**A novel method for determining the small-strain shear modulus of soil
using bender elements technique**

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Abstract

Bender elements technique has become a popular tool for determining shear wave velocity (V_s), hence the small-strain shear modulus of soils (G_{max}), thanks to its simplicity and non-destructive character among other advantages. Several methods were proposed to determine the first arrival of V_s . However, none of them can be widely adopted as a standard and there is still an uncertainty on the detection of the first arrival.

In this study, bender elements tests were performed on lime-treated soil and both shear wave and compression wave velocities at various frequencies were measured. In-depth analysis showed that the S-wave received signal presents an identical travel time and opposite polarity compared with that of the S-wave components in P-wave received signal, especially at high frequency. From this observation, a novel interpretation method based on the comparison between the S-wave and P-wave received signals at high frequency is proposed. This method enables the determination of the arrival time of S-wave objectively, avoiding less reliable arrival pick-up point. Furthermore, the “ π -point” method and cross correlation method were also employed and the obtained results agree well with those from the proposed method, indicating the accuracy and reliability of the latter. The effects of frequency on the shear wave velocity are also discussed.

Keywords: bender elements; signal interpretation; shear wave; compression wave; S+P interpretation method

Introduction

The small-strain shear modulus (G_{max}) is a parameter of paramount importance in describing the elastic properties of soil. It is widely used in the analysis of dynamic problems in anti-seismic engineering (Zhou et al. 2005; Yang et al. 2009; Luke et al. 2013). It is also used to assess the soil stiffness in geo-environmental engineering (Tang et al. 2011; Gu et al. 2013; Hoyos et al. 2015). The value of G_{max} is usually determined from shear wave velocity (V_s) measurements either in the field or in the laboratory.

The measurement of V_s can be obtained from conventional laboratory experiments using resonant column (Hardin and Richart 1963; Anderson and Stokoe 1978; Fam et al. 2002), torsional shear tests (Iwasaki et al. 1978; Youn et al. 2008), flat transducers and accelerometers (Brignoli et al. 1996; Mulmi et al. 2008; Wicaksono et al. 2008) and piezoelectric bender elements technique (Dyvik and Madhus 1985; Viggiani and Atkinson 1995; Brignoli et al. 1996; Jovičić et al. 1996; Pennington et al. 2001; Lings and Greening 2001; Leong et al. 2005; Yamashita et al. 2009; Clayton 2011). The bender elements technique was first introduced by Shirley and Hampton (1978) and Shirley (1978) to soil testing, and due to its ability to measure the shear wave velocity of soil in a small-strain range, less than 0.001%, and in a wide range of stress conditions, it has taken a lot of interest of researchers. Nowadays, the bender elements transducers have been becoming a common laboratory tool, and have been incorporated in many geotechnical testing devices, such as oedometers (Dyvik and Olsen 1991; Zeng and Grolewski 2005; Sukolrat 2007), cubical and conventional triaxial apparatuses (Gajo et al. 1997; Jovičić and Coop 1998; Kuwano et al. 1999; Pennington et al. 2001; Fioravante and Capoferri 2001; Sukolrat et al. 2006; Leong et

al. 2009; Finno and Cho 2011; Aris et al. 2012; Styler and Howie 2013), resonant column apparatuses (Ferreira et al. 2007; Youn et al. 2008; Gu et al. 2013).

Despite the popularity of its use, difficulties of signal interpretation of bender elements technique, mainly in the determination of the wave arrival time, are still remaining. Different methods and frameworks for the signal interpretation were reported (Lee and Santamarina 2005; Viana da Fonseca et al. 2009; Leong et al. 2009; Arroyo et al. 2010), but none has been widely accepted as a standard. In fact, the complex phenomena of wave propagation in a soil sample have not been clearly understood yet, which represents an obstacle to the determination of the arrival point accurately and objectively (Camacho-Tauta et al. 2013).

In the field of geotechnical engineering, most tested materials with bender elements are soils with low stiffness, and generally, the chosen frequency of excitation voltage in most studies ranges from 1 to 30 kHz (Lee and Santamarina 2005; Sukolrat 2007; Youn et al. 2008; Viana da Fonseca et al. 2009; Leong et al. 2009). A few researchers used the bender elements technique for stiffer materials, including sandstones (Alvarado 2007), cement treated clay (Hird and Chan 2008), and argillaceous rocks (Arroyo et al. 2010). In these cases, the working frequency range is different, and higher input frequency should be chosen due to the higher resonant frequency of stiff material (Lee and Santamarina 2005).

