Assessing and ordering investments in polluting fossil-fueled and zero-carbon capital

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Abstract

Climate change mitigation requires to replace preexisting carbon-intensive capital with different types of cleaner capital. Coal power and inefficient thermal engines may be phased out by gas power and efficient thermal engines or by renewable power and electric vehicles. We derive the optimal timing and costs of investment in a low- and a zero-carbon technology, under an exogenous ceiling constraint on atmospheric pollution. Producing output from the low-carbon technology requires to extract an exhaustible resource. A general finding is that investment in the expensive zero-carbon technology should always be higher than, and can optimally start before, investment in the cheaper low-carbon technology. We then provide illustrative simulations calibrated with data from the European electricity sector. The optimal investment schedule involves building some gas capacity that will be left unused before it naturally depreciates, a process known as *mothballing* or *early scrapping*. Finally, the levelized cost of electricity (LCOE) is a misleading metric to assess investment in new capacities. Optimal LCOEs vary dramatically across technologies. Ranking technologies according to their LCOE would bring too little investment in renewable power, and too much in the intermediate gas power.

Keywords: levelized costs of electricity; lifecycle cost; climate change mitigation; path dependence; transition to a low-carbon economy; technology policy; optimal timing; capital utilization rate
JEL classification: O21, Q32, Q41, R4, Q54, Q58

1. Introduction

The European Union aims at decarbonizing almost completely the power sector by 2050, and at reducing emissions from transportation by two thirds below the 1990 level by 2050 (UE, 2011). This requires that in both sectors, the preexisting carbon-intensive capital is replaced by one or several types of greener technologies. Cutting emissions from existing coal power plants can for instance be achieved by building gas power plants (gas is less carbon-intensive than coal), or more-expensive but almost-carbon-free options such as nuclear or

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renewables (hydro, wind, solar, biomass). In the private transportation sector, legacy inefficient thermal vehicles can be replaced by more-efficient thermal vehicles, or more-expensive but less-emitting plug-in hybrids and electric vehicles. How to assess the optimal cost and timing of investment in different types of low-carbon capital running on different types of fossil fuels?

A popular metric is the *levelized cost* of each technology, i.e. the ratio of discounted costs of installing and using the technology, over discounted production during its lifetime — including the cost of greenhouse gases (GHG) emission and resource depletion. In energy textbooks and studies, for instance, the *Levelized Cost of Electricity* (LCOE) is used to compare various types of power plants (e.g. IPCC, 2007; Alok, 2011; Kost et al., 2012; EIA, 2013). In the private transportation sector, it is frequently assumed that the production, i.e. the distance driven, is exogenous. A common practice is then to compare the *life-cycle cost* of different technologies, i.e. the discounted cost of building and driving a set of cars along a given distance (e.g. Ogden et al., 2004). This life-cycle cost is sometimes labeled *levelized cost* (Delucchi and Lipman, 2001; DOE, 2013). In both sectors, an accepted rule of thumb is that technologies that produce at a lower levelized cost are superior. It is not clear whether the levelized costs define a merit order, i.e. whether "lower-cost" technologies should be built first, and "higher-cost" technologies should wait that the carbon price is sufficiently high to become competitive.[1]

At our best knowledge, the literature lacks a theoretical model to assess the optimal cost and timing of investment in different types of low-carbon capital. Similar questions are however treated in the economic literature.

Chakravorty et al. (2008) consider a social planner who chooses when to extract resources differentiated by their carbon intensities (e.g coal and oil) and when to use a clean backstop (renewable power)[2] in order to maximize discounted welfare while keeping atmospheric pollution below an exogenous threshold. They conclude that the optimal sequencing of resource extraction does not follow the intuitive rule that good (i.e. less polluting) deposits should be used before bad ones (i.e. more polluting), as a hasty generalization of Herfindahl's (1967) findings would suggest. The rationale is that if atmospheric pollution decays naturally, the best strategy to meet a pollution ceiling is to burn more coal in the short-term and benefit from the natural dilution. In this modeling framework, all the dynamics come from the intertemporal share of the various scarcity rents (from fossil resources and clean air); this approach does not account for low-carbon capital accumulation.

