Should marginal abatement costs differ across sectors? The effect of low-carbon capital accumulation

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Abstract

Climate mitigation is largely done through investments in low-carbon capital that will have long-lasting effects on emissions. In a model that represents explicitly low-carbon capital accumulation, optimal marginal investment costs differ across sectors. They are equal to the value of avoided carbon emissions over time, minus the value of the forgone option to invest later. It is therefore misleading to assess the cost-efficiency of investments in low-carbon capital by comparing levelized abatement costs, measured as the ratio of investment costs to discounted abatement. The equimarginal principle applies to an accounting value: the Marginal Implicit Rental Cost of the Capital (MIRCC) used to abate. Two apparently opposite views are reconciled. On the one hand, higher efforts are justified in sectors that will take longer to decarbonize, such as transport and urban planning; on the other hand, the MIRCC should be equal to the carbon price at each point in time and in all sectors. Equalizing the MIRCC in each sector to the social cost of carbon is a necessary condition to reach the optimal pathway, but it is not a sufficient condition. Decentralized optimal investment decisions at the sector level require not only the information contained in the carbon price signal, but also knowledge of the date when the sector reaches its full abatement potential.

Keywords: climate change mitigation; carbon price; path dependence; sectoral policies; optimal timing; inertia; levelized costs
JEL classification: L98, O21, O25, Q48, Q51, Q54, Q58

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1. Introduction

Many countries have set ambitious targets to reduce their Greenhouse Gas (GHG) emissions. To do so, most of them rely on several policy instruments. The European Union, for instance, has implemented an emission trading system, feed-in tariffs and portfolio standards in favor of renewable power, and

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different energy efficiency standards on new passenger vehicles, buildings, home appliances and industrial motors. These sectoral policies are often designed to spur investments that will have long-lasting effects on emissions, but they have been criticized because they result in different Marginal Abatement Costs (MACs) in different sectors.

Existing analytical assessments conclude that differentiating MACs is a second-best policy, justified if multiple market failures cannot be corrected independently (Lipsey and Lancaster, 1956). For instance, if governments cannot use tariffs to discriminate imports from countries where no environmental policies are applied, it is optimal to differentiate the carbon tax between traded and non-traded sectors (Hoel, 1996). A government should tax emissions from households at a higher rate than emissions from the production sector, if labor supply exerts market power (Richter and Schneider, 2003). Also, many abatement options involve new technologies with increasing returns or learning-by-doing (LBD) and knowledge spillovers. This "twin-market failure" in the area of green innovation can be addressed optimally by combining a carbon price with a subsidy on technologies subjected to LBD and an R&D subsidy (e.g., Jaffe et al., 2005; Fischer and Newell, 2008; Grimaud and Lafforgue, 2008; Gerlagh et al., 2009; Acemoglu et al., 2012). But if the only available instrument is a carbon tax, higher carbon prices are justified in sectors with larger learning effects (Rosendahl, 2004).

These studies often model a social planner who takes an abatement cost function as given, and may choose any quantity of abatement at each time step. Within this framework, MACs can be easily computed as the derivative of the cost functions; but the inertia induced by slow capital accumulation is neglected (Vogt-Schilb et al., 2012).

Only a few studies explicitly model inertia or slow capital accumulation; they conclude that higher efforts are required in the particular sectors that will take longer to decarbonize, such as transportation infrastructure (Lecocq et al., 1998; Jaccard and Rivers, 2007; Vogt-Schilb and Hallegatte, 2011); but they do not provide an analytical definition of abatement costs.

In this paper, we assess the optimal cost, timing and sectoral distribution of greenhouse gas emission reductions when abatement is obtained through investments in low-carbon capital. We use an intertemporal optimization model with three characteristics. First, we do not use abatement cost functions; instead, abatement is obtained by accumulating low-carbon capital that has a long-lasting effect on emissions. For instance, replacing a coal power plant by renewable power reduces GHG emissions for several decades. Second, these investments have convex costs : accumulating low-carbon capital faster is more expensive. For instance, retrofitting the entire building stock would be more expensive if done over a shorter period of time. Third, abatement cannot exceed a given maximum potential in each sector. This potential is exhausted when all the emitting capital (e.g fossil fuel power plants) is replaced by non-emitting capital (e.g. renewables).

We find that it is optimal to invest more dollars per unit of low-carbon capital in sectors that will take longer to decarbonize, as for instance sectors with greater baseline emissions. Indeed, with maximum abatement potentials, investing in low-carbon capital reduces both emissions and future investment opportunities. The optimal investment costs can be expressed as the value of avoided carbon, minus the value of the forgone option to abate later in the same sector. Since the latter term is different in each sector, it leads to different investment costs and levels.

There are multiple possible definitions of marginal abatement costs in a model where abatement is obtained through low-carbon capital accumulation. Here, we define the Marginal Levelized Abatement Cost (MLAC) as the marginal cost of low-carbon capital (compared to the cost of conventional capital) divided by the discounted emissions that it abates. This metric has been simply labeled as "Marginal Abatement Cost" (MAC) by scholars and government agencies, suggesting that it should be equal to the price of carbon. We find that MLACs should not be equal across sectors, and should not be equal to the carbon price.

Instead, the equimarginal principle applies to an accounting value: the Marginal Implicit Rental Cost of the Capital (MIRCC) used to abate. MIRCCs generalize the concept of implicit rental cost of capital proposed by Jorgenson (1967) to the case of endogenous investment costs. On the optimal pathway, MIRCCs – expressed in dollars per ton – are equal to the current carbon price and are thus equal in all sectors. If the abatement cost is defined as the implicit rental cost of the capital used to abate, a necessary condition for optimality is that MACs equal the carbon price. It is not a sufficient condition, as many sub-optimal investment pathways also satisfy this condition. Optimal investment decisions to decarbonize a sector require combining the information contained in the carbon price signal with knowledge of the date when the sector reaches its full abatement potential.

The rest of the paper is organized as follows. We present our model in section 2. In section 3, we solve it in a particular setting where investments in low-carbon capital have a permanent impact on emissions. In section 4, we solve the model in the general case where low-carbon capital depreciates at a non-negative rate. In section 5 we define the implicit marginal rental cost of capital and compute it along the optimal pathway. Section 6 concludes.

2. A model of low-carbon capital accumulation to cope with a carbon budget

We model a social planner (or any equivalent decentralized procedure) that chooses when and how (i.e., in which sector) to invest in low-carbon capital in order to meet a climate target at the minimum discounted cost.

