Layerwise models for the study of hybrid multilayered structures

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LAYERWISE MODELS FOR THE STUDY OF HYBRID MULTILAYERED STRUCTURES.

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Abstract
In this paper, we present The 3D FE calculation of this structure is extremely complex due to the hollow geometry of the object but also due to the structural and material anisotropy. In particular, the wood has a very weak transverse rolling shear modulus. The effects of these transverse shears are important and cannot be fully accounted by an equivalent homogeneous layer that can be found in composite codes. A model based on a layerwise approach that improves the shear behavior predictions, and examines local responses, especially at the interface between the layers, was developed in the Navier laboratory. Named LS1, it is a Layerwise Stress approach with first-order approximations. Its finite element code called MPFEAP allows estimating the intensity of 3D singularities using a 2D plane mesh, even for this complex 3D structure. The performance of this approach is compared with Abaqus, 2D composite shell element, and with a beam analytical model. We also compare to experimental results.

1. Introduction

The modeling of a hybrid, innovative and complex structure for civil engineering is proposed here. A concrete slab is connected to an openwork (hollow) crossed plywood panel PANOBLOC [11] (cf Fig.1). The 3D finite element of this kind of structure is extremely complex due to the non-continuous aspects and due also to the strong anisotropy of the pieces of wood, especially in term of transverse shear behavior (a very weak rolling shear). The effects of these transverse shears are extremely important for the panel stiffness and have to be correctly taken into account. From a resistance point of view, the stress singularities along the holes of the hollow structure are difficult to estimate with 3D finite element, even with very refined mesh, since non convergent. Scale:1 tests, on 6 meter long panels, have shown that the rupture initiates precisely at these locations, the free edges of wood parts. The estimation of these stress intensities remain consequently an important goal for the design of these structures.

Several approaches and models are compared in this paper, in term of both stiffness and resistance, a shell composite element of Abaqus, an analytical beam method, and MPFEAP (Multi particular Finite Element Approach Program) from Navier laboratory. Based on a layerwise approach named LS1, it permits a 2D description, a 2D meshing, of 3D structures, as shown in Fig. 2. For the MPFEAP approach, two variants are tested: one with an equivalent homogeneous and continuous description of the hollow multilayer, and another one, complete, more precise, where the hollow parts are integrated in the thickness of the local 2D description. Experimental results are also used for comparisons.

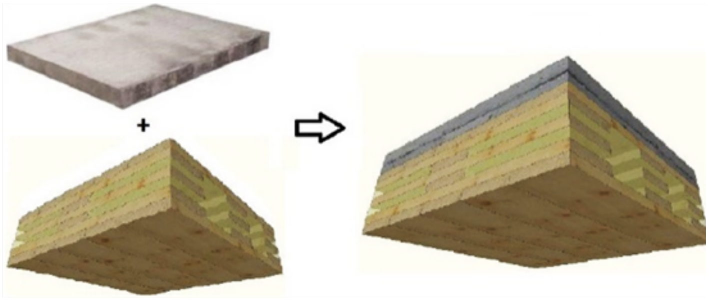

Figure 1: An openwork (hollow) plywood crossply (Panobloc [11]) connected to a concrete slab

The objective is to promote this 2D approach versus 3D calculations, since the cost of 3D calculations is too high for a pre-design phase. In addition, 3D FE are not always fully relevant in the presence of singularities, for example at the interfaces and in the vicinity of the free edges, since not convergent and mesh dependent. After a brief bibliography, the LS1 model will be presented and confrontations are carried out.