In an unrelated approach, Vogt-Schilb et al. (2012) consider a social planner who accumulates one type of carbon-free capital in several sectors to meet a carbon budget at the lowest discounted cost. They find that the optimal cost

[1]Of course, the *levelized costs* provide only part of the relevant information to assess different technologies. In particular, ranking technologies according to their levelized costs leaves aside any benefits of early investment from learning-by-doing (LBD) effects. However, several existing studies suggest that those effects are negligible. Goulder and Mathai (2000) investigate the impact of LBD on the optimal timing of GHG reductions in an aggregated model and find little difference with the simulation without LBD. Fischer and Newell (2008) investigate the optimal costs of producing electricity from renewable power subject to LBD and find that LBD justifies only a 10 % increase in the optimal cost.

[2] They implicitly assume that renewable power should not be used before all the fossil resources are exhausted.

and timing of GHG reductions differ considerably from those obtained with more classic models relying on abatement cost curves. More precisely, capital accumulation means that (i) the timing of abatement (in tCO_2/yr) is steeper than generally found — less abatement in the short term and more abatement in the long term — and (ii) optimal economic efforts to curb emissions (in \$/yr) are concentrated over the short-term and decrease in time. In their framework, the shape of the carbon price, the optimal schedule of investment in low-carbon capital, and actual emission reductions are very different. As a consequence, assessing the cost-efficiency of investment in low-carbon capital can be tricky Vogt-Schilb et al. (2013).[3] This framework does not represent several types of low-carbon capital within a sector, or any required consumption of fossil fuels.

In this paper, we attempt to merge the two approaches. As in Chakravorty et al. (2008), a social planner can choose between two different polluting resources (e.g. coal or gas) that have different carbon intensities and different availabilities (coal is more polluting and much more abundant than gas). She can also use a completely clean substitute (e.g. renewable energy). She has to cope with a carbon budget. As in Vogt-Schilb et al. (2012), reducing GHG emissions requires that the planner slowly accumulates capital at a convex investment costs; our original contribution is that we study with several competing technologies within a sector: low- or zero-carbon capital (e.g. gas power plants, renewable power plants).

We find that the optimal ordering of investment in green technologies does not follow an intuitive ranking. For instance, expensive renewable power may be used to phase out dirty coal before lower-cost gas power plants start to be built. Also, renewables may be used before fossil fuels are exhausted. Further, it may be optimal to build large amounts of gas power plants, and leave them partly unused before they depreciate (a process known as *mothballing* or *early scrapping*).

We also find that levelized costs should not be equal for each technology, confirming a similar finding by Vogt-Schilb et al. (2013). For instance, it may be justified to replace an old coal power plant by a windmill instead of a gas power plant even if the former appears as much as six time more expensive than the latter in terms of discounted costs over discounted production (LCOE). The reason is that, under a carbon budget, each new gas plant comes with the obligation of eventually being replaced by renewable power, while renewable power remains in the long-term optimal production park. The LCOE does not capture this difference. For the same reason, it may be justified to replace an old thermal vehicle by a plug-in hybrid rather than a new efficient thermal vehicle even if the former appears more expensive than the latter in terms of discounted costs over discounted driven distance.

Our results suggest that decisions taken by comparing the levelized costs

[3] Beginning with an early suggestion by Ploeg and Withagen (1991), other contributions study the link between low-carbon capital accumulation and the optimal timing of GHG emission reductions. Among them, Fischer et al. (2004) study the optimal carbon tax in a model where clean capital accumulation reduces GHG emissions and environmental damages lower current welfare. Gerlagh et al. (2009) and Acemoglu et al. (2012) add knowledge accumulation to a similar framework. Rozenberg et al. (2013) study the intertemporal distribution of abatement efforts implied by several mitigation strategies (under-using existing brown capital or focusing on emissions embedded in new capital) to meet an emission ceiling constraint.

of various technologies would favor intermediate technologies (gas plants, efficient thermal vehicles) to the detriment of more-expensive but lower-carbon technologies (renewable power, electric vehicles), leading to a suboptimal investment schedule.

The remaining of the paper is structured as follows: we describe the model in Section 2. In Section 3 we derive the first-order conditions and discuss the equimarginal principle. Section 4 characterizes the various phases of the investment dynamics. In Section 5 we calibrate our model with data from the European electricity sector. Section 6 concludes.

2. Model

A social planner controls the supply of an energy service (e.g, private mobility) or non-storable commodity (electricity), referred to as *output* in this article. It builds green capacity, which emits less GHG than preexisting high-carbon technologies — e.g., conventional gasoline cars or coal power stations — treated as an aggregated overabundant dirty backstop. It uses green and preexisting brown capacities to meet an exogenous inelastic demand, and cope with a given carbon budget.