This technical note puts forward an objective method for the signal interpretation of bender elements technique, with respect to the determination of the first arrival time of shear wave. This method is based on the comparison of both S-wave and P-wave received signals obtained on the same sample with a single pair of bender elements to determine a unique arrival point. In this study, the bender elements tests were performed on compacted

lime-treated silt samples extending, thus, this technique to stiff soils. The results obtained by the proposed method are validated by comparing with those of other methods.

Background

Working principal of Bender Elements

The bender elements are piezo-electrical transducers, which consist of two thin piezo-ceramic bimorph sheets with external conducting surfaces, mounted together with a conductive metal shim at the centre. Details of the connection are given in Figure 1a. When an input waveform voltage is applied on the S-wave transmitter, one piezoceramic sheet extends and the other contracts, leading the transmitter to bend and generate a shear wave signal. The S-wave receiver bends when the shear wave arrives, propagating an electrical signal that can be visualised and measured by a digital oscilloscope. The operation of bender elements for S-wave transmission was well described by other researchers (Dyvik and Madshus 1985; Lings and Greening 2001; Lee and Santamarina 2005; Camacho Tauta et al. 2012).

Lings and Greening (2001) introduced a bender-extender element to transmit and receive both S-wave and P-wave with a single pair of transducers. The bender (or S-wave) receiver and transmitter in bender element testing also act as an extender (or P-wave) transmitter and receiver, respectively. Specifically, when an input voltage is applied on the extender transmitter, both two piezoceramic sheets with opposite polarisations extend or contract at the same time, causing the propagation of P-wave in a longitudinal direction. The P-wave wiring and transmitting are illustrated in Figure 1b. Note that the measurement of P-wave velocity is

mainly useful for unsaturated soils. Since in the saturated soils, P-wave usually travels much faster through water than through soil skeleton (Leong et al. 2009). Nevertheless, in this study, the degree of saturation of the tested material is lower than 95%. Bardet and Sayed (1993) who studied the Ottawa sand reported that the P-wave velocity became close to the value of dry sample when the degree of saturation decreased from 100% to 95%.

Determination of shear wave velocity

During bender elements testing, both the transmitted and received signals are recorded to determine the travel time, t , of S-wave through a sample. The shear wave velocity, V_s , can then be calculated from the tip-to-tip travel length between the bender elements, L_{tt} , and travel time, t , as follows (Viggiani and Atkinson, 1995):

$$V_s = \frac{L_{tt}}{t} \quad (1)$$

According to the theory of shear wave propagation in an elastic body, the small strain shear modulus, $G_{\max}$, can be determined by the following formula:

$$G_{\max} = \rho V_s^2 \quad (2)$$

where ρ is the density of soil sample.

Interpretation of bender element tests usually takes the advantage of the knowledge and experience gained into the development of in situ geophysical tests such as Down-Hole and Cross-Hole tests or even surface tests such as SASW tests (Stokoe et al. 2004; Viana da Fonseca et al. 2006).

Determination of travel time

An accurate determination of the travel time is a crucial issue to get a reliable value of V_s ,

hence $G_{\max}$. The existing common methods for determining travel time can be classified into two categories: time domain and frequency domain methods.

The time domain methods determine the travel time directly from the time lag between the transmitted and received signals. Referring to different characteristic points, the time domain methods can be divided into “arrival-to-arrival” method, “peak-to-peak” method and “cross correlation method”.

The first method, based on the visual inspection of the received signal, is the most commonly used one; however, the determination of an accurate arrival point using this method is still controversial and quite subjective due to the complex received signal, wave’s reflexion and the near field effect. Many studies reported that the near field effect decreases with the increase of frequency or the ratio of the wave path length to wavelength, L_{tt}/λ . Arulnathan et al. (1998) reported that the near field effect disappears when this ratio is larger than 1. Pennington et al. (2001) pointed out that when the L_{tt}/λ values range from 2 to 10, a good signal can be obtained. Wang et al. (2007) advocated a ratio greater than or equal to 2 to avoid the near field effect. Similarly, a value of 3.33 was recommended by Leong et al. (2005) to improve the signal interpretation.

