise system to guide the workers during assembly and inclined pilot holes to insure that screws are set in the middle plane of the members. Due to the complex geometry of the pavilion, the geometry of each assembly is unique. This geometry is defined to satisfied recommendations for distances to the edges and distances between screws and to maximise the lever arm between the screws. The integration of the connection design into a completely parametric environment allowed for a verification of each connection individually based on the effective forces induced by external forces, the actual eccentricities between the member axes and the actual angles of the connecting members. This verification was based on the strength given by the fabricator recommendations for the specific kind of screws used in the pavilion (VG Plus screws by Würth that allow for traction forces in end-grain connections), strength which were confirmed experimentally by destructive tests conducted in the laboratory (see figure 4 right).

Figure 4: (left) Typical screwed T-joint between two members; (right) Rupture test of T-joint.
Hence, at the end, one can say that the pavilion is a safe wooden temporary building that could pass external control check. The goal was indeed here to demonstrate that the proposed design methodology, the associated robotic manufacturing and the proposed structural system comply with recommendations and can be used for realistic applications.

4. Construction of the pavilion

4.1 Design and fabrication of the elements

The fabrication process presented here is based on several constraints. No lifting equipment was available, which limited the possibilities to have large prefabricated components: all the beams were to be assembled on-site manually. The structure was also to be checked through classical calculations, derived from Eurocode 5 and technical agreements. Screwed connections are suited for these matters, but they require guides during assembly. We choose to use tenons and mortises to place the beams correctly. Pilot holes had to be made in the beams to avoid timber splitting and to facilitate the assembly. Additional grooves had also to be milled on the top surface to insert the bracing panels. This all lead to a significant complexity of the resulting beams whose geometry solves all the problem of the structure and envelope. All members are therefore unique produced with help of the robotic Build’in platform of the Ecole des Ponts Paristech.

The platform layout is constituted of two robots, shown in figure 5. One is mounted on a 9 m track and referred as gripper robot. The other is fixed and referred as fixed robot. Those two robots can work together or independently. All end effectors are constituted by automated tool changer and can mount specific tools, which allows us to easily swap the tools between the two robots if needed. For the project we use a milling head mounted on fixed robot and a pneumatic gripper mounted on gripper robot. Various stationary tools are dispatched along a side of the track where gripper robot has access.

In detail, eight independent operations are identified in the fabrication of the beams. (1) At picking step, the gripping robot get the rough material on a dispenser. This step allows the alignment of the beam to the local coordinate system of the robot end effector. (2) Directly after, the shaping, where each beam is cut to an approximate size, is done on a stationary circular saw. Then the gripping robot moves the beam to the common work space for a collaborative machining sequence. Several operations occur as follow: (3) the two tenons milling, (4) the two mortises milling, (5) the ruled surface milling, (6) the groove milling and finally (7) the bottom-surface milling. After those operations, the gripping robot moves for (8) drilling operations to the wood rooter. Finally, the beam is brought back to the dispenser. Because of the robot's mobility and the robustness of the clamping, the beam can navigate between the different workshops without compromising machining accuracy (± 1 mm).
4.2 Assembly of the structure
The pavilion consists of 102 beams and 48 plywood panels, manufactured off-site and assembled in-situ, so that the respect of fabrication tolerances is crucial for the assembly process of this highly hyperstatic structure. Indeed, the chosen connection does not allow for any tuning; geometric control is ensured through the tenon/mortise system and the adjustment of the bracing panels (milled with a tolerance of 2 mm on each side). The beams and panels weigh less than 5 kg each and can be assembled by two people without difficulty. The assembly sequence starts from the construction of a tripod issued from the central node, which requires temporary supports, as seen in figure 6. The structure is then built by cantilevering beams from the tripod.

![Figure 6. Different stages of construction: tripod with temporary supports (left), cantilevering structure (middle) and completed structure (right)](image)

The topographic location of the supports was set weeks before the construction and anchor plates with slotted holes were loosely screwed to the ground. While removing the temporary supports of the tripods, the structure matches these anchor plates within a range of few millimetres which could be recaptured easily by the slotted holes. This illustrates the precision of robotic fabrication and its potential in terms of sequencing of assembly and manufacturing. Only four days were required to mount the structure, with four people and no lifting equipment.

5. Proofing of the structure
5.1. Verification of the built geometry
The high hyperstaticity of the pavilion required a careful geometric control of each fabrication step:

- at member scale, about 8 measures (4 lengths and 4 angles) were checked with a millimetric accuracy (or equivalent 10 mrad);
- at connection scale, screws location and orientation were set off-site and 40 mm pre-drilled holes were open during the robotic manufacturing on each side of the connection;
- at panel scale, before screwing, the geometry of the four edge members lining the panel was adjusted so that the panel (whose diagonal length accuracy were systematically checked and within ± 1 mm) fitted within the 2 mm tolerance of the grooves;
- at structure scale, the anchor plates were fixed in advance on the ground by traditional triangulation method and to make up for position accuracy, the plates were equipped with slotted holes insuring ± 3 mm in each direction.