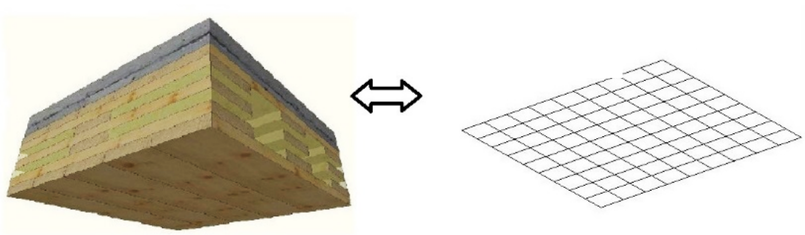

Figure 2: Equivalent 2D meshing (right) of the 3D structure (left)

2. Presentation and description of the LS1 model

Composite structures, due to their intrinsic complexity, require for their design, numerical methods including finite element FE. However, the use of 3D FE models, optimal in terms of physical description of the object is very costly in computation time. Moreover, if nonlinear effects, or singularities or other local effects have to be considered, 3D FEs quickly become inappropriate for large and real structures. In addition, it doesn't always represent the relevant way in terms of mechanics: a beam, even if inhomogeneous, is better described by the concepts of bending moments, shear forces, deflection or curvature than by the concepts of 3d strains or Cauchy stresses. And the singularities in the vicinity of the free edges make often non-converging 3d calculations, even with very fine meshing. Here is proposed a 2d (plate) alternative vision based on a layerwise approach. These kind of models is used to study local responses, especially at the interface between layers. The laboratory Navier, inspired by Pagano's works [3], proposed a family of such approaches [10,7]. The model represents the laminate as a stack of Reissner–Mindlin plates that are connected through interfacial stresses and is rigorously justified from an energetic point of view. Based on the variational formulation of Hellinger-Reissner HR of 3D elastic problems [2] and named LS1 as layerwise stresses with first-order membrane approximations, this approach examines local responses especially at the interface between layers; it has been validated by 3D FE and comparisons with analytical solutions [4,9,10]. The LS1 model, 5 kinematic fields per layer, shows efficiency in representing edges effects and singularities. Consequently LS1 is relevant to calculate complex 3d multilayers since using a 2d plane description, easier to manage. Mpfeap is the associated finite element.

2.1. 3D stress fields approximations

Let us consider a multilayered plate composed of n orthotropic elastic layers perfectly bonded together. Cartesian coordinates are taken so that x is the plane z = 0. A current point in this system is denoted by (x1, x2, x3). e^i is the layer's thickness (i=1,n), h_i^+ , h_i^- the z coordinates of the top and the bottom of the layer i.
The in-plane stress components $\sigma_{\alpha\beta}(x,y,z)$, ($\alpha,\beta$=1,2) are postulated to vary linearly over each layer's thickness e^i (first-order model). They are expressed in terms of the classical generalized forces acting in each layer, i.e., the in-plane force resultants $N^i_{\alpha\beta}(x,y)$, and the moment resultants $M^i_{\alpha\beta}(x,y)$, as follows:

$$\sigma_{\alpha\beta}(x,y,z) = N^i_{\alpha\beta}(x,y)\frac{P_0^i(z)}{e^i} + \frac{12}{e^{i2}} M^i_{\alpha\beta}(x,y)\, P_1^i(z) \qquad \text{(Eq. 1a)}$$

with :

$$P_0^i = 1,\ P_1^i = \frac{z-\overline{h_\iota}}{e^i} \qquad \text{(Eq. 1b)}$$

and $\overline{h_\iota}$ the z coordinate of the center of the layer i. These definitions are coherent with the classical definitions of the resultants $N_{\alpha\beta}{}^i$ (x,y), and the moment resultants $M_{\alpha\beta}{}^i$ (x,y), in the layer i:

$$N_{\alpha\beta}{}^i(x,y) = \int_{h_i^-}^{h_i^+} \sigma_{\alpha\beta}(x,y,z)dz \qquad \text{(Eq. 2a)}$$

$$M_{\alpha\beta}{}^i(x,y) = \int_{h_i^-}^{h_i^+} (z-\overline{h_i})\sigma_{\alpha\beta}(x,y,z)dz \qquad \text{(Eq. 2b)}$$

For the transverse stress approximations we choose a description focusing on the interface. In the analytical expressions of the 3D off-axis stresses $\sigma_{\alpha 3}^i$ and σ_{33}^i , are introduced the exact values of these stresses at the interface i,i+1 (it means between layers i and i+1, at the z-coordinate h_i^+ or h_{i+1}^-),

$$\tau_\alpha^{i,i+1}(x,y) \quad \text{and} \quad \nu^{i,i+1}(x,y),$$

respectively the interface shear stresses and the normal interface stress, defined as follows (eq.3a).