Figure 2 shows a typical single sinusoidal S-wave transmitted signal and its corresponding received signal. Generally, point “a” (the first deflection) is taken as the arrival of the near field component of received signal (Brignoli et al. 1996). Both point “b” (the first reversal) and point “c” (zero after first reversal) are chosen as the arrival points of S-wave by most researchers (Brignoli et al. 1996; Lee and Santamarina 2005; Youn et al. 2008; Yamashita et al. 2009; Arroyo et al. 2010).

The “peak-to-peak” method is also widely applied in the signal interpretation. In this method, time delay between the peak of transmitted signal and the first major peak of received signal (point “d” in Figure 2) is regarded as the travel time (Clayton et al. 2004; Ogino et al. 2015). Note that, since the frequency of received signal may be slightly different from that of transmitted signal, and that the nature of the soil and size of the sample often affect the shape of the signal which could present more than one peak, great attention should be paid to the calculation of travel time by the “peak-to-peak” method.

“Cross-correlation” method is another kind of time domain method. It assumes the travel time as the time shift corresponding to the peak value of cross-correlation function between the transmitted and received signals. The cross-correlation approach, first adopted by Viggiani and Atkinson (1995), is based on the following expression::

$$CC_{xy}(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T x^*(t)y(t+\tau)dt \quad (3)$$

where $x(t)$ and $y(t)$ are the received and transmitted signals respectively, T is the time record and τ is the time shift between two signals. First, the transmitted and received signals are converted to their linear spectrums using Fast Fourier Transform. Then the cross-power spectrum can be built based on the linear spectrum of received signal and the complex conjugate of the linear spectrum of transmitted signal. Eventually, the maximum of the cross-correlation function gives the travel time of shear wave. However, the accuracy of the cross-correlation method is largely dependent on the quality of received signals. Many limitations due to complex characteristics of received signal or incompatible transformation have been reported by several researchers (Arulnathan et al. 1998; Viana da Fonseca et al. 2009; Chan 2012).

Frequency domain methods estimate the travel time according to the relationship between the change in the phase angle, which corresponds to the phase shift between the transmitter and receiver signals, and input frequency (Greening and Nash 2004; Viana da Fonseca et al. 2009; Ogino et al. 2015). These methods can be applied using discrete method called “ π -point” method (Greening and Nash 2004; Viana da Fonseca et al. 2009), or continuous method such as Frequency Spectral Analysis (Greening and Nash 2004; Viana da Fonseca et al. 2009; Kim et al. 2015). They were first performed on rock and mud samples by Kaarberg (1975) and later widely accepted by other researchers (Greening et al. 2003; Gutierrez 2007; Viana da Fonseca et al. 2009), thanks to their negligible effect of extraneous signals.

In the “ π -point” method, a continuous sinusoidal wave at a single frequency is used as an input signal, the continuous sinuous wave transmitter and receiver are displayed in an X-Y plot on an oscilloscope, and the phase shift between these two signals is measured. The frequency of transmitter is increased very slightly from a low value, inducing a phase shift between the two signals. When these signals are in phase or out of phase, i.e. the phase differences are multiple N of π or $(-\pi)$, the corresponding frequency, f , and the number of wavelength, N , are recorded.

It is well known that velocity, V , can be determined from the wavelength, λ , and the frequency (Viana da Fonseca et al. 2009):

$$V = \lambda f = f \frac{L}{N} \quad (4)$$

where the travel time, t , can be deduced from:

$$t = \frac{N}{f} \quad (5)$$

This indicates that the slope of the N - f plot represents the travel time.

Result given by this method is more objective than that obtained by time domain method.

However, it has the drawbacks of time consuming and limited interpretable points (Viana da Fonseca et al. 2009).

Compared with the time-consuming “ π -point” method, continuous method (Frequency Spectral Analysis) provides more available information in a short period of time with less effort. Continuous method applies a sweep signal which has a wide frequency spectrum (for example: 0 – 20 kHz) as input wave and uses a spectrum analyzer to establish the relationship between the frequency and the phase change, as introduced by Greening et al. (2003), Greening and Nash (2004) and explained in details by Kim et al. (2015). Specifically, the spectrum analyzer computes the coherence function between the transmitted and received signals. Based on the coherence function and phase angle it provides, the travel time can be determined directly from the slope of the linear line representing the relationship between the frequency and phase angle. However, further analysis is necessary when applying this method if the result convergence cannot be reached quickly (Viana da Fonseca et al. 2009).