The fact that the structure could be mounted and fitted its support is already a proof of its general faithfulness to the initial design. Nevertheless, the building sequence starting from the tripod and progressing toward the free edges of the structure was chosen because in this way the accumulation of defaults will be easier to accommodate. It was thus necessary to check if the accuracy achieved on the fixed boundaries was still valid on the whole structure. A measuring campaign was thus carried out with a laser scanner and photogrammetric 3D reconstruction by our partners from ENSG. The two techniques were used for pedagogical purposes and comparison of their relative accuracy. The first 3D point cloud from the laser scanner was established after four positions of the machine to get a complete picture of
the inside and outside of the pavilion. The resulting accuracy is about \( \pm 1 \) mm. The second 3D point cloud was done from a set of 134 photos (5400x3600) and with the free photogrammetry software Micmac developed by our partners (see http://micmac.ensg.eu). The resulting accuracy is also about \( \pm 1 \) mm in every direction. A set of 25 stationary targets had been set around the pavilion in order to be localised by traditional tacheometer techniques and to allow for geo-referencing of both point clouds. Globally the two point clouds are identical with an average gap close to 1 mm.

The point cloud are then compared to the initial 3D model in Rhino. The average gap is about -6 mm (the 3D model being above the cloud). From the analysis of the signed distance between the mesh and cloud, one can clearly see that:

- the plywood panels which were unprotected have creep and present a residual deflection in the middle of approximately 4 mm relatively to the initial panel plane;
- the 3D model being unloaded, even by the self-weight, part of this gap can be explained by the action of gravity (2 mm at crown according to the mechanical model with Karamba3d);
- Deviation from the theoretical form increased as expected along the free-edge and around the extreme supports which were installed last.

Considering hence creep of panels and self-weight deflection, the quality of the execution at structural scale is globally very satisfactory.

### 5.2 Loading tests

Once the accuracy had been verified, loading tests were conducted. The structure being designed according to Eurocode recommendations, it is very stiff; displacements under service loads are below 1/200 of the span, which corresponds here to 3 cm approximately. This magnitude of displacement is only one order of magnitude higher than traditional 3D displacement measures obtained from tacheometer. In order to minimise the weight to be installed on the structure and to maximise the displacements, it was decided to submit the structure to an antisymmetric loading with ten water bags distributed on one of the three legs of the tripod (see figure 7). The load is applied progressively and displacements are measured at each step (on targets and prism with tacheometers and on the whole structure with laser and photogrammetry):

- first 20 L in each bag, uniformly (total load 200 kg);
- then 50 L more in the four bags on the right (total load 400 kg);
- then 50 L more in the two central bags (total load 500 kg);
- finally, 50 L are added to the last four bags (total load 700 kg).

![Figure 7: Structure under loading tests: [left] Load distribution. [right] Signed distance between structure and model by photogrammetry.](image)

It is worth remembering that numerically it was demonstrated that, thanks to the panels and the induced shell-like behaviour, the maximal deflection with the panel is about 5 times lower under non-symmetric loading and about 15 times lower under symmetric loading than the deflection of the structure without panels. It is worth also mentioning that the beams and panels have stayed unprotected under rain during 6 months before the test, and that therefore their Young modulus have dropped. Reverse engineering on
the loss of Young modulus based on calculated and measured displacements shows that the actual $k_{\text{def}}$ is close to 80\%, i.e. $E_{\text{actual}} = (1+k_{\text{def}})E_{\text{init}}$ (note that numerically it can be shown that as the structure is braced, the stiffness of the connectors do not play any significant role here). Comparison of the results shows also lower horizontal displacements experimentally than numerically (especially in the middle of the structure). Further investigation on the displacements and influence of connection are still under progress to explain this default in the prediction of the direction. This being said, one can say that globally the amplitude of displacement is well estimated by the model so that it can be concluded that the bracing with the panels is effective. It validates hence the relevance of shell-nexorade hybrid structure.

6. Conclusions
The authors presented an illustration of the cross influence of the theoretical formulation of problems and of the technological choices foreseen for practical applications on the creativity in structural design. They presented first a reformulation of the form-finding of nexorade by the translation method, showed that this method leads a linear problem under linear constraints for which usual optimisation method can be used. They have shown that this framework allows for the design of nexorade braced by planar quads, a structural concept that they called shell-nexorade hybrid. They presented then the design, construction and proofing of a 50m² wooden pavilion, based on the proposed methodology. They demonstrating hence the potential and feasibility of shell-nexorade hybrid structures.

Acknowledgment
The timber pavilion presented here is the first prototype built in the context of a teaching and research agreement in the Build’in platform of the Co-Innovation Lab of l'Ecole des Ponts. It could not have been built without the support of our industrial partners: ABB France, Hal Robotics Ltd, Simonin SAS, Schunk France, Würth France.

References