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Assumption 1 in our original paper can be replaced with the following, less stringent assumption.

Assumption 1. *There exists $\lambda > 0$ such that at any point $\mathbf{x} \in \Omega$, the eigenvalues of $[\mathbf{C}(\mathbf{x}) - \mathbf{C}_0]$ are greater than λ in absolute value.*

Proof of Theorem 4 requires that the local stiffness be bounded from below *and* above. Therefore, Assumption 2 must be altered as follows

Assumption 2. *There exists $\kappa_{\max} > \kappa_{\min} > 0$ and $\mu_{\max} > \mu_{\min} > 0$ such that at any point $\mathbf{x} \in \Omega$*

$$\kappa_{\min} \leq \kappa(\mathbf{x}) \leq \kappa_{\max}, \quad \mu_{\min} \leq \mu(\mathbf{x}) \leq \mu_{\max}.$$

Then, the end of the proof of Theorem 4 (starting from “Taking advantage of the isotropy”) must be modified as follows.

Proof of Theorem 4. [...] Taking advantage of the isotropy of both local and reference materials, the above volume averages can be expanded

$$|\Omega|\,\overline{\boldsymbol{\tau}^+ : (\mathbf{C} - \mathbf{C}_0)^{-1} : \boldsymbol{\tau}^+} = \int_{\kappa(\mathbf{x})>\kappa_0} \frac{\|\boldsymbol{\tau}^{\mathrm{hyd}}(\mathbf{x})\|^2}{d\,[\kappa(\mathbf{x}) - \kappa_0]}\,\mathrm{d}\,\Omega + \int_{\mu(\mathbf{x})>\mu_0} \frac{\|\boldsymbol{\tau}^{\mathrm{dev}}(\mathbf{x})\|^2}{2\,[\mu(\mathbf{x}) - \mu_0]}\,\mathrm{d}\,\Omega,$$

and

$$|\Omega|\,\overline{\boldsymbol{\tau}^- : \mathbf{S}_0 : (\mathbf{S} - \mathbf{S}_0)^{-1} : \mathbf{S}_0 : \boldsymbol{\tau}^-} = \int_{\kappa(\mathbf{x})<\kappa_0} \frac{\kappa(\mathbf{x})\,\|\boldsymbol{\tau}^{\mathrm{hyd}}(\mathbf{x})\|^2}{d\kappa_0\,[\kappa_0 - \kappa(\mathbf{x})]}\,\mathrm{d}\,\Omega + \int_{\mu(\mathbf{x})<\mu_0} \frac{\mu(\mathbf{x})\,\|\boldsymbol{\tau}^{\mathrm{dev}}(\mathbf{x})\|^2}{2\mu_0\,[\mu_0 - \mu(\mathbf{x})]}\,\mathrm{d}\,\Omega.$$

From Assumption 2, we first have

$$|\Omega|\,\overline{\boldsymbol{\tau}^+ : (\mathbf{C} - \mathbf{C}_0)^{-1} : \boldsymbol{\tau}^+} \geq \frac{\int_{\kappa(\mathbf{x})>\kappa_0} \|\boldsymbol{\tau}^{\mathrm{hyd}}(\mathbf{x})\|^2\,\mathrm{d}\,\Omega}{d\,(\kappa_{\max} - \kappa_0)} + \frac{\int_{\mu(\mathbf{x})>\mu_0} \|\boldsymbol{\tau}^{\mathrm{dev}}(\mathbf{x})\|^2\,\mathrm{d}\,\Omega}{2\,(\mu_{\max} - \mu_0)},$$

then

$$|\Omega|\,\overline{\boldsymbol{\tau}^- : \mathbf{S}_0 : (\mathbf{S} - \mathbf{S}_0)^{-1} : \mathbf{S}_0 : \boldsymbol{\tau}^-} \geq \frac{\kappa_{\min} \int_{\kappa(\mathbf{x})<\kappa_0} \|\boldsymbol{\tau}^{\mathrm{hyd}}(\mathbf{x})\|^2\,\mathrm{d}\,\Omega}{d\kappa_0\,(\kappa_0 - \kappa_{\min})} + \frac{\mu_{\min} \int_{\mu(\mathbf{x})<\mu_0} \|\boldsymbol{\tau}^{\mathrm{dev}}(\mathbf{x})\|^2\,\mathrm{d}\,\Omega}{2\mu_0\,(\mu_0 - \mu_{\min})}.$$

from which the following bound results

$$a\,(\boldsymbol{\tau}, \boldsymbol{\varpi}) \geq \frac{\alpha}{|\Omega|} \int_\Omega \left[\|\boldsymbol{\tau}^{\mathrm{hyd}}(\mathbf{x})\|^2 + \|\boldsymbol{\tau}^{\mathrm{dev}}(\mathbf{x})\|^2\right] \mathrm{d}\,\Omega = \alpha \|\boldsymbol{\tau}\|_{\mathbb{V}}^2, \tag{49}$$

with[1]

$$\alpha = \min\Big\{ [d\,(\kappa_{\max} - \kappa_0)]^{-1}, [2\,(\mu_{\max} - \mu_0)]^{-1}\ \kappa_{\min}\,[d\kappa_0\,(\kappa_0 - \kappa_{\min})]^{-1}, \mu_{\min}\,[2\mu_0\,(\mu_0 - \mu_{\min})]^{-1} \Big\}.$$

The proof of the first statement is complete, since $\|\boldsymbol{\varpi}\|_{\mathbb{V}} = \|\boldsymbol{\tau}\|_{\mathbb{V}}$, and $\alpha > 0$. □

[1] In this definition of α, it is assumed that $\kappa_{\min} < \kappa_0 < \kappa_{\max}$ and $\mu_{\min} < \mu_0 < \mu_{\max}$. The proof remains valid if any of these inequalities are not verified. For example, if $\mu_{\min} \geq \mu_0$, then, from Assumption 2, $\mu(\mathbf{x}) \geq \mu_0$ at any point $\mathbf{x} \in \Omega$. In other words, the integral over the set of points $\mathbf{x} \in \Omega$ such that $\mu(\mathbf{x}) < \mu_0$ is null. Then Eq. (49) still holds, provided that α is defined as follows

$$\alpha = \min\Big\{ [d\,(\kappa_{\max} - \kappa_0)]^{-1}, [2\,(\mu_{\max} - \mu_0)]^{-1}, \kappa_{\min}\,[d\kappa_0\,(\kappa_0 - \kappa_{\min})]^{-1} \Big\}.$$

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