$$\tau_\alpha^{i,i+1}(x,y) = \sigma_{\alpha 3}(x,y,h_j^+) \qquad \upsilon^{i,i+1}(x,y) = \sigma_{33}(x,y,h_j^+) \qquad \text{(Eq. 3a)}$$

The global Reissner Mindlin shear force, defines as follows (eq.3b) is also introduced in these expressions:

$$Q_\alpha^i(x,y) = \int_{h_i^-}^{h_i^+} \sigma_{\alpha 3}(x,y,z)dz \qquad \text{(Eq. 3b)}$$

$\tau_\alpha^{i,i+1}$ and $\upsilon^{i,i+1}$ ensures consequently and automatically the continuity of these stresses through the thickness, even through the interfaces. Details may be found in [10,7]

2.2. The associated generalized displacement and strain field

For each layer i, the membrane displacements U_α^i, the rotations of the section Φ_α^i and the vertical deflection U_3^i are the generalized displacements conjugated to the five generalized forces defined in eqs. (2a,b), and (3a,b) (details in [7]):

$$U_\alpha^i(x,y) = \int_{h_i^-}^{h_i^+} \frac{P_0^i(z)}{e^i} u_\alpha(x,y,z)dz \quad \text{(Eq. 4a)}$$

$$\Phi_\alpha^i(x,y) = \int_{h_i^-}^{h_i^+} \frac{12}{e^{i2}} P_1^i(z) u_\alpha(x,y,z)dz \quad \text{(Eq. 4b)}$$

$$U_3^i(x,y) = \int_{h_i^-}^{h_i^+} \frac{P_0^i(z)}{e^i} u_3(x,y,z)dz \quad \text{(Eq. 4c)}$$

It is worthwhile emphasizing that no assumptions are made concerning the 3D displacement fields, and that generalized displacements are completely defined (eqs.4) through the variational statement once the stress fields has been postulated as in Eqs. (1a). Finally, integration by parts of the equilibrium equations issued from the HR functional leads to the definition of the generalized strains conjugated to the generalized forces, membrane strains, curvatures, transverse shear strains, and less classically, three interface strains, $D_\alpha^{j,j+1}$ and $D_3^{j,j+1}$, respectively shear and normal strains:

$$N_{\alpha\beta}^i \leftrightarrow \varepsilon_{\alpha\beta}^i(x,y) = \frac{1}{2}(U_{\alpha,\beta}^i + U_{\beta,\alpha}^i) \quad \text{(Eq. 5a)}$$

$$M_{\alpha\beta}^i \leftrightarrow \kappa_{\alpha\beta}^i(x,y) = \frac{1}{2}(\Phi_{\alpha,\beta}^i + \Phi_{\beta,\alpha}^i) \quad \text{(Eq. 5b)}$$

$$Q_\alpha^i \leftrightarrow \gamma_{\alpha 3}^i(x,y) = \Phi_\alpha^i + U_{3,\alpha}^i \quad \text{(Eq. 5c)}$$

$$\tau_\alpha^{j,j+1} \leftrightarrow D_\alpha^{j,j+1}(x,y) = (U_\alpha^{j+1} - U_\alpha^j) - (\frac{e^j}{2}\Phi^j + \frac{e^{j+1}}{2}\Phi^{j+1}) \quad \text{(Eq. 5d)}$$

$$\upsilon_\alpha^{j,j+1} \leftrightarrow D_3^{j,j+1}(x,y) = (U_3^{j+1} - U_3^j) \quad \text{(Eq. 5e)}$$