Viana da Fonseca et al. (2009) also proposed a practical framework which combines both time-domain and frequency-domain methods for an enhanced interpretation of the bender element testing results. Nevertheless, the differences of the travel time determined by the time domain method and frequency domain method are quite large, and the causes are still not well understood (Greening et al. 2003; Greening and Nash 2004; Viana da Fonseca et al. 2009; Ogino et al. 2015). Therefore, there is still a strong need of searching a simple and objective approach for the bender element testing interpretation with a reliable determination

of the arrival time.

In this study, the “arrival-to-arrival” method, “peak-to-peak” method, and “ π -point” method are evaluated through the interpretation of signals obtained on compacted lime treated soils. Furthermore, based on the observation that the S-wave received signal presents an identical travel time and opposite polarity compared with that of the S-wave components in P-wave received signal, especially at high frequency, a novel method namely S+P method is proposed and its accuracy is proved by the comparison with “ π -point” method.

Experimental methods

Tested Material

The soil used in this study was a plastic silt, taken from an experimental embankment with the ANR project TerDOUEST (Terrassements Durables - Ouvrages en Sols Traités, 2008 - 2012) at Héricourt, France. This soil was first air-dried, ground and sieved to 0.4 mm. Quicklime was used in the treatment. The soil powder was first mixed thoroughly with 2% lime, and then humidified to reach two target water contents (at dry or wet side of optimum). More details about the geotechnical properties of this silt and the preparation process of samples can be found in Wang et al. (2015). In this study, four compacted lime-treated samples (50 mm in diameter and 50 mm in height) with degree of saturation, $S_r = 72\%$ at dry side and $S_r = 93\%$ at wet side, were tested. The specific sample information is listed in Table 1.

Experimental Techniques

235 The bender element system used in this study consists of two bender elements (one S-wave
236 transmitter/P-wave receiver and one S-wave receiver/P-wave transmitter, as shown in Figure
237 3), installed at the two extremities of the soil sample. Beforehand, a slot was carried out on
238 the surface of each sample extremity with the same direction to facilitate the insertion of the
239 protruded part of the bender elements, and a good alignment of the latter. Afterwards, the
240 sample (50 mm in diameter and 50 mm in height) was placed on a home-made wooden
241 sample holder (see Figure 4a) specially designed to forbid any wave transmission outside the
242 soil sample, which may interfere with the signal arrival (Brignoli et al. 1996; Lee and
243 Santamarina 2005). Additional force was provided by the holder to enhance a good contact
244 between the benders and the sample. Special care was taken to avoid any cross-talk by
245 improving the shielding and grounding of the system. The output signal (± 20 V sine pulse)
246 was generated by a function generator (*TTi* TG1010A) and amplified by a power amplifier.
247 Both the transmitter and the receiver signals were captured with an oscilloscope (Agilent
248 DSO-X 2004A). The set-up of the system used is presented in Figure 4a and details of the
249 arrangement of devices are illustrated in Figure 4b. All the four different interpretation
250 methods presented previously (“arrival-to-arrival”, “peak-to-peak”, “ π -point” and S+P
251 methods) were applied for each sample. As for the conventional “arrival-to-arrival” and
252 “peak-to-peak” methods, a single pulse S-wave with various input frequency was used as
253 transmitted signal. In S+P method, both S-wave and P-wave signals transmitted through the
254 same sample by modifying the connection between the two benders, as illustrated in Figure
255 4b. A continuous sweep signal (frequency increased slowly from 20 kHz to 50 kHz) was
256 applied in the “ π -point” method.

Prior to testing, calibration of bender elements (tip-to-tip calibration) was carried out by holding the two benders (transmitter and receiver) in contact with each other directly without sample. Both S-wave transmitter and P-wave transmitter were generated to measure the delay times for these two transmitters ($t_{d_S} = 5.5 \mu s$ and $t_{d_P} = 3.5 \mu s$). These delay times were accounted for in the calculation of travel time when applying the time domain method.