2.1. Investing in and using capital

At each time t, the social planner chooses positive investment $x_{i,t}$ in a set of technologies indexed by i. The investment adds to the installed capacity $k_{i,t}$, which otherwise depreciates at the constant rate δ (dotted variables denote temporal derivatives):

$$\dot{k}_{i,t} = x_{i,t} - \delta k_{i,t} \tag{1}$$

$$x_{i,t} \geq 0 \tag{2}$$

Without loss of generality , we assume green capacities are nil at the beginning ($k_{i,t=0} = 0$).[4] Investment is made at a cost $c_i(x_{i,t})$ assumed increasing and convex ($c_i' > 0, c_i'' > 0$). This captures the increasing opportunity cost to use scarce resources (skilled workers and appropriate capital) in order to build more green capacities.[5]

The social planner then chooses how much output to produce from each technology. We assume that the production process exhibits constant returns to scale: two gas plants can produce twice the power that one gas plant can produce. The positive production $q_{i,t}$ with technology i cannot exceed the installed capacity $k_{i,t}$:

$$0 \leq q_{i,t} \leq k_{i,t} \tag{3}$$

We assume that overabundant brown capital is inherited at the beginning of the period (e.g, inefficient coal plants, or thermal engines). At each point, the

[4] In Section 5 we tackle the case of the European electricity sector with preexisting low-carbon capacity.

[5] Unlike Vogt-Schilb et al. (2012), we allow investment made infinitesimally slowly to be costly: $c_i'(0) \geq 0$.

total production (including from preexisting brown technologies) has to meet an exogenous demand D assumed constant for simplicity:

$$\sum_i q_{i,t} = D \tag{4}$$

The social planner can produce output from preexisting high-carbon capital.

2.2. Resources: carbon budget and fossil fuel deposits

Let R_i be the carbon intensity (or emission rate) of technology i. The stock of cumulative emissions m_t grows with emissions $R_i\, q_{i,t}$:

$$\dot{m}_t = \sum_i R_i\, q_{i,t} \tag{5}$$

The social planner is subject to a so-called carbon budget, i.e., cumulative emissions cannot exceed a given ceiling $\bar{M}$:

$$m_t \leq \bar{M} \tag{6}$$

Cumulative emissions have been found to be a good proxy for climate change (Allen et al., 2009; Matthews et al., 2009).[6] Some policy instruments, such as an emission trading scheme with unlimited banking and borrowing, set a similar constraint on firms.

Finally, using fossil fuel (e.g coal, gas) i requires to extract some exhaustible resource from the initial stock S_i^0, such that the current stock $S_{i,t}$ classically satisfies:

$$\begin{aligned} S_{i,0} &= S_i^0 \\ \dot{S}_{i,t} &= -q_{i,t} \\ S_{i,t} &\geq 0 \end{aligned} \tag{7}$$

2.3. Low and zero-carbon technologies

For analytical tractability, we assume the social planner can choose only two green technologies: a fossil-fueled low-carbon technology (ℓ or LCT) and an inexhaustible zero-carbon technology (z or ZCT).

The ZCT (e.g, renewable power or electric vehicles) is completely carbon-free. As operating the ZCT does not require fossil fuels to be extracted, the ZCT is not affected by (7).

$$R_z = 0 \text{ and } S_z = \infty \tag{8}$$

We model a single preexisting high-carbon technology (h or HCT). It could represent coal power or old gasoline and is more carbon-intensive than the low-carbon technology:

$$R_h > R_\ell > 0 \tag{9}$$

[6] Many models assume the atmospheric carbon naturally decays at a constant rate. We chose not to include this to keep the analysis as simple as possible.

We assume that low-carbon capacity is cheaper than zero-carbon capacity in the sense that:

$$\forall x \quad c'_\ell(x) < c'_z(x) \tag{10}$$

Finally, we focus on the case where high carbon resources $S_{h,0}$ are also overabundant and where the ceiling on GHG concentration is binding. This corresponds for instance to a case where h represents coal, too abundant to reach the 2°C target.[7]