2.1. Low-carbon capital accumulation

The economy is partitioned in a set of sectors indexed by i. For simplicity, we assume that abatement in each sector does not interact with the others.[1] Without loss of generality, the stock of low-carbon capital in each sector i starts at zero, and at each time step t, the social planner chooses a positive amount of physical investment $x_{i,t}$ in abating capital $a_{i,t}$, which depreciates at rate δ_i (dotted variables represent temporal derivatives):

$$a_{i,0} = 0 \tag{1}$$

$$\dot{a}_{i,t} = x_{i,t} - \delta_i a_{i,t} \tag{2}$$

[1] This is not completely realistic, as abatement realized in the power sector may actually reduce the cost to implement abatement in other sectors using electric-powered capital. In order to keep things simple, we let this issue to further research.

For simplicity, abating capital is directly measured in terms of avoided emissions (Tab. 1).[2] Investments in low-carbon capital cost $c_i(x_i)$, where the functions c_i are positive, increasing, differentiable and convex.

The cost convexity bears on the investments flow, to capture increasing opportunity costs to use scarce resources (skilled workers and appropriate capital) to build and deploy low-carbon capital. For instance, $x_{i,t}$ could stand for the pace — measured in buildings per year — at which old buildings are being retrofitted at date t (the abatement $a_{i,t}$ would then be proportional to the share of retrofitted buildings in the stock). Retrofitting buildings at a given pace requires to pay a given number of scarce skilled workers. If workers are hired in the merit order and paid at the marginal productivity, the marginal price of retrofitting buildings $c_i'(x_i)$ is a growing function of the pace x_i.

In each sector, a sectoral potential $\bar{a}_i$ represents the maximum amount of GHG emissions (in $\mathrm{GtCO_2}$ per year) that can be abated in this sector:

$$a_{i,t} \leq \bar{a}_i$$

For instance, if each vehicle is replaced by a zero-emission vehicle, all the abatement potential in the private mobility sector has been realized.[3] The sectoral potential may be roughly approximated by sectoral emissions in the baseline, but they may also be smaller (if some fatal emissions occur in the sector) or could even be higher (if negative emissions are possible). We make the simplifying assumption that the potentials $\bar{a}_i$ and the abatement cost functions c_i are constant over time and let the cases of evolving potentials and induced technical change to further research.

2.2. Carbon budget

The climate policy is modeled as a so-called *carbon budget* for emissions above a safe level (Fig. 1). We assume that the environment is able to absorb a constant flow of GHG emissions $E_s \geq 0$. Above E_s, emissions are dangerous. The objective is to maintain cumulative dangerous emissions below a given ceiling B. Cummulative emissions have been found to be a good proxy for climate change (Allen et al., 2009; Matthews et al., 2009).[4] For simplicity, we assume that all dangerous emissions are abatable, and, without loss of generality, that doing so requires to use all the sectoral potentials. Denoting E^d the emissions above E_s, this reads $E^d = \sum_i \bar{a}_i$. In other words, $(\bar{a}_i - a_{i,t})$ stands for the high carbon capital that has not been replaced by low carbon capital yet in sector i,

[2] Investments $x_{i,t}$ are therefore measured as additional abating capacity (in $\mathrm{tCO_2/yr}$) per unit of time, i.e. in $(\mathrm{tCO_2/yr})/\mathrm{yr}$.

[3] This modeling approach may remind the literature on the optimal extraction rates of non renewable resources. Our maximal abatement potentials are similar to the different deposits in Kemp and Long (1980), or the stocks of different fossil fuels in the more recent literature (e.g, Chakravorty et al., 2008; Smulders and Van Der Werf, 2008; van der Ploeg and Withagen, 2012). Our results are also similar, as these authors find in particular that different reservoirs or different types of non-renewable resources should not necessarily be extracted at the same marginal cost.

[4] Our conclusions are robust to other representations of climate policy objectives such as an exogenous carbon price, e.g. a Pigouvian tax; or a more complex climate model.

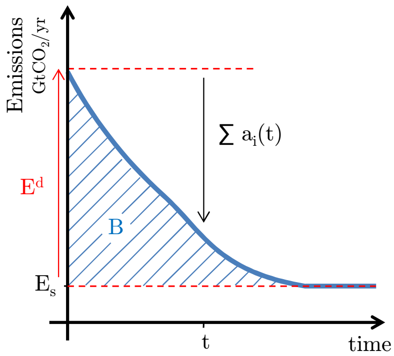

Figure 1: An illustration of the climate constraint. The cumulative emissions above E_s are capped to a carbon budget B (this requires that the long-run emissions tend to E_s). Dangerous emissions E^d are measured from E_s.

as measured in emissions. The carbon budget reads:

$$\int_0^\infty \sum_i (\bar{a}_i - a_{i,t})\, dt \leq B \tag{3}$$

2.3. The social planner's program

The full social planner's program reads:

$$\min_{x_{i,t}} \int_0^\infty e^{-rt} \sum_i c_i(x_{i,t})\, dt \tag{4}$$

$$\begin{aligned} \text{subject to} \quad & \dot{a}_{i,t} = x_{i,t} - \delta_i a_{i,t} && (\nu_{i,t}) \\ & a_{i,t} \leq \bar{a}_i && (\lambda_{i,t}) \\ & \int_0^\infty \sum_i (\bar{a}_i - a_{i,t})\, dt \leq B && (\mu) \end{aligned}$$

The Greek letters in parentheses are the respective Lagrangian multipliers (notations are summarized in Tab. 1).

The social cost of carbon (SCC) μ does not depend on i nor t, as a ton of GHG saved in any sector i at any point in time t contributes equally to meet the carbon budget. In every sector, the optimal timing of investments in low-carbon capital is partly driven by the current price of carbon μe^{rt}, which follows an Hotelling's rule by growing at the discount rate r.

The multipliers $\lambda_{i,t}$ are the sector-specific social costs of the sectoral potentials. They are null before the potentials $\bar{a}_i$ are reached (slackness condition). The costate variable $\nu_{i,t}$ may be interpreted as the present value of investments in low-carbon capital in sector i at time t.

3. Marginal investment costs (MICs) with infinitely-lived capital

In a first step, we solve the model in the simple case where $\delta_i = 0$. This case helps to understand the mechanisms at sake. However, with this assumption, marginal abatement costs cannot be defined: one single dollar invested produces an infinite amount of abatement (if a MAC was to be defined, it would be null). This issue is discussed further in the following sections.

