Thick periodic plates homogenization, application to sandwich panels including chevron folded core

Arthur Lebée, Karam Sab

Laboratoire Navier (UMR CNRS 8205)
Université Paris-Est - Ecole des Ponts ParisTech - IFSTTAR

May 10, 2011
Shear forces effects in heterogeneous plates?

- Raises many difficulties in laminated plates:
  - wrong shear deflection (transverse shear correction factors in laminates)
  - no accurate estimate of local stress generated ("free faces" effect)

- Almost no simple method when the plate is periodic

⇒ apply a homogenization scheme derived from a new plate theory (Lebée and Sab, 2010) to a sandwich panel including a folded core
Folded Cores for sandwich panels
The classical approach for sandwich panels
Basics of the Bending-Gradient theory (Lebée and Sab, 2010)
Application to sandwich panels
Contents

Folded Cores for sandwich panels
The classical approach for sandwich panels
Basics of the Bending-Gradient theory (Lebée and Sab, 2010)
Application to sandwich panels
Folded Cores for sandwich panels

A cheaper substitute to honeycomb:

- Many possible materials (if foldable...: paper, metals...)
- Continuous process (Kehrle, 2004)
Folded Cores for sandwich panels

A cheaper substitute to honeycomb:

- Many possible materials (if foldable...: paper, metals...)
- Continuous process (Kehrle, 2004)

⇒ the chevron pattern is investigated: assessment?
Folded Cores for sandwich panels
The classical approach for sandwich panels
Sandwich panel simplified model
Closed-form bounds for shear forces stiffness
Finite Element bounds for shear forces stiffness
Basics of the Bending-Gradient theory (Lebée and Sab, 2010)
Application to sandwich panels
Contents

Folded Cores for sandwich panels
The classical approach for sandwich panels
Sandwich panel simplified model
Closed-form bounds for shear forces stiffness
Finite Element bounds for shear forces stiffness
Basics of the Bending-Gradient theory (Lebée and Sab, 2010)
Application to sandwich panels
Sandwich panel simplified model

- Relevant loadings
  - Skins involved in bending:
    - traction/compression of the skins
Sandwich panel simplified model

- Relevant loadings
  - Skins involved in bending:
    - traction/compression of the skins
  - Core involved with shear forces:
    quantity of interest
Sandwich panel simplified model

- Relevant loadings
  - Skins involved in bending:
    - traction/compression of the skins
  - Core involved with shear forces:
    quantity of interest

- Implicit contrast assumption:
  “skins are stiff compared to the core”
Sandwich panel simplified model

- **Relevant loadings**
  - Skins involved in bending:
    - traction/compression of the skins
  - Core involved with shear forces:
    - quantity of interest

- **Implicit contrast assumption:**
  “skins are stiff compared to the core”

- **Bounds from Kelsey et al. (1958) for shear forces stiffness**
  - Apply uniform stress/displacement on a unit cell of the core, replacing skins.
Contents

Folded Cores for sandwich panels
The classical approach for sandwich panels
Sandwich panel simplified model
Closed-form bounds for shear forces stiffness
Finite Element bounds for shear forces stiffness
Basics of the Bending-Gradient theory (Lebée and Sab, 2010)
Application to sandwich panels
Closed-form bounds for shear forces stiffness

- Parameters (pattern reduced to a tilted parallelogram):
  - Facet thickness $t_f$
  - Shape ratio $a_0/b_0$
  - 2 orientation angles $\delta$ et $\zeta$
Closed-form bounds for shear forces stiffness

- Parameters (pattern reduced to a tilted parallelogram):
  - Facet thickness $t_f$
  - Shape ratio $a_0/b_0$
  - 2 orientation angles $\delta$ et $\zeta$

- Assumptions:
  - Plane-stress in facets
  - Piecewise uniform fields
Closed-form bounds for shear forces stiffness

Results

- Manufactured cores (Nguyen et al., 2005):
  - $\delta = 72^\circ$ and $\zeta = 34^\circ$
  - Shape ratio: $0.5 < a_0/b_0 < 1.5$

- Normalized bounds $\mathcal{E} = F_{11}/\rho G_m h$:
  - $0.23 < \mathcal{E} < 0.71$: very loose