3. First-order conditions and optimal investment costs

3.1. Social planner's program, first-order conditions and complementarity slackness conditions

The program of the social planner consists in determining the trajectories of investment $x_{i,t}$ and production $q_{i,t}$ that minimize discounted costs while satisfying the demand D and complying with the carbon budget $\bar{M}$ (r is the constant discount rate and the Greek letters in parentheses are the costate variables and Lagrange multipliers):

$$\min_{x_{i,t},q_{i,t}} \int_0^\infty e^{-rt} \sum_i c_i(x_{i,t}) dt \tag{11}$$

$$\begin{aligned}
\text{s.t.}\quad \dot{k}_{i,t} &= x_{i,t} - \delta k_{i,t} && (\nu_{i,t})\\
q_{i,t} &\leq k_{i,t} && (\gamma_{i,t})\\
\sum_i q_{i,t} &= D && (\omega_t)\\
q_{i,t} &\geq 0 && (\lambda_{i,t})\\
x_{i,t} &\geq 0 && (\xi_{i,t})\\
\dot{m}_t &= \sum_i R_i\, q_{i,t} && (\mu_t)\\
m_t &\leq \bar{M} && (\eta_t)\\
\dot{S}_{i,t} &= -q_{i,t} && (\alpha_{i,t})\\
S_{i,t} &\geq 0 && (\beta_{i,t})
\end{aligned}$$

Notations are gathered in Table 1.

Before presenting simplified and easy-to-understand first order conditions at the next subsection, we methodically write the Hamiltonian, full FOCs and complementary slackness conditions.[8]

[7] We end up with the same options than Coulomb and Henriet (2013): our social planner may choose between a dirty backstop (e.g. coal), a clean backstop (renewable power), or a polluting exhaustible resource (gas).

[8] The transversality condition is replaced by the terminal condition that at some point the atmospheric carbon reaches its ceiling (6).

	Description	Power	Transportation
i	technology index		
h	high-carbon technology (HCT)		
l	low-carbon technology (LCT)		
z	zero-carbon technology (ZCT)		
$k_{i,t}$	capacity of technology i at time t	GW	km/yr
$q_{i,t}$	production of technology i at time t	GW	km/yr
$x_{i,t}$	investment in technology i at time t	GW/yr	km/yr^2
$\nu_{i,t}$	shadow price of new capacities i	\$/(GW· yr)	\$/km
μ_t	present cost of emissions	$\$/tCO_2$	$\$/tCO_2$
$\alpha_{i,t}$	present cost of resource used by technology i	\$/MWh	\$/km
$\gamma_{i,t}$	shadow rental cost of existing capacity i	\$/(GW· yr)	\$/km
ω_t	output price	\$/GWh	\$/km
$c_i(\cdot)$	investment costs in technology i	\$/yr	\$/yr
m_t	stock of atmospheric carbon	tCO_2	tCO_2
δ	depreciation rate	yr^{-1}	yr^{-1}
r	discount rate	yr^{-1}	yr^{-1}
R_i	emission rate of technology i	tCO_2/GWh	
$\bar{M}$	carbon budget	tCO_2	tCO_2
D	Demand	GW	km/yr

Table 1: Variables and parameters notations used in the model. The last column gives possible units for the electricity sector.

The Hamiltonian reads:

$$\begin{aligned}\mathcal{H} = e^{-rt}\sum_i c_i(x_{i,t}) - \sum_i \nu_{i,t}\,(x_{i,t} - \delta\, k_{i,t}) + \mu_t \sum_i R_i\, q_{i,t} \\ + \eta_t\left(m_t - \bar{M}\right) + \omega_t\left(D - \sum_i q_{i,t}\right) + \sum_i \gamma_{i,t}\,(q_{i,t} - k_{i,t}) \\ - \sum_i \lambda_{i,t}\, q_{i,t} - \sum_i \xi_{i,t}\, x_{i,t} - \sum_i \lambda_{i,t}\, q_{i,t} - \sum_i \xi_{i,t}\, x_{i,t} \\ + \sum_i \alpha_{i,t}\, q_{i,t} - \sum_i \beta_{i,t}\, S_{i,t}\end{aligned} \tag{12}$$

The first-order conditions are:

$$\frac{\partial \mathcal{H}}{\partial x_i} = 0 \iff c_i'(x_{i,t}) = e^{rt}(\nu_{i,t} + \xi_{i,t}) \tag{13}$$

$$\frac{\partial \mathcal{H}}{\partial q_i} = 0 \iff \gamma_{i,t} = \lambda_{i,t} - \mu_t R_i - \alpha_{i,t} + \omega_t \tag{14}$$

$$\dot{\nu}_{i,t} + \frac{\partial \mathcal{H}}{\partial k_i} = 0 \iff \dot{\nu}_{i,t} - \delta \nu_{i,t} = \gamma_{i,t} \tag{15}$$

$$\dot{\mu}_t + \frac{\partial \mathcal{H}}{\partial m_t} = 0 \iff \dot{\mu}_t = -\eta_t \tag{16}$$

$$\dot{\alpha}_i + \frac{\partial \mathcal{H}}{\partial S_i} = 0 \qquad \Longleftrightarrow \qquad \dot{\alpha}_i = \beta_{i,t} \tag{17}$$