Name	Description	Unit
c_i	Cost of investment in sector i	$/yr
$\bar{a}_i$	Sectoral potential in sector i	tCO_2/yr
δ_i	Depreciation rate of abating capital in sector i	yr^{-1}
r	Discount rate	yr^{-1}
$a_{i,t}$	Current abatement in sector i	tCO_2/yr
$x_{i,t}$	Current investment in abating activities in sector i	(tCO_2/yr)/yr
$\nu_{i,t}$	Costate variable (present social value of green investments)	$/($tCO_2$/yr)
$\lambda_{i,t}$	Social cost of the sectoral potential	$/$tCO_2$
μ	Social cost of carbon (SCC) (present value)	$/$tCO_2$
μe^{rt}	Current carbon price	$/$tCO_2$
c_i'	Marginal investment cost (MIC) in sector i	$/($tCO_2$/yr)
$\ell_{i,t}$	Marginal levelized abatement cost (MLAC) in sector i	$/$tCO_2$
$p_{i,t}$	Marginal implicit rental cost of capital (MIRCC) in sector i	$/$tCO_2$

Table 1: Notations (ordered by parameters, variables, multipliers and marginal costs).

Definition 1. *We call* Marginal Investment Cost (MIC) *the cost of the last unit of investment in low-carbon capital* ${c_i}'(x_{i,t})$.

MICs measure the economic efforts being oriented towards building and deploying low-carbon capital in a given sector at a given point in time. While one unit of investment at time t in two different sectors produces two similar goods – a unit of low-carbon capital that will save GHG from t onwards – they should not necessarily be valued equally.

Proposition 1. *In the case where low-carbon capital is infinitely-lived ($\delta_i = 0$), optimal MICs are not equal across sectors. Optimal MICs equal the value of the carbon they allow to save less the value of the forgone option to abate later in the same sector.*
Equivalently, sectors should invest up to the pace at which MICs are equal to the total social cost of emissions avoided before the sectoral potential is reached.

Proof. With infinitely-lived capital ($\delta_i = 0$), the generalized Lagrangian reads:

$$\begin{aligned} L(x_{i,t}, a_{i,t}, \lambda_i, \nu_i, \mu) = &\int_0^\infty e^{-rt} \sum_i c_i(x_{i,t})\, dt + \int_0^\infty \sum_i \lambda_{i,t} \left(a_{i,t} - \bar{a}_i\right)\, dt \\ &+ \mu \left(\int_0^\infty \sum_i \left(\bar{a}_i - a_{i,t}\right) dt - B \right) \\ &- \int_0^\infty \sum_i \dot{\nu}_{i,t} a_{i,t}\, dt - \int_0^\infty \nu_{i,t} x_{i,t}\, dt \end{aligned}$$

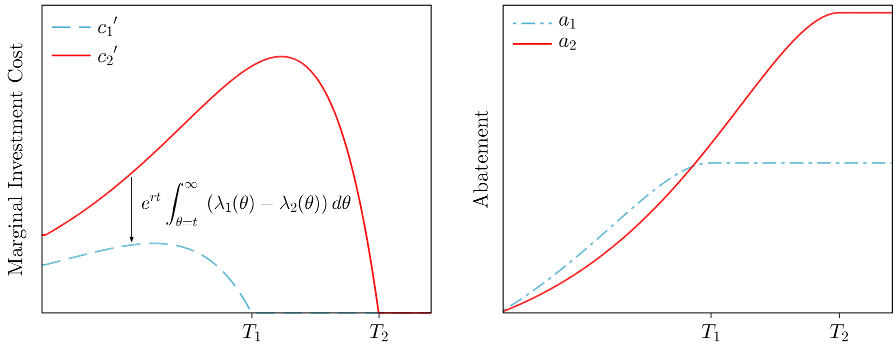

Figure 2: Left: Optimal marginal investment costs (MIC) in low-carbon capital in a case with two sectors ($i \in \{1, 2\}$) with infinitely-lived capital ($\delta_i = 0$). The dates T_i denote the endogenous date when all the emitting capital in sector i has been replaced by low-carbon capital; after this date, additional investment would bring no benefit. Optimal MICs differ across sectors, because of the social costs of the sectoral potentials ($\lambda_{i,t}$).
Right: corresponding optimal abatement pathways.

The first-order conditions are:[5]

$$\forall (i, t),\ \frac{\partial L}{\partial a_{i,t}} = 0 \iff \dot{\nu}_{i,t} = \lambda_{i,t} - \mu \tag{5}$$

$$\forall (i, t),\ \frac{\partial L}{\partial x_{i,t}} = 0 \iff e^{-rt} c_i{}'(x^*_{i,t}) = \nu_{i,t} \tag{6}$$

The optimal MIC can be written as:[6]

$$c_i{}'(x^*_{i,t}) = e^{rt} \int_t^{\infty} (\mu - \lambda_{i,\theta})\, d\theta \tag{7}$$

The complementary slackness condition states that the positive social cost of the sectoral potential $\lambda_{i,t}$ is null when the sectoral potential $\bar{a}_i$ is not binding:

$$\forall (i, t),\ \lambda_{i,t} \geq 0,\ \text{and}\ \lambda_{i,t} \cdot (\bar{a}_i - a_{i,t}) = 0 \tag{8}$$

Each investment made in a sector brings closer the endogenous date, denoted T_i, at which all the production of this sector will come from low-carbon capital (Fig. 3). After this date T_i, the option to abate global GHG emissions thanks to investments in low-carbon capital in sector i is removed. The value of this option is the integral from t to ∞ of the social cost of the sectoral potential $\lambda_{i,\theta}$; it is subtracted from the integral from t to ∞ of μ (the value of abatement) to obtain the value of investments in low-carbon capital.

Using (7) and (8) allows to express the optimal marginal investment costs

[5] The same conditions can be written using a Hamiltonian.
[6] We integrated $\dot{\nu}_{i,t}$ as given by (5) between t and ∞; used the relation $\lim_{t\to\infty} \nu_{i,t} = 0$ (justified latter); and replaced $\nu_{i,t}$ in (6).