$0 < \mathcal{E} < 1$

$\rho G_m h$: Voigt upper bound
Contents

Folded Cores for sandwich panels

The classical approach for sandwich panels

Sandwich panel simplified model

Closed-form bounds for shear forces stiffness

Finite Element bounds for shear forces stiffness

Basics of the Bending-Gradient theory (Lebée and Sab, 2010)

Application to sandwich panels
Finite Element bounds for shear forces stiffness

- Shell elements

![Finite Element bounds for shear forces stiffness](image)
The classical approach for sandwich panels

Finite Element bounds

- Shell elements
- Consistent with analytical bounds

\[
\frac{a_0}{b_0} = 1.5
\]

\[
\frac{a_0}{b_0} = 0.5
\]

\[
\frac{a_0}{b_0} = 1
\]

\[
\varepsilon_1^{+, FE}
\]

\[
\varepsilon_1^{-, FE}
\]

\[
\varepsilon_1^{+, FE}
\]

\[
\varepsilon_1^{-, FE}
\]

\[
\varepsilon_1^{+, FE}
\]

\[
\varepsilon_1^{-, FE}
\]

\[
\varepsilon_1^{+, FE}
\]

\[
\varepsilon_1^{-, FE}
\]

\[
\varepsilon_1^{+, FE}
\]

\[
\varepsilon_1^{-, FE}
\]

\[
\varepsilon_1^{+, FE}
\]

\[
\varepsilon_1^{-, FE}
\]

\[
\varepsilon_1^{+, FE}
\]

\[
\varepsilon_1^{-, FE}
\]

\[
\varepsilon_1^{+, FE}
\]

\[
\varepsilon_1^{-, FE}
\]

\[
\varepsilon_1^{+, FE}
\]

\[
\varepsilon_1^{-, FE}
\]

\[
\varepsilon_1^{+, FE}
\]

\[
\varepsilon_1^{-, FE}
\]

\[
\varepsilon_1^{+, FE}
\]

\[
\varepsilon_1^{-, FE}
\]

\[
\varepsilon_1^{+, FE}
\]

\[
\varepsilon_1^{-, FE}
\]

\[
\varepsilon_1^{+, FE}
\]

\[
\varepsilon_1^{-, FE}
\]

\[
\varepsilon_1^{+, FE}
\]

\[
\varepsilon_1^{-, FE}
\]

\[
\varepsilon_1^{+, FE}
\]

\[
\varepsilon_1^{-, FE}
\]

\[
\varepsilon_1^{+, FE}
\]

\[
\varepsilon_1^{-, FE}
\]

\[
\varepsilon_1^{+, FE}
\]

\[
\varepsilon_1^{-, FE}
\]

\[
\varepsilon_1^{+, FE}
\]

\[
\varepsilon_1^{-, FE}
\]

\[
\varepsilon_1^{+, FE}
\]

\[
\varepsilon_1^{-, FE}
\]

\[
\varepsilon_1^{+, FE}
\]

\[
\varepsilon_1^{-, FE}
\]

\[
\varepsilon_1^{+, FE}
\]

\[
\varepsilon_1^{-, FE}
\]

\[
\varepsilon_1^{+, FE}
\]

\[
\varepsilon_1^{-, FE}
\]

\[
\varepsilon_1^{+, FE}
\]

\[
\varepsilon_1^{-, FE}
\]

\[
\varepsilon_1^{+, FE}
\]

\[
\varepsilon_1^{-, FE}
\]

\[
\varepsilon_1^{+, FE}
\]

\[
\varepsilon_1^{-, FE}
\]

\[
\varepsilon_1^{+, FE}
\]

\[
\varepsilon_1^{-, FE}
\]

\[
\varepsilon_1^{+, FE}
\]

\[
\varepsilon_1^{-, FE}
\]

\[
\varepsilon_1^{+, FE}
\]

\[
\varepsilon_1^{-, FE}
\]

\[
\varepsilon_1^{+, FE}
\]

\[
\varepsilon_1^{-, FE}
\]

\[
\varepsilon_1^{+, FE}
\]

\[
\varepsilon_1^{-, FE}
\]

\[
\varepsilon_1^{+, FE}
\]

\[
\varepsilon_1^{-, FE}
\]

\[
\varepsilon_1^{+, FE}
\]

\[
\varepsilon_1^{-, FE}
\]

\[
\varepsilon_1^{+, FE}
\]