2.3. The constitutive law

The behavior of the 2D model that is variationally consistent with the stress assumptions is formulated in compliance form and the constitutive relations link the generalized strains to the generalized forces as follow:

$$\varepsilon_{\alpha\beta}^i(x,y) = \frac{1}{e^i} S_{\alpha\beta\gamma\delta}^i N_{\gamma\delta}^i(x,y) \quad \text{(Eq. 6a)}$$

$$\kappa_{\alpha\beta}^i(x,y) = \frac{12}{(e^i)^3} S_{\alpha\beta\gamma\delta}^i M_{\gamma\delta}^i(x,y) \quad \text{(Eq. 6b)}$$

$$\gamma_{\alpha 3}^i(x,y) = \frac{6}{5e^i} S_{\alpha 3\beta 3}^i Q_\beta^i - \frac{1}{10} S_{\alpha 3\beta 3}^i(\tau_\beta^{i,i+1} + \tau_\beta^{i-1,i}) \quad \text{(Eq. 6c)}$$

$$D_\alpha^{i,i+1}(x,y) = -\frac{1}{10} S_{\alpha 3\beta 3}^i Q_\beta^i - \frac{1}{10} S_{\alpha 3\beta 3}^{i+1} Q_\beta^{i+1} - \frac{e^i}{30} S_{\alpha 3\beta 3}^i \tau_\beta^{i-1,i} + \frac{2}{15}(e^i S_{\alpha 3\beta 3}^i + e^{i+1} S_{\alpha 3\beta 3}^{i+1}) \tau_\beta^{i,i+1} - \frac{e^{i+1}}{30} S_{\alpha 3\beta 3}^{i+1} \tau_\beta^{i+1,i+2} \quad \text{(Eq. 6d)}$$

$$D_3^{i,i+1}(x,y) = \frac{9}{70} e^i S_{3333}^i \upsilon^{i-1,i} + \frac{13}{35}(e^i S_{3333}^i + e^{i+1} S_{3333}^{i+1}) \upsilon^{i,i+1} - \frac{9}{70} e^{i+1} S_{3333}^{i+1} \upsilon^{i+1,i+2} \quad \text{(Eq. 6e)}$$

With α, β, γ, δ = {1,2}, et S_{ijkl}^i the components of the 3D compliance of the layer *i.*

2.3. Finite element approximation MPFEAP

Since the proposed LS1 model has *for each layer* the same degrees of freedom of a classical Reissner model (equ.4, 5 dof), the standard C0 FE interpolation scheme can be employed. The present implementation MPFEAP, consists of a standard isoparametric eight-node element as discussed in [5] with 5n dof per node (n layers). It is well known that this FE approach suffers transverse shear locking, which dramatically deteriorates the convergence rate towards thin plate (Kirchhoff–Love) solutions. In this work, a reduced numerical quadrature technique is employed with four Gauss points which appeared sufficient for the scope of the present research [7].

Consequently MPFEAP uses a plane mesh, so 2D, and despite this plane description, allows to estimate the intensity of the singularities inside. The interface stresses are also a direct output, and without any post processing.

3. The floor modeling

The floor is 6 meters length and 1 meter width. The 6 layers stacking sequence is relatively complex. The first layer, on the top of the floor, is in high performance concrete and 5 cm thick. The 5 next layers are constituted by wooden strips, non-contiguous, and alternatively in the axial and transverse directions.

The loading case is a 3 points bending test and the materials properties experimentally identified and used in the simulations, are for concrete:

$E_1 = E_2 = E_3 = 37\,000$ MPa
$G_{12} = G_{13} = G_{23} = 15\,417$ MPa
$\nu_{12} = \nu_{13} = \nu_{23} = 0.2$

and for wood:

$E_1 = 11\,000$ MPa, $E_2 = E_3 = 500$ MPa
$G_{12} = G_{13} = 700$ MPa, $G_{23} = 40$ MPa
$\nu_{12} = \nu_{13} = \nu_{23} = 0.35$

Note the very weak rolling transverse shear $G_{23,}$ which plays an important role in the design, and from the test results, may induces the first rupture of the multilayer.