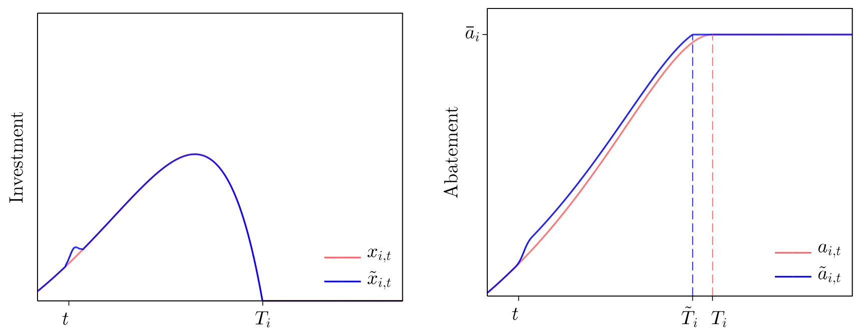

Figure 3: From a given investment pathway ($x_{i,t}$) leading to the abatement pathway ($a_{i,t}$), one supplementary unit of investment at time θ, as in ($\tilde{x}_{i,t}$), has two effects: it saves more GHG ($\tilde{a}_{i,t}$), but it also brings closer the date when the maximum sectoral potential is reached ($T_i \to \tilde{T}_i$). Both effects shape optimal marginal investments costs (7,9).

as a function of T_i and μ:[7]

$$c_i{}'(x^*_{i,t}) = \begin{cases} \mu e^{rt}(T_i - t) & \text{if } t < T_i \\ 0 & \text{if } t \geq T_i \end{cases} \tag{9}$$

Optimal MICs equal the total social cost of the carbon — expressed in current value (μe^{rt}) — that will be saved thanks to the abatement *before the sectoral potential is reached* — the time span ($T_i - t$).

The following lemma fulfills the proof. □

Lemma 1. *When the abating capital is infinitely-lived ($\delta_i = 0$), for any cost of carbon μ, the decarbonizing date T_i is an increasing function of the sectoral potential $\bar{a}_i$.*

Proof. See Appendix A.

Since potentials $\bar{a}_i$ differ across sectors, the dates T_i also differ across sectors, and optimal MICs are not equal. The general shape of the optimal MICs is displayed in Fig. 2. Vogt-Schilb et al. (2012) provide some numerical simulations calibrated with IPCC (2007) estimates of costs and abating potentials of seven sectors of the economy.

4. Marginal levelized abatement costs (MLACs)

In this section, we solve for the optimal marginal investment costs in the general case when the depreciation rate is positive ($\delta_i > 0$). Then, we show that the levelized abatement cost is not equal across sectors along the optimal path, and in particular is not equal to the carbon price.

[7] $\forall t < T_i,\ a_{i,t} < \bar{a}_i \implies \lambda_{i,t} = 0$ (8); for $t \geq T_i$ the abatement $a_i(t)$ is constant, equal to $\bar{a}_i$, thus $x_i(t) = \dot{a}_{i,t}$ is null, and, using (6) $\forall t \geq T_i,\ \nu_{i,t} = 0, \implies \dot{\nu}_{i,t} = 0 \implies \lambda_{i,t} = \mu$ (5). This last equality means that once the sectoral potential is binding, the associated shadow cost equals the value of the carbon that it prevents to abate.

Optimal marginal investment costs

Proposition 2. *Along the optimal path, abatement in each sector i increases until it reaches the sectoral potential $\bar{a}_i$ at a date denoted T_i; before this date, marginal investment costs can be expressed as a function of $\bar{a}_i$, T_i, the depreciation rate of the low-carbon capital δ_i, and the social cost of carbon μ:*

$$\forall t \leq T_i, \quad c_i{}'(x_{i,t}^*) = \mu \int_t^{T_i} e^{-(r+\delta_i)(\theta-t)}\, d\theta + e^{-(r+\delta_i)(T_i-t)}\, c_i'\left(\delta_i \bar{a}_i\right) \tag{10}$$

$$c_i{}'(x_{i,t}^*) = \mu e^{rt} \int_t^{T_i} e^{-\delta_i \theta}\, d\theta + e^{-(r+\delta_i)(T_i-t)}\, c_i'\left(\delta_i \bar{a}_i\right) \tag{11}$$

$$c_i{}'(x_{i,t}^*) = \underbrace{\mu e^{rt}\, \frac{1 - e^{-\delta_i (T_i - t)}}{\delta_i}}_{\mathcal{K}} + e^{-(r+\delta_i)(T_i-t)}\, c_i'\left(\delta_i \bar{a}_i\right) \tag{12}$$

Proof. See Appendix B.

Equation 12 states that at each time step t, each sector i should invest in low-carbon capital up to the pace at which marginal investment costs (Left-hand side term) are equal to marginal benefits (RHS term).

In the marginal benefits, the current carbon price μe^{rt} appears multiplied by a positive time span: $\frac{1}{\delta_i}\left(1 - e^{-\delta_i(T_i-t)}\right)$. This term is the equivalent of $(T_i - t)$ in the case of infinitely-lived abating capital (9). We interpret $\mathcal{K}$ as the marginal benefit of building new low-carbon capital. The longer it takes for a sector to reach its potential (i.e. the further T_i), the more expensive should be the last unit of investments directed toward low-carbon capital accumulation.

$\mathcal{K}$ reflects a complex trade-off: investing soon allows the planner to benefit from the persistence of abating efforts over time, and prevents investing too much in the long-term; but it brings closer the date T_i, removing the option to invest later, when the discount factor is higher. This results in a bell-shaped distribution of mitigation costs over time: in the short term, the effect of discounting may dominate[8] and the effort may grow exponentially; in the long term, the effect of the limited potential dominates and new capital accumulation decreases to zero (Fig. 4).

Marginal benefits also have another component, that tends to the marginal cost of maintaining abatement at its maximal level: $e^{-(\delta_i+r)(T_i-t)} c_i'(\bar{a}_i)$. As the first term in (12) tends to 0, the optimal MIC tends to the cost of maintaining low-carbon capital at its maximal level:

$$c_i{}'(x_{i,t}^*) \xrightarrow[t \to T_i]{} c_i'(\delta_i \bar{a}_i) \tag{13}$$

After T_i, the abatement is constant ($a_{i,t} = \bar{a}_i$) and the optimal MIC in sector i is simply constant at $c_i'\left(\delta_i \bar{a}_i\right)$.