\[
\varepsilon_1^{-, FE}
\]

\[
\varepsilon_1^{+, FE}
\]

\[
\varepsilon_1^{-, FE}
\]

\[
\varepsilon_1^{+, FE}
\]

\[
\varepsilon_1^{-, FE}
\]

\[
\varepsilon_1^{+, FE}
\]

\[
\varepsilon_1^{-, FE}
\]

\[
\varepsilon_1^{+, FE}
\]

\[
\varepsilon_1^{-, FE}
\]

\[
\varepsilon_1^{+, FE}
\]

\[
\varepsilon_1^{-, FE}
\]

\[
\varepsilon_1^{+, FE}
\]

\[
\varepsilon_1^{-, FE}
\]

\[
\varepsilon_1^{+, FE}
\]

\[
\varepsilon_1^{-, FE}
\]

\[
\varepsilon_1^{+, FE}
\]

\[
\varepsilon_1^{-, FE}
\]

\[
\varepsilon_1^{+, FE}
\]

\[
\varepsilon_1^{-, FE}
\]

\[
\varepsilon_1^{+, FE}
\]

\[
\varepsilon_1^{-, FE}
\]

\[
\varepsilon_1^{+, FE}
\]

\[
\varepsilon_1^{-, FE}
\]

\[
\varepsilon_1^{+, FE}
\]

\[
\varepsilon_1^{-, FE}
\]

\[
\varepsilon_1^{+, FE}
\]

\[
\varepsilon_1^{-, FE}
\]

\[
\varepsilon_1^{+, FE}
\]

\[
\varepsilon_1^{-, FE}
\]

\[
\varepsilon_1^{+, FE}
\]

\[
\varepsilon_1^{-, FE}
\]

\[
\varepsilon_1^{+, FE}
\]

\[
\varepsilon_1^{-, FE}
\]

\[
\varepsilon_1^{+, FE}
\]

\[
\varepsilon_1^{-, FE}
\]

\[
\varepsilon_1^{+, FE}
\]

\[
\varepsilon_1^{-, FE}
\]

\[
\varepsilon_1^{+, FE}
\]

\[
\varepsilon_1^{-, FE}
\]

\[
\varepsilon_1^{+, FE}
\]

\[
\varepsilon_1^{-, FE}
\]

\[
\varepsilon_1^{+, FE}
\]

\[
\varepsilon_1^{-, FE}
\]

\[
\varepsilon_1^{+, FE}
\]

\[
\varepsilon_1^{-, FE}
\]

\[
\varepsilon_1^{+, FE}
\]

\[
\varepsilon_1^{-, FE}
\]

\[
\varepsilon_1^{+, FE}
\]

\[
\varepsilon_1^{-, FE}
\]

\[
\varepsilon_1^{+, FE}
\]

\[
\varepsilon_1^{-, FE}
\]

\[
\varepsilon_1^{+, FE}
\]

\[
\varepsilon_1^{-, FE}
\]

\[
\varepsilon_1^{+, FE}
\]

\[
\varepsilon_1^{-, FE}
\]

\[
\varepsilon_1^{+, FE}
\]

\[
\varepsilon_1^{-, FE}
\]

\[
\varepsilon_1^{+, FE}
\]

\[
\varepsilon_1^{-, FE}
\]

\[
\varepsilon_1^{+, FE}
\]

\[
\varepsilon_1^{-, FE}
\]

\[
\varepsilon_1^{+, FE}
\]

\[
\varepsilon_1^{-, FE}
\]

\[
\varepsilon_1^{+, FE}
\]

\[
\varepsilon_1^{-, FE}
\]

\[
\varepsilon_1^{+, FE}
\]

\[
\varepsilon_1^{-, FE}
\]

\[
\varepsilon_1^{+, FE}
\]

\[
\varepsilon_1^{-, FE}
\]

\[
\varepsilon_1^{+, FE}
\]

\[
\varepsilon_1^{-, FE}
\]

\[
\varepsilon_1^{+, FE}
\]

\[
\varepsilon_1^{-, FE}
\]
Finite Element bounds for shear forces stiffness

- Shell elements
- Consistent with analytical bounds
- Still loose: more discrepancy than for honeycombs

\[ \frac{E_1^{-, FE}}{E_1^{+, FE}} \]

\[ a_0/b_0 = 1.5 \]

\[ a_0/b_0 = 0.5 \]

\[ a_0/b_0 = 1 \]
Finite Element bounds for shear forces stiffness

- Shell elements
- Consistent with analytical bounds
- **Still loose**: more discrepancy than for honeycombs
- Engineers often refer to the upper bound (rigid skins)

