



HAL
open science

Corrigendum to “Combining Galerkin approximation techniques with the principle of Hashin and Shtrikman to derive a new FFT-based numerical method for the homogenization of composites” [Comput. Methods Appl. Mech. Engrg. 217–220 (2012) 197–212]

Sébastien Brisard, Luc Dormieux

► **To cite this version:**

Sébastien Brisard, Luc Dormieux. Corrigendum to “Combining Galerkin approximation techniques with the principle of Hashin and Shtrikman to derive a new FFT-based numerical method for the homogenization of composites” [Comput. Methods Appl. Mech. Engrg. 217–220 (2012) 197–212]. Computer Methods in Applied Mechanics and Engineering, 2015, 283, pp.1587-1588. 10.1016/j.cma.2014.10.021 . hal-01773735

HAL Id: hal-01773735

<https://hal-enpc.archives-ouvertes.fr/hal-01773735>

Submitted on 23 Apr 2018

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L’archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Corrigendum to “Combining Galerkin approximation techniques with the principle of Hashin and Shtrikman to derive a new FFT-based numerical method for the homogenization of composites” [Comput. Methods Appl. Mech. Engrg. 217-220 (2012) 197–212]

S. Brisard^{a,*}, L. Dormieux^a

^aUniversité Paris-Est, Laboratoire Navier (UMR 8205), CNRS, ENPC, IFSTTAR, F-77455 Marne-la-Vallée

Assumption 1 in our original paper can be replaced with the following, less stringent assumption.

Assumption 1. *There exists $\lambda > 0$ such that at any point $\mathbf{x} \in \Omega$, the eigenvalues of $[\mathbf{C}(\mathbf{x}) - \mathbf{C}_0]$ are greater than λ in absolute value.*

Proof of Theorem 4 requires that the local stiffness be bounded from below and above. Therefore, Assumption 2 must be altered as follows

Assumption 2. *There exists $\kappa_{\max} > \kappa_{\min} > 0$ and $\mu_{\max} > \mu_{\min} > 0$ such that at any point $\mathbf{x} \in \Omega$*

$$\kappa_{\min} \leq \kappa(\mathbf{x}) \leq \kappa_{\max}, \quad \mu_{\min} \leq \mu(\mathbf{x}) \leq \mu_{\max}.$$

Then, the end of the proof of Theorem 4 (starting from “Taking advantage of the isotropy”) must be modified as follows.

Proof of Theorem 4. [...] Taking advantage of the isotropy of both local and reference materials, the above volume averages can be expanded

$$\overline{|\Omega| \boldsymbol{\tau}^+ : (\mathbf{C} - \mathbf{C}_0)^{-1} : \boldsymbol{\tau}^+} = \int_{\kappa(\mathbf{x}) > \kappa_0} \frac{\|\boldsymbol{\tau}^{\text{hyd}}(\mathbf{x})\|^2}{d[\kappa(\mathbf{x}) - \kappa_0]} d\Omega + \int_{\mu(\mathbf{x}) > \mu_0} \frac{\|\boldsymbol{\tau}^{\text{dev}}(\mathbf{x})\|^2}{2[\mu(\mathbf{x}) - \mu_0]} d\Omega,$$

and

$$\overline{|\Omega| \boldsymbol{\tau}^- : \mathbf{S}_0 : (\mathbf{S} - \mathbf{S}_0)^{-1} : \mathbf{S}_0 : \boldsymbol{\tau}^-} = \int_{\kappa(\mathbf{x}) < \kappa_0} \frac{\kappa(\mathbf{x}) \|\boldsymbol{\tau}^{\text{hyd}}(\mathbf{x})\|^2}{d\kappa_0 [\kappa_0 - \kappa(\mathbf{x})]} d\Omega + \int_{\mu(\mathbf{x}) < \mu_0} \frac{\mu(\mathbf{x}) \|\boldsymbol{\tau}^{\text{dev}}(\mathbf{x})\|^2}{2\mu_0 [\mu_0 - \mu(\mathbf{x})]} d\Omega.$$

From Assumption 2, we first have

$$\overline{|\Omega| \boldsymbol{\tau}^+ : (\mathbf{C} - \mathbf{C}_0)^{-1} : \boldsymbol{\tau}^+} \geq \frac{\int_{\kappa(\mathbf{x}) > \kappa_0} \|\boldsymbol{\tau}^{\text{hyd}}(\mathbf{x})\|^2 d\Omega}{d(\kappa_{\max} - \kappa_0)} + \frac{\int_{\mu(\mathbf{x}) > \mu_0} \|\boldsymbol{\tau}^{\text{dev}}(\mathbf{x})\|^2 d\Omega}{2(\mu_{\max} - \mu_0)},$$

then

$$\overline{|\Omega| \boldsymbol{\tau}^- : \mathbf{S}_0 : (\mathbf{S} - \mathbf{S}_0)^{-1} : \mathbf{S}_0 : \boldsymbol{\tau}^-} \geq \frac{\kappa_{\min} \int_{\kappa(\mathbf{x}) < \kappa_0} \|\boldsymbol{\tau}^{\text{hyd}}(\mathbf{x})\|^2 d\Omega}{d\kappa_0 (\kappa_0 - \kappa_{\min})} + \frac{\mu_{\min} \int_{\mu(\mathbf{x}) < \mu_0} \|\boldsymbol{\tau}^{\text{dev}}(\mathbf{x})\|^2 d\Omega}{2\mu_0 (\mu_0 - \mu_{\min})}.$$

from which the following bound results

$$a(\boldsymbol{\tau}, \boldsymbol{\varpi}) \geq \frac{\alpha}{|\Omega|} \int_{\Omega} [\|\boldsymbol{\tau}^{\text{hyd}}(\mathbf{x})\|^2 + \|\boldsymbol{\tau}^{\text{dev}}(\mathbf{x})\|^2] d\Omega = \alpha \|\boldsymbol{\tau}\|_{\nabla}^2, \quad (49)$$

with¹

$$\alpha = \min \left\{ [d(\kappa_{\max} - \kappa_0)]^{-1}, [2(\mu_{\max} - \mu_0)]^{-1}, \kappa_{\min} [d\kappa_0 (\kappa_0 - \kappa_{\min})]^{-1}, \mu_{\min} [2\mu_0 (\mu_0 - \mu_{\min})]^{-1} \right\}.$$

The proof of the first statement is complete, since $\|\boldsymbol{\varpi}\|_{\nabla} = \|\boldsymbol{\tau}\|_{\nabla}$, and $\alpha > 0$. □

*Corresponding author.

Email addresses: sebastien.brisard@ifsttar.fr (S. Brisard), luc.dormieux@enpc.fr (L. Dormieux)

¹In this definition of α , it is assumed that $\kappa_{\min} < \kappa_0 < \kappa_{\max}$ and $\mu_{\min} < \mu_0 < \mu_{\max}$. The proof remains valid if any of these inequalities are not verified. For example, if $\mu_{\min} \geq \mu_0$, then, from Assumption 2, $\mu(\mathbf{x}) \geq \mu_0$ at any point $\mathbf{x} \in \Omega$. In other words, the integral over the set of points $\mathbf{x} \in \Omega$ such that $\mu(\mathbf{x}) < \mu_0$ is null. Then Eq. (49) still holds, provided that α is defined as follows

$$\alpha = \min \left\{ [d(\kappa_{\max} - \kappa_0)]^{-1}, [2(\mu_{\max} - \mu_0)]^{-1}, \kappa_{\min} [d\kappa_0 (\kappa_0 - \kappa_{\min})]^{-1} \right\}.$$