Marginal levelized abatement costs

Our model does not feature an abatement cost function that can be differentiated to compute the marginal abatement costs (MACs). In this section, we

[8] if either T_i or δ_i is sufficiently large

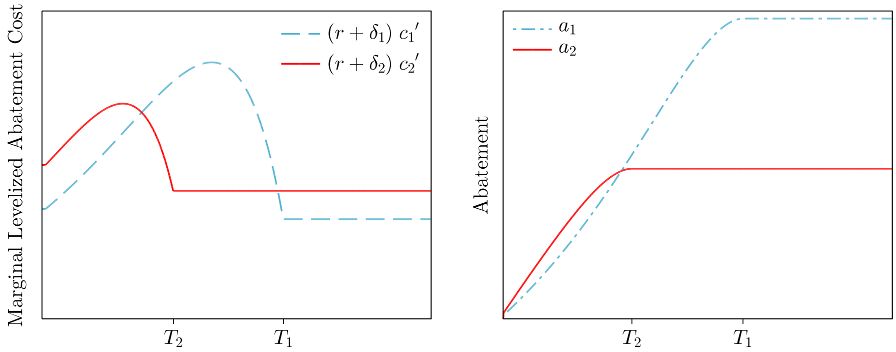

Figure 4: Left: ratio of marginal investments to abated GHG (MLACs) along the optimal trajectory in a case with two sectors ($i \in \{1, 2\}$). In a first phase, the optimal timing of sectoral abatement comes from a trade-off between (i) investing later in order to reduce present costs thanks to the discounting, (ii) investing sooner to benefit from the persistent effect of the abating efforts over time, and (iii) smooth investment over time, as investment costs are convex. This results in a bell shape. After the dates T_i when the potentials $\bar{a}_i$ have been reached, marginal abatement costs are constant to $\delta_i\, c_i' \left(\delta_i \bar{a}_i\right)$.
Right: corresponding optimal abatement paths.

compute the levelized abatement cost of the low-carbon capital. This metric is sometimes labeled "marginal abatement costs" by some scholars and institutions. We find that marginal levelized abatement costs should not be equal to the carbon price, and should not be equal across sectors.

Definition 2. *We call* Marginal Levelized Abatement Cost (MLAC) *and denote $\ell_{i,t}$ the ratio of marginal investment costs to discounted abatements. MLACs can be expressed as:*

$$\forall x_{i,t}, \quad \ell_{i,t} = (r + \delta_i)\, c_i{}'(x_{i,t}) \tag{14}$$

Proof. See Appendix D.

MLACs may be interpreted as MICs annualized using $r + \delta_i$ as the discount rate. This is the appropriate discount rate because, taking a carbon price as given, one unit of investment in low-carbon capital generates a flow of real revenue that decreases at the rate $r + \delta_i$.

Practitioners may use MLACs when comparing different technologies.[9] Let us take an illustrative example: building electric vehicles (EV) to replace conventional cars. Let us say that the social cost of the last EV built at time t, compared to the cost of a classic car, is 7 000 \$/EV. This figure may include, in addition to the higher upfront cost of the EV, the lower discounted operation and maintenance costs — complete costs computed this way are sometimes called *levelized costs*. If cars are driven 13 000 km per year and electric cars emit 110 gCO_2/km less than a comparable internal combustion engine vehicle, each EV allows to save 1.43 tCO_2/yr. The MIC in this case would be

[9] We defined *marginal* levelized costs. The gray literature simply uses levelized costs; they equal marginal levelized costs if investment costs are linear (Appendix E).

4 900 \$/(tCO$_2$/yr). If electric cars depreciate at a constant rate such that their average lifetime is 10 years ($1/\delta_i = 10$ yr), then $r + \delta_i = 15\%$/yr and the MLAC is 734 \$/tCO$_2$.[10]

Proposition 3. *Optimal MLACs are not equal to the carbon price.*

PROOF. Combining the expression of optimal MICs from Prop. 2 and in the expression of MLACs from Def. 2 gives the expression of optimal MLACs:

$$\forall t \leq T_i, \quad \ell^*_{i,t} = (r + \delta_i)\, c_i{}'(x^*_{i,t})$$
$$\ell^*_{i,t} = \mu e^{rt} \left(1 - e^{-\delta_i(T_i - t)}\right) \frac{r + \delta_i}{\delta_i} + e^{-(r+\delta_i)(T_i - t)} (r + \delta_i)\, c_i' \left(\delta_i \bar{a}_i\right) \tag{15}$$

□

Corollary 1. *In general, optimal MLACs are different in different sectors.*

PROOF. We use a proof by contradiction. Let two sectors be such that they exhibit the same investment cost function, the same depreciation rate, but different abating potentials:

$$\forall x > 0,\ c_1'(x) = c_2'(x),\ \delta_1 = \delta_2,\ \bar{a}_1 \neq \bar{a}_2$$

Suppose that the two sectors take the same time to decarbonize (i.e. $T_1 = T_2$). Optimal MICs would then be equal in both sectors (12). This would lead to equal investments, hence equal abatement, in both sectors at any time (1,2), and in particular to $a_1(T_1) = a_2(T_2)$. By assumption, this last equality is not possible, as:

$$a_1(T_1) = \bar{a}_1 \neq \bar{a}_2 = a_2(T_2)$$

In conclusion, different potentials $\bar{a}_i$ have to lead to different optimal decarbonizing dates T_i, and therefore to different optimal MLACs $\ell^*_{i,t}$.

A similar reasoning can be done concerning two sectors with the same investment cost functions, same potentials, but different depreciation rates; or two sectors that differ only by their investment cost functions. □

This finding does not necessarily imply that mitigating climate change requires other sectoral policies than those targeted at internalizing learning spillovers. Well-tried arguments plead in favor of using few instruments (Tinbergen, 1956; Laffont, 1999). In our case, a unique carbon price may induce different MLACs in different sectors.

5. Marginal implicit rental cost of capital (MIRCC)

The result from the previous section may seem to contradict the equimarginal principle: two similar goods, abatement in two different sectors, appear

[10] The MIC was computed as 7 000 \$/(1.43 tCO$_2$/yr) = 4 895 \$/(tCO$_2$/yr); and the MLAC as 0.15 yr^{-1} · 4 895 \$/(tCO$_2$/yr)= 734 \$/tCO$_2$.