![Diagram showing ratios and bounds](image-url)
Contents

Folded Cores for sandwich panels
The classical approach for sandwich panels
Basics of the Bending-Gradient theory (Lebée and Sab, 2010)
From 3D to a plate model
Reissner’s approach for a homogeneous plate
The Bending-Gradient plate theory
Application to sandwich panels
Contents

Folded Cores for sandwich panels
The classical approach for sandwich panels
Basics of the Bending-Gradient theory (Lebée and Sab, 2010)
From 3D to a plate model
Reissner’s approach for a homogeneous plate
The Bending-Gradient plate theory
Application to sandwich panels
The 3D problem

\[
\begin{align*}
\sigma_{ij,j} &= 0 \quad \text{on } \Omega. \\
\sigma_{ij} &= C_{ijkl}(x_3)\varepsilon_{kl} \quad \text{on } \Omega. \\
\sigma_{i3} &= T_i^\pm \quad \text{on } \omega^\pm. \\
\varepsilon_{ij} &= \frac{1}{2} (u_{i,j} + u_{j,i}) \quad \text{on } \Omega. \\
u_i &= 0 \quad \text{on } \partial\omega \times [-h/2, h/2].
\end{align*}
\]

- Laminated plate
- Clamped plate
- Only out-of-plane loading, per unit surface

\[
T^\pm = \frac{p_3}{2} e_3
\]
Plate stress and equilibrium equations

- Plate stresses

\[
\begin{align*}
M_{\alpha\beta}(x_1, x_2) &= \int_{-\frac{h}{2}}^{\frac{h}{2}} x_3 \sigma_{\alpha\beta} dx_3 \\
Q_\alpha(x_1, x_2) &= \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_3 dx_3
\end{align*}
\]
Plate stress and equilibrium equations

- **Plate stresses**
  \[
  \begin{align*}
  M_{\alpha\beta}(x_1, x_2) &= \int_{-\frac{h}{2}}^{\frac{h}{2}} x_3 \sigma_{\alpha\beta} dx_3 \\
  Q_{\alpha}(x_1, x_2) &= \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{\alpha} dx_3
  \end{align*}
  \]

- **Equilibrium equations**:
  \[
  \begin{align*}
  \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{\alpha} dx_3 &\quad \Rightarrow \quad Q_{\alpha,\alpha} + p_3 = 0 \\
  \int_{-\frac{h}{2}}^{\frac{h}{2}} x_3 \sigma_{\alpha\beta,\beta} dx_3 &\quad \Rightarrow \quad M_{\alpha\beta,\beta} - Q_{\alpha} = 0
  \end{align*}
  \]
  Boussinesq (1871); Mindlin (1951)...
Plate stress and equilibrium equations

- Plate stresses

\[
\begin{align*}
M_{\alpha\beta}(x_1, x_2) &= \int_{-\frac{h}{2}}^{\frac{h}{2}} x_3 \sigma_{\alpha\beta} \, dx_3 \\
Q_{\alpha}(x_1, x_2) &= \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{\alpha} \, dx_3
\end{align*}
\]

- Equilibrium equations:

\[
\begin{align*}
\int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{\alpha3,3} \, dx_3 & \Rightarrow \begin{cases} 
Q_{\alpha,\alpha} + p_3 = 0 \\
M_{\alpha\beta,\beta} - Q_{\alpha} = 0
\end{cases} \\
\int_{-\frac{h}{2}}^{\frac{h}{2}} x_3 \sigma_{\alpha\beta,\beta} \, dx_3
\end{align*}
\]

Boussinesq (1871); Mindlin (1951)...