3.1. The three models

Several methods of calculation are used for the modelling of the plywood-concrete floor, and comparisons are made between these models. The rigidity of the floor is firstly estimated with homogeneous approaches (which are sufficient). The resistance estimation needs more precise approaches since the transverse shear at the interfaces between the layers has to be estimated, taking into account the porosity and the actual position of the strips of wood in the 2 longitudinal and transverse directions. The three models are described now:

a) Analytical estimation: the Timoshenko beam model is used to calculate structural stiffness and resistance of the multilayer since the beam is quite thick, and since large differences exist between the different material constants, increasing the transverse shear effects. Regarding stiffness, and to estimate the deflection of the floor, the porosity is just integrated by an ad-hoc and proportional decrease of equivalent layers stiffnesses. For shear strength, the approach of Jourasky (1856) allows to estimate the distribution of shear in the thickness of a continuous multilayered beam. In the case of an uniform porosity along the beam, the method may propose an estimation of transverse shears (not taking into account the singularities)

b) Standard Finite Element calculation: the S8R (Abaqus) standard thick plate element for composite materials is used for the floor stiffness estimation. The porosity is again integrated by an ad-hoc and proportional decrease of equivalent layers stiffnesses. The 3D finite element has not been used since very heavy to implement for this quite complex structure, and according to the author's experience, gives very close results to MPFEAP's ones, as demonstrated several times in publications [4,5,6,7].

c) MPFEAP analysis with equivalent homogeneous layers description: Similar in spirit to the b) approach (Abaqus), this model integrates however more precisely transverse shear behaviors, introducing no correction factors on these stiffnesses. However, as previous models, it provides no information on the stress concentrations due to porosities it doesn't describe.

d) MPFEAP complete analysis: describing porosities and the wooden strips in the 2 longitudinal and transverse directions, it is a rich 2D model, but easier to manage than 3D approach, and allowing the stress concentrations estimation.

3.2. Floor rigidity

A calculation of floor stiffness is achieved with the a), b) and c) approaches and compared with the results of experimental tests. A force is applied in the middle of the floor, up to 64 KN provoking the rupture of the floor. Fig. 3 shows the different mid-pan deflections obtained with the different models and measured on the intrados of the panel.

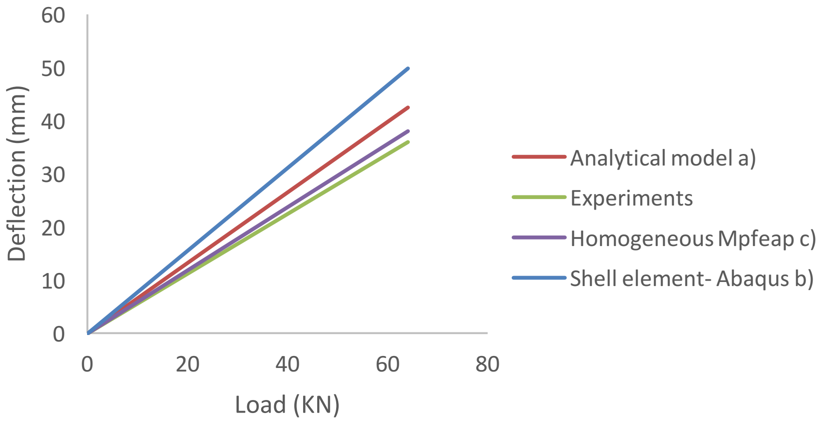

Figure 3: Experimental and simulated mid-pan deflections (mm).

The best fit is realized with the c) model, the homogeneous equivalent MPFEAP without the precise openwork description. It takes accurately into account the high gradient of properties between the different layers, especially concerning the transverse shear phenomenon. The model d), the complete MPFEAP, not reported here, doesn't improve this rigidity prediction. The analytical approach provides a not so bad estimation, but this is only for very simple cases, 1D beams with simple loadings, elastic behavior, and uniform distribution of strips and porosities. The more interesting aspect is perhaps the difficulty for the 2D plate elements from ABAQUS to simulate this situation. It's mainly due to the *hazardous* estimation of correction factors for transverse shear stiffnesses, depending, moreover, on the case loading.

3.3. Resistance and transverse shear stresses estimation.

Here we compare the numerical results a), c) and d), with the experimental rupture of the beam. The figure 4 shows the floor after rupture. The rupture appears between layers 2 and 3 (layer 5 is the intrados one). The ultimate load is 64 KN. The maximum transverse shear stresses σ_{13} calculated for this loading, with the different models a), c) and d) and for each interface, are reported in table 1.
Some tests were also made on the wood strips alone, to estimate the transverse shear strengths of the wood, and especially in the Transverse Normal direction (rolling shear). An average value of 1.73 MPa has been found.