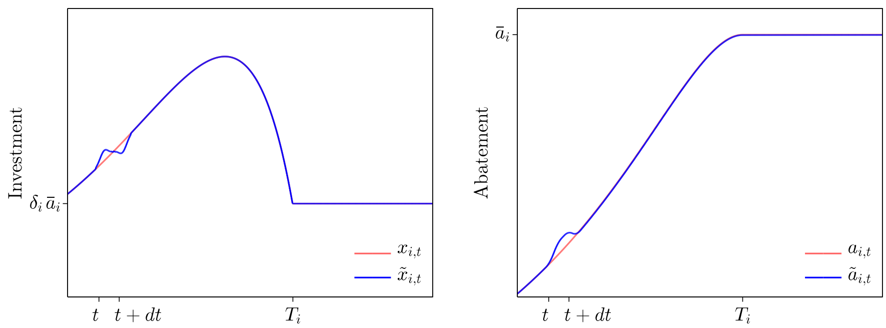

Figure 5: From a given investment pathway $(x_{i,t})$ leading to the abatement pathway $(a_{i,t})$, saving one more unit of GHG at a date θ without changing the rest of the abatement pathway, as in $(\tilde{a}_{i,t})$, requires to invest one more unit at θ and $(1 - \delta_i\, d\theta)$ less at $\theta + d\theta$, as $(\tilde{x}_{i,t})$ does.

to have different prices. In fact, investment in low-carbon capital produce different goods in different sectors because they have two effects: avoiding GHG emissions and removing an option to invest later in the same sector (section 3).

Here we consider an investment strategy that increases abatement in a sector at one date while keeping the rest of the abatement trajectory unchanged. It consequently leaves unchanged any opportunity to invest later in the same sector.

From an existing investment pathway $(x_{i,t})$ leading to an abatement pathway $(a_{i,t})$, the social planner may increase investment by one unit at time θ and immediately reduce investment by $1 - \delta\, d\theta$ at the next period $\theta + d\theta$. The resulting investment schedule $(\tilde{x}_{i,t})$ leads to an abatement pathway $(\tilde{a}_{i,t})$ that abates one supplementary unit of GHG between θ and $\theta + d\theta$ (Fig. 5). Moving from $(x_{i,t})$ to $(\tilde{x}_{i,t})$ costs:

$$\mathcal{P} = \frac{1}{d\theta}\left[{c_i}'(x_{i,\theta}) - \frac{(1-\delta_i\, d\theta)}{(1 + r\, d\theta)}\; c_i'(x_{i,\theta + \, d\theta})\right] \tag{16}$$

For marginal time lapses, this tends to :

$$\mathcal{P} \xrightarrow[d\theta \to 0]{} (r + \delta_i)\; {c_i}'(x_{i,\theta}) - \frac{d{c_i}'(x_{i,\theta})}{d\theta} \tag{17}$$

Definition 3. *We call* marginal implicit rental cost of capital (MIRCC) *in sector i at a date t, denoted $p_{i,t}$ the following value:*

$$p_{i,t} = (r + \delta_i)\; {c_i}'(x_{i,t}) - \frac{d{c_i}'(x_{i,t})}{dt} \tag{18}$$

This definition extends the concept of the *implicit rental cost of capital* to the case where investment costs are an endogenous functions of the investment pace.[11] It corresponds to the market rental price of low-carbon capital in a

[11] We defined *marginal* rental costs. This differs from the proposal by Jorgenson (1963, p. 143), where investment costs are linear, and no distinction needs to be done between average and marginal costs (Appendix E).

competitive equilibrium, and ensures that there are no profitable trade-offs between: (i) lending at a rate r; and (ii) investing at time t in one unit of capital at cost $c_i{}'(x_{i,t})$, renting this unit during a small time lapse dt, and reselling $1 - \delta dt$ units at the price $c_i'(x_{i,t+dt})$ at the next time period.

Proposition 4. *In each sector i, before the date T_i, the optimal implicit marginal rental cost of capital equals the current carbon price:*

$$\forall i,\ \forall t \leq T_i, \quad p^*_{i,t} = (r + \delta_i)\ c_i{}'(x^*_{i,t}) - \frac{dc_i{}'(x^*_{i,t})}{dt} = \mu e^{rt} \tag{19}$$

Proof. In Appendix C we show that this relation is a consequence of the first order conditions.

Equation 19 also gives a sufficient condition for the marginal levelized abatement costs (MLACs from Def. 2) to be equal across sectors to the carbon price: this happens when marginal investment costs are constant along the optimal path: $dc_i{}'(x^*_{i,t})/dt = 0$. When this condition is satisfied, MLACs can be used labeled as MACs, and should be equal across sectors to the carbon price. However, if investment costs are convex functions of the investment pace, marginal investment costs change in time and MLACs differ from MIRCCs (9,12).

Proposition 5. *Equalizing MIRCC to the social cost of carbon (SCC) is not a sufficient condition to reach the optimal investment pathway.*

Proof. Equations (18) and (19) define a differential equation that $c_i{}'(x_{i,t})$ satisfies when the IRRC are equalized to the SCC. This differential equation has an infinity of solutions (those listed by equation B.6 in the appendix). In other words, many different investment pathways lead to equalize MIRCC and the SCC. Only one of these pathways leads to the optimal outcome; it can be selected using the fact that at T_i, abatement in each sector has to reach its maximum potential (the boundary condition used from B.7). □

The cost-efficiency of investments is therefore more complex to assess when investment costs are endogenous than when they are exogenous. In the latter case, as Jorgenson (1967, p. 145) emphasized: "*It is very important to note that the conditions determining the values [of investment in capital] to be chosen by the firm [...] depend only on prices, the rate of interest, and the rate of change of the price of capital goods for the current period.*"[12] In other words, when investment costs are exogenous, current price signals contain all the information that private agents need to take socially-optimal decisions. In contrast, in our case – with endogenous investment costs and maximum abating potentials – the information contained in prices should be complemented with the correct expectation of the date T_i when the sector is entirely decarbonized.

6. Discussion and conclusion

We used three metrics to assess the social value of investments in low-carbon capital made to decarbonize the sectors of an economy: the marginal investment

[12] In our model, these correspond respectively to the current price of carbon μe^{rt}, the discount rate r, and the *endogenous* current change of MIC $dc_i{}'(x_{i,t})/dt$.

costs (MIC), the marginal levelized abatement cost (MLAC), and the marginal implicit rental cost of capital (MIRCC).

We find that along the optimal path, the marginal investment costs and the marginal levelized abatement costs differ from the carbon price, and differ across sectors. This may bring strong policy implications, as agencies use levelized abatement costs labeled as "marginal abatement cost" (MAC), and existing sectoral policies are often criticized because they set different MACs (or different carbon prices) in different sectors. Our results show that levelized abatement costs should be equal across sectors only if the costs of investments in low-capital do not depend on the date or the pace at which investments are implemented.

In the optimal pathway, the marginal implicit rental cost of capital (MIRCC) equals the current carbon price in every sector that has not finished its decarbonizing process. In other words, if abatement costs are defined as the implicit rental cost of the low-carbon capital required to abate, MACs should be equal across sectors.