- \( \sigma^{(M)} \): asymptotic expansion

- \( \sigma^{(Q)} \): ??
Contents

- Folded Cores for sandwich panels
- The classical approach for sandwich panels
- Basics of the Bending-Gradient theory (Lebée and Sab, 2010)
  - From 3D to a plate model
  - Reissner’s approach for a homogeneous plate
  - The Bending-Gradient plate theory
- Application to sandwich panels
Reissner’s approach for a homogeneous plate

- Bending stress fields (asymptotic expansion: $\varepsilon_{\alpha\beta} = x_3 \kappa_{\alpha\beta}$):

$$\sigma^{(M)} = \frac{12x_3}{h^3} \begin{pmatrix} M_{11} & M_{12} & 0 \\ M_{12} & M_{22} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$
Reissner’s approach for a homogeneous plate

- Bending stress fields (asymptotic expansion: \( \varepsilon_{\alpha\beta} = x_3 \kappa_{\alpha\beta} \)):
  \[
  \sigma^{(M)} = \frac{12x_3}{h^3} \begin{pmatrix}
  M_{11} & M_{12} & 0 \\
  M_{12} & M_{22} & 0 \\
  0 & 0 & 0
  \end{pmatrix}
  \]

- Volume force related to shear forces:
  \[
  \sigma^{(M)}_{ij,j} = \frac{12x_3}{h^3} \begin{pmatrix}
  M_{1\alpha,\alpha} \\
  M_{2\alpha,\alpha} \\
  0
  \end{pmatrix} = \frac{12x_3}{h^3} \begin{pmatrix}
  Q_1 \\
  Q_2 \\
  0
  \end{pmatrix} = f_i^{(Q)}
Reissner’s approach for a homogeneous plate

- Bending stress fields (asymptotic expansion: $\varepsilon_{\alpha\beta} = x_3 \kappa_{\alpha\beta}$):

$$\sigma^{(M)} = \frac{12x_3}{h^3} \begin{pmatrix} M_{11} & M_{12} & 0 \\ M_{12} & M_{22} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

- Volume force related to shear forces:

$$\sigma^{(M)}_{ij,j} = \frac{12x_3}{h^3} \begin{pmatrix} M_{1\alpha,\alpha} \\ M_{2\alpha,\alpha} \\ 0 \end{pmatrix} = \frac{12x_3}{h^3} \begin{pmatrix} Q_1 \\ Q_2 \\ 0 \end{pmatrix} = f_i^{(Q)}$$

- Transverse shear unit-load problem (Reissner, 1945):

$$\begin{cases} \sigma_{ij,j}^{(Q)} + f_i^{(Q)} = 0 \\ \sigma_{i3}^{(Q)} = 0 \quad \text{on} \quad x_3 = \pm h/2 \end{cases} \Rightarrow \sigma_{ij}^{(Q)} = \frac{3}{2h} \left(1 - \frac{4x_3^2}{h^2}\right) \begin{pmatrix} 0 & 0 & Q_1 \\ 0 & 0 & Q_2 \\ Q_1 & Q_2 & 0 \end{pmatrix}$$
Contents

Folded Cores for sandwich panels
The classical approach for sandwich panels
Basics of the Bending-Gradient theory (Lebée and Sab, 2010)
   From 3D to a plate model
   Reissner’s approach for a homogeneous plate
The Bending-Gradient plate theory
Application to sandwich panels
Revisiting Reissner’s approach

- Building the body force imposes the introduction of the full bending gradient:

\[ f^Q \] becomes \( f^R \), where: \( R_{\alpha\beta\gamma} = M_{\alpha\beta,\gamma} \)
Revisiting Reissner’s approach

- Building the body force imposes the introduction of the full bending gradient:

\[ f(Q) \text{ becomes } f(R), \text{ where: } R_{\alpha \beta \gamma} = M_{\alpha \beta, \gamma} \]

- We define the full bending gradient local stress field:

\[ \sigma^{BG} = \sigma^{(N)} + \sigma^{(M)} + \sigma^{(R)} \]
Revisiting Reissner’s approach

- Building the body force imposes the introduction of the full bending gradient:

\[ f(Q) \text{ becomes } f(R), \text{ where: } R_{\alpha\beta\gamma} = M_{\alpha\beta,\gamma} \]

- We define the full bending gradient local stress field:

\[ \sigma^{BG} = \sigma^{(N)} + \sigma^{(M)} + \sigma^{(R)} \]

- Mechanical meaning of \( R \):

\[
\begin{cases}
Q_1 = R_{111} + R_{122} = M_{11,1} + M_{12,2} \\
Q_2 = R_{121} + R_{222} = M_{21,1} + M_{22,2}
\end{cases}
\]

Thus:

- \( R_{111} = M_{11,1} \): Cylindrical Bending part of \( Q_1 \)
- \( R_{221} = M_{22,1} \): Pure warping
- \( R_{121} = M_{12,1} \): Torsion part of \( Q_2 \)
- \( R_{112} = M_{11,2} \): Pure warping
- \( R_{222} = M_{22,2} \): Cylindrical Bending part of \( Q_2 \)
- \( R_{122} = M_{12,2} \): Torsion part of \( Q_1 \)
Major features of the Bending-Gradient theory

- Enables the distinction between each component of $R_{\alpha\beta\gamma}$:

- The exact extension of RM model to laminated plates:
  - If the plate is homogeneous, BG is turned into RM model
  - The restriction of the BG to a RM model is not unique (except if homogeneous)

- A successful application to highly anisotropic laminated plates
  - Excellent estimate of transverse shear fields and deflection
  - Local fields converge with slenderness (St Venant Solution)
Contents

Folded Cores for sandwich panels
The classical approach for sandwich panels
Basics of the Bending-Gradient theory (Lebée and Sab, 2010)
Application to sandwich panels
  Extension to periodic plates
  Justification of the classical approach for sandwich panels
Application to the chevron pattern
Contents

Folded Cores for sandwich panels
The classical approach for sandwich panels
Basics of the Bending-Gradient theory (Lebée and Sab, 2010)
Application to sandwich panels
Extension to periodic plates
Justification of the classical approach for sandwich panels
Application to the chevron pattern
Extension to periodic plates

- Unit-cell and average estimates
Extension to periodic plates

- Unit-cell and average estimates
- Bending auxiliary problem (Caillerie, 1984)

\[ \begin{aligned}
\mathcal{P}(\kappa) &= \\
\begin{cases}
\tilde{\sigma}^{(\kappa)} \cdot \tilde{\nabla} = 0 \\
\tilde{\sigma}^{(\kappa)} = \tilde{\mathcal{C}}(\tilde{y}) : \tilde{\varepsilon}^{(\kappa)} \\
\tilde{\varepsilon}^{(\kappa)} = y_3 \tilde{\kappa} + \tilde{\nabla} \otimes \tilde{u}^{\text{per}} \\
\tilde{\sigma}^{(\kappa)} \cdot \tilde{e}_3 = 0 \text{ on free faces } \partial Y_3^\pm \\
\tilde{\sigma}^{(\kappa)} \cdot \tilde{n} \text{ skew-periodic on lateral edge } \partial Y_l \\
\tilde{u}^{\text{per}}(\tilde{y}) \ (y_1, y_2)\text{-periodic on lateral edge } \partial Y_l
\end{cases}
\end{aligned} \]

→ gives:

Localization related to curvature \( \kappa \)

Bending compliance tensor: \( \mathcal{D} \)

→ enable the derivation of \( f^{(R)} \)
Extension to periodic plates

- Unit-cell and average estimates
- Bending auxiliary problem (Caillerie, 1984)
- Shear auxiliary problem

\[
\mathcal{P}^{(R)} \begin{cases} 
\tilde{\boldsymbol{\sigma}}^{(R)} \cdot \nabla \tilde{v} + \tilde{\mathbf{f}}^{(R)} (\tilde{y}) = 0 \\
\tilde{\boldsymbol{\sigma}}^{(R)} = \tilde{\mathcal{C}} (\tilde{y}) : \left( \nabla \otimes \tilde{\mathbf{u}}^{(R)} \right) \\
\tilde{\boldsymbol{\sigma}}^{(R)} \cdot \tilde{\mathbf{e}}_3 = 0 \text{ on free faces } \partial Y_3^\pm \\
\tilde{\boldsymbol{\sigma}}^{(R)} \cdot \tilde{\mathbf{n}} \text{ skew-periodic on lateral edge } \partial Y_1 \\
\tilde{\mathbf{u}}^{(R)} (\tilde{y}) (y_1, y_2) \text{-periodic on lateral edge } \partial Y_1 \end{cases}
\]

→ gives:

Localization related to \( \mathbb{R} \)
Shear compliance tensor: \( \tilde{\mathcal{C}} (\tilde{y}) \)
Contents