The complementary slackness conditions are:

$$\forall i,t, \quad \xi_{i,t} \geq 0, \quad x_{i,t} \geq 0 \qquad \text{and} \qquad \xi_{i,t}\, x_{i,t} = 0 \tag{18}$$

$$\forall i,t, \quad \lambda_{i,t} \geq 0, \quad q_{i,t} \geq 0 \qquad \text{and} \qquad \lambda_{i,t}\, q_{i,t} = 0 \tag{19}$$

$$\forall i,t, \quad \eta_t \geq 0, \quad \bar{M} - m_t \geq 0 \qquad \text{and} \qquad \eta_t \left(\bar{M} - m_t\right) = 0 \tag{20}$$

$$\forall i,t, \quad \beta_{i,t} \geq 0, \quad S_{i,t} \geq 0 \qquad \text{and} \qquad \beta_{i,t}\, S_{i,t} = 0 \tag{21}$$

$$\forall i,t, \quad \gamma_{i,t} \geq 0, \quad k_{i,t} - q_{i,t} \geq 0 \qquad \text{and} \qquad \gamma_{i,t}\, \left(k_{i,t} - q_{i,t}\right) = 0 \tag{22}$$

$$\forall t, \quad \omega_t \geq 0, \quad D - \sum_i q_{i,t} = 0 \qquad \text{and} \qquad \omega_t \left(D - \sum_i q_{i,t}\right) = 0 \tag{23}$$

3.2. Optimal costs when production and investment are positive

When production and investment are strictly positive, the multipliers associated with their respective positivity constraints are nil (18,19), and first-order conditions (13,14,15) simplify to:

$$c_i'(x_{i,t}) = e^{rt} \nu_{i,t} \tag{24}$$

$$\dot{\nu}_{i,t} - \delta \nu_{i,t} = \omega_t - \mu_t R_i - \alpha_{i,t} \tag{25}$$

The simplified FOCs imply that when production and investment are strictly positive, the optimal investment schedules $x_{i,t}$ satisfy the following differential equation:[9]

$$(\delta + r)\; c_i'(x_{i,t}) - \frac{d}{dt} c_i'(x_{i,t}) = e^{rt}\; \left(\omega_t - \mu_t\, R_i - \alpha_{i,t}\right) \tag{26}$$

The Left Hand Side term corresponds to what Vogt-Schilb et al. (2013) have called the *marginal implicit rental cost of capital* (MIRCC) $p_{i,t}$:

$$p_{i,t} = (\delta + r)\; c_i'(x_{i,t}) - \frac{d}{dt} c_i'(x_{i,t}) \tag{27}$$

The MIRCC extends the concept of the implicit rental cost of capital proposed by Jorgenson (1967) to the case of endogenous capacity prices. It corresponds to the efficient market rental price of capacities, where capitalists would be indifferent between (i) buying capital at t at a cost $c_i'(x_{i,t})$, renting it out during one period dt at a price $p_{i,t}$, and selling the depreciated (δ) capacities at $t + dt$ at a price $c_i'(x_{i,t}) + \frac{d}{dt} c_i'(x_{i,t}) dt$ or (ii) simply lend money at the interest rate r.

The RHS of (26) relates to the variable costs and revenues of a producer. The output is sold at its current price $e^{rt}\, \omega_t$ (ω_t interprets as the output price, as it is the shadow cost of the demand constraint (23)). Producing one unit of the output requires to use fuel bought at the current price $\alpha_{i,t}\, e^{rt}$ (see 37 below) and pay for the emitted carbon R_i (5) at the current price $\mu_t\, e^{rt}$ (see 31).

[9] (24)$-\delta \frac{d}{dt}$(24) leads to $e^{rt}\,(\dot{\nu}_{i,t} - \delta \nu_{i,t}) = (\delta + r)\; c_i'(x_{i,t}) - \frac{d}{dt} c_i'(x_{i,t})$; substituting in (25) leads to the desired result.