Experimental Results

Figure 5 presents the results of sample D1 with the time domain methods (conventional arrival-to-arrival method and peak-to-peak method). The S-wave transmitted signals with various frequencies are considered (dashed lines) and the S-wave received signals are presented in solid lines. In the arrival-to-arrival method, the first reversal in the received signal is chosen as the arrival point of S-wave. Besides, the first major peak is highlighted to calculate the time delay by the peak-to-peak method. It is observed that for the relatively low frequencies used here ($f = 5, 10$ and 15 kHz), the interpretation of these received signals is ambiguous because of the evident near field effect. However, the signals at higher frequencies, from 20 kHz to 50 kHz, become quite clear and the near field less marked. These cases correspond to a ratio of wave path length to wavelength, L_{tt}/λ , larger than or equal to 1.9 , and are in good agreement with those reported in the literature (Sanchez-Salinero et al. 1986; Brignoli et al. 1996; Arulnathan et al. 1998; Pennington et al. 2001; Leong et al. 2005; Wang et al. 2007). The results obtained from the “ π -point” method on the sample ($w = 17\%$, with a curing time, $t_c = 25$ h) are shown in Figure 6. A good linear relationship between the number of wavelength and frequency is obtained. The travel time, t , can be determined as 0.1441 ms

279 directly from the slope of the matched line, according to Equation 5. Additionally, the good
280 linear relationship observed also highlights that the frequency ranging from 25 to 50 kHz is
281 reasonable and suitable for the determination of shear wave velocity by the time domain
282 method.

283 In Figure 7, the S-wave and P-wave transmitted signals and the corresponding received
284 signals are gathered. The arrival point of S-wave received signal can be determined by
285 referring to P-wave received signal. It is well known that P-wave component travels the
286 fastest, thus arrives first before the S-wave received signal. Specifically, the arrival point of
287 S-wave received signal (point S) corresponds to the initial main excursion (point S_p), which
288 is in the opposite direction of movement compared to that of point S in the S-wave received
289 signal. Apparently, the curvature of these points (S and S_p) becomes much sharper when the
290 input frequency increases up to 40 or 50 kHz, as shown in Figure 7c and 7d. Furthermore, the
291 arrival point of P-wave received signal (point P) just corresponds to the arrival point of the
292 near field components (point P_s) in the S-wave received signal. This is in good agreement
293 with the observation by Brignoli et al. 1996 - a “near field component” travelling at a similar
294 velocity as P-wave was observed on a dry sample. Therefore, we propose this method,
295 namely S+P method to determine the arrival point of S-wave: the arrival time is defined by
296 point S (corresponding point S_p in the P-wave received signal).

297 To verify the accuracy of this S+P method, Figure 8 collects all data of shear wave velocity
298 obtained from different interpretation methods. The results obtained from the S+P method are
299 compared with those from other methods. Note that in the arrival-to-arrival method, point b
300 (first reversal as mentioned above) is chosen as the arrival point of S-wave. As for the result

by π -point method in Figure 8, the straight line represents a unique value of travel time which is determined as the input frequency slowly increasing from about 20 kHz to 50 kHz in each case. The differences between the shear wave velocities (V_s) obtained from the conventional time domain methods (arrival-to-arrival method and peak-to-peak method) and that from the “ π -point” method are about 30%. Similar ranges of difference are reported by other researchers (Viggiani and Atkinson 1995; Viana da Fonseca et al. 2009; Ogino et al. 2015). Nevertheless, the difference between the results obtained from S+P method and that taken from “ π -point” method is only around 10% in all tests; while this difference shows a good agreement between the results obtained from S+P method and those from cross correlation method, especially in the high frequency range, as illustrated in Figure 8. Therefore, the accuracy of the proposed method (S+P method) can be confirmed.

Discussions

The results obtained in this study show that the S-wave received signals are unreadable at lower frequencies ($f = 5, 10$ and 15 kHz), due to the influence of near field components. The effect of the latter is usually significant insofar as it obscures the shear wave arrival time point when the input frequency is low and the distance between the transmitter and receiver is short. Many researchers used the ratio of wave path length to wavelength L_{tt}/λ as an essential parameter to describe the near field effect. The near field effect can be reduced markedly with the increase of L_{tt}/λ , by improving the input frequency of transmitter or enlarging the distance between the transmitter and the receiver.