This finding required to extend the concept of implicit rental cost of capital to the case of endogenous investment costs – at our best knowledge it had only been used with exogenous investment costs. A theoretical contribution is to show that when investment costs are endogenous and the capital has a maximum production, equalizing the MIRCC to the current price of the output (the carbon price in this application) is not a sufficient condition to reach the Pareto optimum. In other words, current prices do not contain all the information required to decentralize the social optimum; they must be combined with knowledge of the date when the capital reaches its maximum production.

The bottom line is that two apparently opposite views are reconciled: on the one hand, higher efforts are actually justified in the specific sectors that will take longer to decarbonize, such as urban planning and the transportation system; on the other hand, the equimarginal principle remains valid, but applies to an accounting value: the implicit rental cost of the low-carbon capital used to abate.

Our analysis does not incorporate any uncertainty, imperfect foresight or incomplete or asymmetric information. We also disregarded induced technical change, known to impact the optimal timing and cost of GHG abatement; and growing abating potentials, a key factor in developing countries. A program for further research is to investigate the combined effect of these factors in the framework of low-carbon capital accumulation.

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Appendix A. Proof of lemma 1

PROOF. As ${c_i}'$ is strictly growing, it is invertible. Let χ_i be the inverse of ${c_i}'$; applying χ_i to (9) gives:

$$x_{i,t} = \begin{cases} \chi_i \left(e^{rt}(T_i - t)\mu\right) & \text{if } t < T_i \\ 0 & \text{if } t \geq T_i \end{cases} \tag{A.1}$$

The relation between the sectoral potential ($\bar{a}_i$), the MICs (through χ_i), the SCC (μ) and the time it takes to achieve the sectoral potential T_i reads:

$$\begin{aligned} \bar{a}_i &= a_i(T_i) \\ &= \int_0^{T_i} \chi_i \left(e^{rt}(T_i - t)\mu\right) dt \end{aligned}$$

Let us define f_{χ_i} such that:

$$\begin{aligned} f_{\chi_i}(t) &= \int_0^t \chi_i \left(e^{r\theta}(t-\theta)\mu\right)\, d\theta \\ \implies \frac{df_{\chi_i}}{dt}(t) &= \int_0^t e^{r\theta}{\chi_i}'\left(e^{r\theta}(t-\theta)\mu\right)\, d\theta \end{aligned}$$

Let us show that f_{χ_i} is invertible: ${\chi_i}' > 0$ as the inverse of $c' > 0$, thus $\frac{df_{\chi_i}}{dt} > 0$ and therefore f_{χ_i} is strictly growing. Finally:

$$\bar{a}_i \mapsto T_i = {f_{\chi_i}}^{-1}(\bar{a}_i) \quad \text{is an increasing function}$$

When the marginal cost function ${c_i}'$ is given, χ_i and therefore f_{χ_i} are also given. Therefore, T_i can always be found from $\bar{a}_i$. The larger the potential, the longer it takes for the optimal strategy to achieve it. □

Appendix B. Proof of proposition 2

Lagrangian

The Lagrangian associated with (4) reads:

$$\begin{aligned} L(x_i, a_i, \dot{a}_i, \lambda_i, \nu_i, \mu) = &\int_0^\infty e^{-rt} \sum_i c_i(x_{i,t})\, dt + \int_0^\infty \sum_i \lambda_{i,t}\left(a_{i,t} - \bar{a}_i\right)\, dt \\ &+ \mu\left(\int_0^\infty \sum_i \left(\bar{a}_i - a_{i,t}\right) dt - B\right) \\ &+ \int_0^\infty \sum_i \nu_{i,t}\left(\dot{a}_{i,t} - x_{i,t} + \delta_i a_{i,t}\right)\, dt \end{aligned} \tag{B.1}$$

In the last term, $\dot{a}_{i,t}$ can be removed thanks to an integration by parts:

$$\begin{aligned} &\int_t^\infty \sum_i \nu_{i,t}\left(\dot{a}_{i,t} - x_{i,t} + \delta_i a_{i,t}\right) dt \\ &= \sum_i \left(\int_0^\infty \nu_{i,t}\dot{a}_{i,t}\, dt + \int_0^\infty \nu_{i,t}\left(\delta_i a_{i,t} - x_{i,t}\right)\, dt\right) \\ &= \sum_i \left(constant - \int_0^\infty \dot{\nu}_{i,t} a_{i,t}\, dt + \int_0^\infty \nu_{i,t}\left(\delta_i a_{i,t} - x_{i,t}\right)\, dt\right) \end{aligned}$$

The transformed Lagrangian does not depend on $\dot{a}_{i,t}$:

$$\begin{aligned} L(x_{i,t}, a_{i,t}, \lambda_i, \nu_i, \mu) = &\int_0^\infty e^{-rt} \sum_i c_i(x_{i,t})\, dt + \int_0^\infty \sum_i \lambda_{i,t}\left(a_{i,t} - \bar{a}_i\right)\, dt \\ &+ \mu\left(\int_0^\infty \sum_i \left(\bar{a}_i - a_{i,t}\right) dt - B\right) \\ &- \int_0^\infty \sum_i \dot{\nu}_{i,t} a_{i,t}\, dt + \int_0^\infty \nu_{i,t}\left(\delta_i a_{i,t} - x_{i,t}\right)\, dt \end{aligned} \tag{B.2}$$

First order conditions

The first order conditions read:[13]

$$\forall (i,t), \frac{\partial L}{\partial a_{i,t}} = 0 \iff \dot{\nu}_{i,t} - \delta_i \nu_{i,t} = \lambda_{i,t} - \mu \tag{B.3}$$

$$\forall (i,t), \frac{\partial L}{\partial x_{i,t}} = 0 \iff e^{-rt} {c_i}'(x_{i,t}) = \nu_{i,t} \tag{B.4}$$

Slackness condition

For each sector i there is a date T_i such that (slackness condition):

$$\begin{aligned} \forall t < T_i, \quad a_{i,t} < \bar{a}_i \quad \& \quad \lambda_{i,t} = 0 \\ \forall t \geq T_i, \quad a_{i,t} = \bar{a}_i \quad \& \quad \lambda_{i,t} \geq 0 \end{aligned} \tag{B.5}$$

Before T_i, (B.3) simplifies:

$$\begin{aligned} \forall t \leq T_i, \ \dot{\nu}_i(t) - \delta_i \nu_{i,t} = -\mu \\ \implies \nu_{i,t} = V_i \, e^{\delta_i t} + \frac{\mu}{\delta_i} \end{aligned}$$

Where V_i is a constant that will be determined later. The MICs are the same quantities expressed in current value (B.4):

$$\forall t \leq T_i, \ {c_i}'(x_{i,t}) = e^{rt} \left[V_i \, e^{\delta_i t} + \frac{\mu}{\delta_i} \right] \tag{B.6}$$

Any V_i chosen such that $\left[V_i e^{\delta_i t} + \frac{\mu}{\delta_i} \right]$ remains positive defines an investment pathway that satisfies the first order conditions. The optimal investment pathways also satisfies the following boundary conditions.