Proposition 1. *If production and investment are strictly positive during a time interval* (σ_i, τ_i)*, the optimal* marginal implicit rental cost of capital $p_{i,t}$ *is equal to the current price of the output* $\omega_t\, e^{rt}$ *minus the current cost of emissions* $\mu R_i\, e^{rt}$ *and minus the current price of fossil resources* $\alpha_i\, e^{rt}$*.*

$$\forall t \in (\sigma_i, \tau_i), \quad p_{i,t} = e^{rt}\ \left(\omega_t - \mu_t\, R_i - \alpha_{i,t}\right) \tag{28}$$

Prop. 1 can be seen as an application of the equimarginal principle. It provides a simple rule to arbitrate *production* decisions at each moment, by relating the output price, the rental cost of productive capacities and the variable costs. As the equimarginal principle applies to the decision of renting the capital, it does not directly describe trade-offs for *investors*.

Investment decisions follow a much more complex dynamic. In our framework, the quantity that would help an investor to make an investment decision is the optimal Marginal Investment Cost (MIC). In the remaining of this article, we try to express the optimal MIC. The first step is to derive the general expression given by the next corollary.

Corollary 1. *If production and investment are strictly positive during a time interval* (σ_i, τ_i)*, the optimal Marginal Investment Cost (MIC) can be expressed as a sum of two terms: (i) the present value of all future revenues from selling the output minus costs from emission and resource usage* $(\omega_\theta - \mu R_i - \alpha_i)$ *produced by the depreciated marginal unit of capacity* $(e^{-\delta(t-\theta)})$*, plus (ii) a term that tends to* $c_i'(x_{i,\tau_i})$*:*

$$\forall t \in (\sigma_i, \tau_i), \tag{29}$$
$$c_i'(x_{i,t}) = e^{rt} \int_t^{\tau_i} e^{-\delta(t-\theta)} (\omega_\theta - \mu_\theta\, R_i - \alpha_{i,\theta}) d\theta + e^{(r+\delta)(t-\tau_i)} c_i'(x_{i,\tau_i})$$

Proof. Appendix A shows that (29) is the textbook solution of (26). □

Corollary 1 can be seen as a generalization of the previous finding by Vogt-Schilb et al. (2012) that when abatement is obtained by accumulating low-carbon capital, optimal efforts to curb emissions are not necessarily growing over time.

Corollary 2. *If investment in and usage of a technology i never stop after a date* σ_i*, the optimal MIC for that technology simply equals the present value of all future revenues from selling the output minus costs from emission and resource usage produced by the depreciated marginal unit of capacity*

$$\forall t \in (\sigma_i, \infty), \quad c_i'(x_{i,t}) = e^{rt} \int_t^{\infty} e^{-\delta(t-\theta)} (\omega_\theta - \mu_\theta\, R_i - \alpha_{i,\theta})\, d\theta \tag{30}$$

Proof. Taking the limit of (29) when $\tau_i \to \infty$ leads to the result.[10] □

Corollary 2 expresses the optimal investment cost of the technology used during the steady state (we show later that it is the zero-carbon technology).

Corollary 1 and Corollary 2 give a general relation between the optimal MIC and a discounted sum of future revenues during a time period when capacities

[10] We implicitly made the (reasonable) assumption that $c_i'(x_{i,t})$ is bounded if $x_{i,t}$ is bounded.

are used (they do not provide more information than Prop. 1). Assessing the optimal investment costs in practice requires to express more concretely those net revenues, hence the output price ω_t, the carbon price μ_t, the resource costs $\alpha_{i,t}$, and the time period (σ_i, τ_i). Doing so is the purpose of the next section.

4. Transition phases, output price and explicit optimal investment costs

4.1. General phases

The number of inequalities combinations captured by the slackness conditions is large. The different cases may be tackled analytically if we assume that on the optimal path, the system passes through phases (this assumption is confirmed by numerical simulations with standard functional forms, but cannot be proved for general functions).

The following proposition and Fig. 1 summarize our findings concerning those phases.

Proposition 2. *The optimal pathway can be divided into four main phases:*

1. *In a first phase (for $t \in [0, T_\omega]$) HCT production decreases (compensated by the increasing total production of LCT and ZCT) and the output price equals the (constant) emission costs plus the resource costs from the high-carbon technology: $\omega_t = \mu R_h + \alpha_h$.*
   Then (for $t \in [T_\omega, T]$) LCT production decreases and ZCT production increases (LCT production may decrease before T_ω).
2. *In the second phase ($t \in [T_\omega, T_\gamma]$), LCT production decreases slower than the natural rate of replacement of its capacity. Investment in LCT continues even if its production decreases ($x_{\ell,t} < \delta k_{\ell,t}$).*
3. *In a third phase (for $t \in [T_\gamma, T]$) LCT production decreases faster than the natural depreciation rate of its capacity and the output price equals the sum of constant resource costs and constant emission costs from the low-carbon technology: $\omega_t = \mu R_\ell + \alpha_\ell$.*
4. *At T, the system reaches a steady state, all production comes from the ZCT, emissions are nil, atmospheric pollution is at its ceiling. If low-carbon resources were binding they are exhausted at T ($\alpha_{\ell,T}\, S_{\ell,T} = 0$).*