322 The results indicate also that the near field effect diminishes and a much clear received signal
323 appears as the L_{tt}/λ value reaches 1.91 (in the arrival-to-arrival method). Similar conclusion
324 can be made referring to the results of “ π -point” method which shows a good linear
325 relationship between the number of wavelength and frequency when the input frequency
326 increases from 20 kHz (in case of $L_{tt}/\lambda = 1.9$).

327 A higher frequency makes the received signal more readable in case of testing on a stiff
328 material such as compacted lime-treated silt. However, the P-wave component can become
329 significant when the excitation frequency is high (Brignoli et al. 1996). This is different from
330 a near field component (Lee and Santamarina 2005). Bender element can also generate small
331 compressive displacements even though the main displacements produced are of shear nature
332 (Brignoli et al. 1996). More importantly, the small compressive components (P-wave
333 components) can be prominent in case of high frequency being excited on stiff materials,
334 resulting in slight interference with the shear components (S-wave components) which have
335 relatively slower travel velocity. Conversely, some shear displacements would be generated
336 simultaneously with the major excitation of compressive displacements. Note that the polarity
337 of S-wave components is always contrary to that of P-wave components in the same signal. It
338 is even important that the travel velocity of S-wave components in P-wave received signal is
339 identical with that of the S-wave received signals.

340 According to the analyses above, the determination of arrival point of S-wave received
341 signals becomes clear and objective using the S+P method. That is, based on the comparison
342 between S-wave and P-wave received signals, a unique point, S (as illustrated in Figure 7),
343 can be identified and it corresponds to the arrival point of S-wave. This point located in

S-wave received signal, corresponding exactly to the point S_p in P-wave received signal, which shows opposite direction of movement in comparison with the point S in S-wave signal.

The results from S+P method show a small effect of input frequency (in a range from 20 to 50 kHz) on the shear wave velocity. As Figure 8 shows, shear wave velocity increases slightly as the input frequency increases. Similar phenomena were reported by other authors. Youn et al. (2008) pointed out that the shear wave velocity obtained from time domain method presents a small increasing trend with the increase of excitation frequency. Yamashita et al. (2009) also noted that the shear modulus increases with the increase of excitation frequency. To a certain extent, it might be a possible reason to explain why a slight decreasing trend of arrival time is observed when the excitation frequency increases. Specifically, when the frequency is relatively low (as shown in Figure 7a and 7b), point S determined by S+P method as the arrival point is not very clear. By contrast, in the case of high frequency (as shown in Figure 7c and 7d), point S is easy to be distinguished due to a sharp curvature at point S_p in the P-wave received signal. Brignoli et al. (1996) also recommended that high frequency should be used when measuring the P-wave components.

Conclusions

Due to the near field effect, reflected waves and soil properties, determining the shear wave arrival time accurately and reliably in bender elements testing is difficult and still controversial, and new methods need to be developed.

365 In this study, a novel method is proposed for a practical interpretation of the results from
366 bender element tests, for unsaturated or nearly saturated soil specimen. This method, namely
367 S+P method, is mainly based on the comparison between the S-wave and P-wave received
368 signals, and enables the determination of the arrival point of S-wave signal in a more
369 objective fashion. When a high frequency S-wave signal is excited, the P-wave components
370 become evident, and are easy to be distinguished from the S-wave received signal. Note that
371 the P-wave received signal also includes some S-wave components which arrive after the
372 P-wave components. Based on the identical travel velocity of S-wave components in both
373 S-wave received signal and P-wave received signal, and the opposite polarities between these
374 two different S-wave components, a unique arrival point of S-wave can be determined.
375 Comparisons between the results obtained by this method and those by the π -point method
376 and cross correlation method were made, indicating the relevance of the proposed method. It
377 is also worth noting that compared to the π -point method, the proposed method is less time
378 consuming.

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Table 1 Sample characteristics

Sample	Dry density (Mg/m ³)	Water content (%)	Degree of saturation (%)	Curing time (Hours)
D1	1.65	17	72	25
D2	1.65	17	72	528
W1	1.65	22	93	27
W2	1.65	22	93	504

Figure 1. Sketch of bender and extender connection: (a) transmitting S-wave; (b) transmitting P-wave (after Lings and Greening 2001)

Figure 2. Typical S-wave transmitted and received signals

Figure 3. Bender elements transducers: (a) wiring of two bender elements; (b) GDS bender elements waterproofed and encapsulated in pot

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