Boundary conditions

At the date T_i, $a_{i,t}$ is constant and the investment $x_{i,t}$ is used to counterbalance the depreciation of abating capital. This allows to compute V_i:

$$\begin{aligned} & x_i(T_i) = \delta_i \bar{a}_i \qquad\qquad (\text{B.7}) \\ \implies & e^{-rT_i} c_i' \left(\delta_i \bar{a}_i \right) = V_i \, e^{\delta_i \cdot T_i} + \frac{\mu}{\delta_i} \qquad (\text{from eq. B.6}) \\ \implies & V_i = e^{-\delta_i T_i} \left(e^{-rT_i} c_i' \left(\delta_i \bar{a}_i \right) - \frac{\mu}{\delta_i} \right) \end{aligned}$$

Optimal marginal investment costs (MICs)

Using this expression in (B.6) gives:

$$\forall t \leq T_i, \quad {c_i}'(x_{i,t}) = e^{rt} \left[e^{-\delta_i \cdot T_i} \left(e^{-rT_i} c_i' \left(\delta_i \bar{a}_i \right) - \frac{\mu}{\delta_i} \right) e^{\delta_i t} + \frac{\mu}{\delta_i} \right]$$

Simplifying this expression allows to express the optimal marginal investment costs in each sector as a function of δ_i, $\bar{a}_i$, μ and T_i:

$${c_i}'(x^*_{i,t}) = \mu e^{rt} \, \frac{1 - e^{-\delta_i (T_i - t)}}{\delta_i} + e^{-(\delta_i + r)(T_i - t)} \, c_i' \left(\delta_i \bar{a}_i \right) \tag{B.8}$$

After T_i, the MICs in sector i are simply constant to $c_i' \left(\delta_i \bar{a}_i \right)$. $\square$

[13] The same conditions may be written using a Hamiltonian.

Appendix C. Proof of proposition 4

The first order conditions can be rearranged. Starting from (B.4):

$$c_i{}'(x_{i,t}) = e^{-rt}\nu_{i,t} \tag{B.4}$$

$$\implies \frac{dc_i{}'(x_{i,t})}{dt} = e^{rt}\left(\dot{\nu}_{i,t} + r\cdot\nu_{i,t}\right) \tag{C.1}$$

$$= e^{rt}\left(\delta_i\nu_{i,t} - \mu + r\cdot\nu_{i,t}\right) \qquad \text{(from B.3 and B.5)} \tag{C.2}$$

$$= (r+\delta)c_i{}'(x_{i,t}) - \mu e^{rt} \qquad \text{(from B.4)} \tag{C.3}$$

$$\implies \mu e^{rt} = (r+\delta_i)\, c_i{}'(x_{i,t}) - \frac{dc_i{}'(x_{i,t})}{dt} \tag{C.4}$$

Substituting in the definition of the implicit marginal rental cost of capital $p_{i,t}$ (18) leads to $p_{i,t} = \mu e^{rt}$. The solutions of (C.4), where the variable is $c_i{}'(x_{i,t})$, are those listed in (B.6).

Appendix D. Proof of the expression of $\ell_{i,t}$ in Def. 2

Let h be a marginal physical investment in low-carbon capital made at time t in sector i (expressed in tCO_2/yr per year). It generates an infinitesimal abatement flux that starts at h at time t and decreases exponentially at rate δ_i, leading to the total discounted abatement ΔA (expressed in tCO_2):

$$\Delta A = \int_{\theta=t}^{\infty} e^{r\,(\theta-t)} h\, e^{-\delta_i(\theta-t)}\, d\theta \tag{D.1}$$

$$= \frac{h}{r+\delta_i} \tag{D.2}$$

This additional investment h brings current investment from $x_{i,t}$ to $(x_{i,t}+h)$. The additional cost ΔC (expressed in \$) that it brings reads:

$$\Delta C = c_i(x_{i,t}+h) - c_i(x_{i,t}) \underset{h\to 0}{=} h\, c_i{}'(x_{i,t}) \tag{D.3}$$

The MLAC $\ell_{i,t}$ is the division of the additional cost by the additional abatement it allows:

$$\ell_{i,t} = \frac{\Delta C}{\Delta A} \tag{D.4}$$

$$\ell_{i,t} = (r+\delta_i)\, c_i{}'(x_{i,t}) \tag{D.5}$$

□

Appendix E. Levelized costs and implicit rental cost when investment costs are exogenous and linear

Let I_t be the amount of investments made at exogenous unitary cost Q_t to accumulate capital K_t that depreciates at rate δ:

$$\dot{K}_t = I_t - \delta\, K_t \tag{E.1}$$

Let $F(K_t)$ be a classical production function (where the price of output is normalized to 1). Jorgenson (1967) defines current receipts R_t as the actual cash flow:

$$R_t = F(K_t) - Q_t\, I_t \tag{E.2}$$

he finds that the solution of the maximization program

$$\max_{I_t} \int_0^\infty e^{-rt} R_t\, dt \tag{E.3}$$

does not equalize the marginal productivity of capital to the investment costs Q_t:

$$F_K(K_t^*) = (r+\delta)\, Q_t - \dot{Q}_t \tag{E.4}$$

He defines the implicit rental cost of capital C_t, as the accounting value:

$$C_t = (r+\delta)\, Q_t - \dot{Q}_t \tag{E.5}$$

such that the solution of the maximization program is to equalize the marginal productivity of capital and the rental cost of capital:

$$F_K(K_t^*) = C_t \tag{E.6}$$

He shows that this is consistent with maximizing discounted economic profits, where the current profit is given by the accounting rule:

$$\Pi_t = F(K_t) - C_t\, K_t \tag{E.7}$$

In this case, the (unitary) levelized cost of capital L_t is given by:

$$L_t = (r+\delta)\, Q_t \tag{E.8}$$

And the levelized cost of capital matches the optimal rental cost of capital if and only if investment costs are constant:

$$\dot{Q}_t = 0 \iff F_K(K_t^*) = L_t \tag{E.9}$$