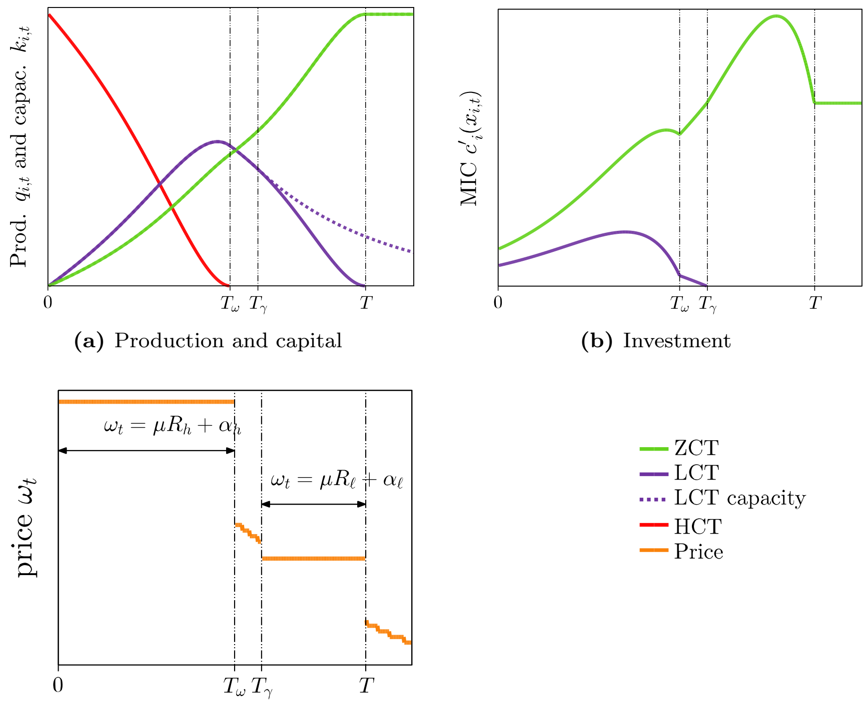
In the remaining of this subsection we present four lemmas that build a proof for Prop. 2.

Definition 1. *Let T_{atm} be the date when the ceiling on atmospheric carbon is reached.*

Before T_{atm}, the social cost of carbon μ_t is constant (16,20):

$$\forall t < T_{atm}, \quad \mu_t = \mu > 0 \tag{31}$$

The carbon-free atmosphere can be seen as a non renewable resource depleted by GHG emissions. In this context, the optimal current carbon price μe^{rt} follows the Hotelling's rule, i.e. grows at the discount rate, as abatement realized at any time contributes equally to meet the carbon budget. The carbon price μ is strictly positive as we focus on the case where the carbon budget is binding.



(a) Production and capital

(b) Investment

(c) Output price

Figure 1: Illustration of the four main phases.

Definition 2. *Let T be the date when the system reaches a steady state.*

During the steady state, the ZCT produces all the output. Indeed, atmospheric carbon is stable, hence emissions from the HCT and LCT must be nil (3,4):

$$\forall t \geq T,\ \dot{m}_t = 0 \implies q_{\ell,t} = q_{h,t} = 0 \tag{32}$$

HCT production can stop before the system reaches a steady state.

Definition 3. *Let $T_\omega \leq T$ be the date when high-carbon production stops.*

$$\forall t \geq T_\omega,\ q_{h,t} = 0 \tag{33}$$

Lemma 1. *Before the HCT is phased out, the optimal output price ω_t is equal to the sum of variable cost and emission costs from the high-carbon technology:*

$$\forall t \leq T_\omega, \quad \omega_t = \mu R_h + \alpha_h \tag{34}$$

Proof. By assumption, the HCT capacity is always underused ($q_{h,t} < k_{h,t}$), hence the multiplier associated with the capacity constraint is nil $\gamma_{h,t} = 0$ (22). While h is used to produce the output, the multiplier associated with the positivity constraint is also nil $\lambda_{h,t} = 0$ (19). The output price ω_t can then be obtained from (14). □

In the power sector, Lemma 1 means that as long as the marginal power plant is a coal power plant, the price of electricity is the cost of coal plus the carbon price times the emission rate of coal.