The three approaches are consistent and locate correctly the rupture. The analytical approach is however reserved to very simple cases, 1D beams with simple loadings, elastic behavior, and uniform distribution of strips and porosities. For more complex cases, local densification of strips, 2D slabs, complex loadings or boundary conditions, the homogeneous MPFEAP model c) permits a good first estimation of the ultimate limit state of the floor. It will be also possible soon to take into account inelastic phenomenon, as the compression damage of concrete for instance (to be published).

Finally the complete model d), which describes the exact geometrical positions of the strips of wood in the 2 longitudinal and transverse directions as well as porosity, locates correctly too the concentration location and the weak area of the floor, but allows also a better estimation of the transverse shear stress in these quite difficult conditions. The experimental strength is very close to the numerical estimation, 1.77 MPa. We highlight once again that this model remains a 2 FE approach, involving only plane elements.

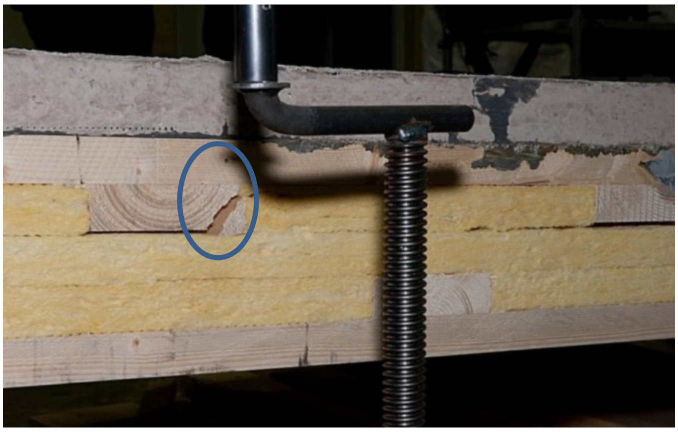

Figure 5 : Transverse shear cracking (blue circle) between layers 2 and 3 in a transverse wooden strip.

Table 1. maximum transverse shear stresses σ_{13} (MPa), for each interface and for the different models a), c) and d).

Interface	Analytical model a)	Equivalent MPFEAP c)	Complete MPFEAP d)
0-1	0.46	0.48	0.52
1-2	0.68	0.63	0.81
2-3	2.01	1.86	1.77
3-4	1.71	1.55	1.48
4-5	0.55	0.52	0.7

4. Conclusion

In this paper, the relevance of an original 2D plate approach is demonstrated, through the calculation of a highly complex structure combining several materials, wood, high performance concrete and an openwork design [11]. The model is Layerwise, and named LS1 [7] since it involves first-order membrane stresses. The 2D associated finite element code MPFEAP allows to describe easily the 3D complex structure and to estimate the transverse shear effects and the intensity of the singularities along the free edges.

The performance of this approach is compared with classical homogeneous solutions, obtained analytically or numerically (Abaqus). For the estimation of the structure rigidity, the homogeneous equivalent MPFEAP without the precise openwork description (c), takes accurately into account the high gradient of properties between the different layers, especially concerning the transverse shear phenomenon. The analytical approach can be only for very simple cases, 1D beams with simple loadings, elastic behavior, and uniform distribution of strips and porosities, and the 2D plate elements from ABAQUS doesn't estimate correctly the transverse shear behavior.

Concerning the ultimate behavior, all the models locate correctly the rupture, but once again, the analytical approach can treat only very simple cases. For real and complex cases, local densification of strips, 2D slabs, complex loadings or boundary conditions, the homogeneous MPFEAP model permits a good first estimation of the ultimate limit state of the floor. If a more precise estimation is needed the model d), which describes the exact geometrical positions of the wooden strips as well as porosity, allows a better estimation of the transverse shear stress in these quite difficult conditions. The experimental strength is very close to the numerical estimation. Note that this fine description remains a 2D plane description, a quite interesting alternative to the heavy 3D descriptions, and relevant for real and complex material and geometries.

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