Folded Cores for sandwich panels
The classical approach for sandwich panels
Basics of the Bending-Gradient theory (Lebée and Sab, 2010)
Application to sandwich panels
Extension to periodic plates
Justification of the classical approach for sandwich panels
Application to the chevron pattern
Justification of the classical approach (sandwich theory)

- Divide in 3 layers
  (homogeneous skins and heterogeneous core)
Justification of the classical approach (sandwich theory)

- Divide in 3 layers
  (homogeneous skins and heterogeneous core)
- Bending auxiliary problem
  - Contrast assumption $\Leftrightarrow t_f \ll t_s$:
    $\rightarrow t_s / t_f$ Contrast ratio
  $\Rightarrow$ Skins under traction/compression
  $\Rightarrow$ Core not involved in Bending stiffness
Justification of the classical approach (sandwich theory)

- Divide in 3 layers
  (homogeneous skins and heterogeneous core)
- Bending auxiliary problem
- Shear auxiliary problem
  \( \sim^{(R)} \) becomes \( \sim^{(Q)} \)
  - The BG is degenerated into RM model
  - \( \sim^{(Q)} \) confirms the classical intuition
  - Proof of the bounds from Kelsey et al. (1958)
Folded Cores for sandwich panels
The classical approach for sandwich panels
Basics of the Bending-Gradient theory (Lebée and Sab, 2010)
Application to sandwich panels
Extension to periodic plates
Justification of the classical approach for sandwich panels
Application to the chevron pattern
Shear forces localization $\sigma^{(Q)}$

- Overall shearing of the core
Application to the chevron pattern

Shear forces localization $\sigma^{(Q)}$

- Overall shearing of the core
- Out-of-plane skins distortion
Application to the chevron pattern

Shear forces localization $\sigma^{(Q)}$

- Overall shearing of the core
- Out-of-plane skins distorsion
- Critically influence shear force stiffness
Comparaison with full 3D simulation
Comparaison with full 3D simulation

\[ h = 30\text{mm}, \quad t_s = 1, \quad t_f = 0.1\text{mm} \]
Is skin distortion really critical?

- mid-span deflection:

\[ U_3 = U_3^{KL} \left( 1 + \left( \frac{L^*}{L} \right)^2 \right) \]

where \( L^* = \pi \sqrt{\frac{D_{1111}}{F_{11}}} \)
Is skin distortion really critical?

- mid-span deflection:

\[ U_3 = U_3^{KL} \left( 1 + \left( \frac{L^*}{L} \right)^2 \right) \]

where \( L^* = \pi \sqrt{\frac{D_{1111}}{F_{11}}} \)

- The shift “stiff/compliant skins” occurs for usual contrast ratios:

\[ \frac{t_s}{t_f} = 20 \text{ and } \frac{L}{h} = 20 \]

\[ \Rightarrow 25\% < \left( \frac{L^*}{L} \right)^2 < 60\% \]
Conclusion

- Analytical bounds from Kelsey et al. (1958)
  - useful (optimization, preliminary design)
  - but limited (loose bounds): neglects core/skin interaction
Conclusion

- Analytical bounds from Kelsey et al. (1958)
  - useful (optimization, preliminary design)
  - but limited (loose bounds): neglects core/skin interaction
- Application of the Bending-Gradient theory to sandwich panels
  - Quantification of the contrast assumption
  - The Bending-Gradient is turned into a Reissner-Mindlin
  - Proof of Kelsey et al. (1958) bounds
Conclusion

- Analytical bounds from Kelsey et al. (1958)
  - useful for optimization, preliminary design
  - but limited (loose bounds): neglects core/skin interaction

- Application of the Bending-Gradient theory to sandwich panels
  - Quantification of the contrast assumption
  - The Bending-Gradient is turned into a Reissner-Mindlin
  - Proof of Kelsey et al. (1958) bounds

- Application to the chevron pattern
  - Brings out the critical effect of skin/core interaction
Conclusion

- Analytical bounds from Kelsey et al. (1958)
  - useful (optimization, preliminary design)
  - but limited (loose bounds): neglects core/skin interaction
- Application of the Bending-Gradient theory to sandwich panels
  - Quantification of the contrast assumption
  - The Bending-Gradient is turned into a Reissner-Mindlin
  - Proof of Kelsey et al. (1958) bounds
- Application to the chevron pattern
  - Brings out the critical effect of skin/core interaction
- Outlooks
  - Strength analysis of sandwich panels under shear forces?