It is possible that at one point, production from the LCT declines. One possible reason is if fossil deposit are almost depleted. Another reason relates to GHG emissions. From T_ω, the demand constraint (4) makes it impossible to reduce further emissions by using either additional ℓ or z. Total emissions can be reduced further by producing more with the ZCT and less with the LCT (since $R_z < R_\ell$). Therefore, from T_ω on, LCT production may decline to allow for more ZCT production. In particular, it may become beneficial to use less ℓ than allowed by installed capacities.

Definition 4. *Let $T_\gamma \leq \min(T_{S_\ell}, T_{atm})$ be the date when LCT production is lower than its capacity:*

$$\forall t \geq T_\gamma,\ q_{\ell,t} < k_{\ell,t}$$

Lemma 2. *Along the optimal path, when low-carbon capacities are used, but under full capacity, variable costs from LCT determine the output price:*

$$\forall t \in [T_\gamma, T] \quad \omega_t = \mu R_\ell + \alpha_\ell \tag{35}$$

Proof. The proof is similar to that of Lemma 1. □

Corollary 3. *Along the optimal path, the low-carbon capacities may not be underused before high-carbon production is phased out.*

$$T_\gamma > T_\omega$$

Proof. In general, the output price cannot be equal to the variable costs of both the HCT and the LCT (i.e. both Lemma 1 and Lemma 2 cannot hold at the same time). For instance, in the power sector, the marginal power plant may be coal or gas, but not both at the same time.[11] □

Lemma 3. *In the steady state, the output price equals the rental cost of the zero-carbon capacity:*

$$\forall t \geq T \quad e^{rt}\omega_t = p_{z,t} \tag{36}$$

Proof. From Prop. 1, as the zero-carbon technology does not require to burn any resource ($\alpha_z = 0$), nor pay for any emission ($R_z = 0$). □

In the power sector, Lemma 3 means that when all the electricity is produced from windmills, the market price of electricity equals the rental price of windmills.

Definition 5. *Let T_{S_i} be the date when deposit i is depleted.*

Fossil fuel prices follow the Hotelling rule, their present price $\alpha_{i,t}$ is constant before they are exhausted (17,21):

$$\forall t < T_{S_i},\ \alpha_{i,t} = \alpha_i > 0 \tag{37}$$

Lemma 4. *Along the optimal path, if low-carbon resources are depleted before the steady state, they are exhausted at the moment when atmospheric pollution reaches its ceiling and when the productive system reaches its steady state.*

$$T_{S_\ell} \leq T \implies T_{S_\ell} = T_{atm} = T$$

If high-carbon resources are depleted before the steady state, then

$$T_{S_h} = T_\omega$$

Proof. If low-carbon resources are eventually depleted, it happens at the date T_{S_ℓ}. This date arrives after low-carbon capacity is underused, hence after the HCT is phased out ($T_{S_\ell} \geq T_\gamma \geq T_\omega$) (Definition 4 and Corollary 3). Once low-carbon resources are exhausted, all the demand must be satisfied by the zero carbon technology, hence the atmospheric carbon m_t remains stable. As we assumed the carbon budget is binding, the ceiling $\bar{M}$ is reached at this moment ($T_{atm} = T_{S_\ell}$).

Concerning the high carbon resource: if they get depleted, this happens at T_{S_h}. At this moment, production from the HCT stops, hence $T_{S_h} = T_\omega$. □

Lemma 4 and (37) ensure that fossil fuel prices $\alpha_{i,t}$ are constant during the whole time when the corresponding fossil fuels are used.

This subsection has provided insights on the general shape of the transition from high-carbon production to low-carbon and to zero-carbon production, and on the price of the output.We still lack a characterization of the period during which the optimal marginal investment costs may be calculated explicitly (the (σ_i, τ_i) from Corollary 1). The next subsection attempts to provide one.

[11] We disregard the case where fuel costs compensate exactly differences in carbon intensities $\alpha_\ell - \alpha_h = \mu\,(R_h - R_\ell)$ as it requires a very restrictive set of assumptions.

4.2. Sub-phases and ordering

Definition 6. *Let T_i be the date when investment in capacity i starts. Let T_ℓ^e be the date when investment in the LCT ends.*[12]

The HCT is phased out only after investment in one of the green technologies started:

$$T_\omega \geq \min(T_\ell, T_z) \tag{38}$$

If the low-carbon capacity is underused, investment in new low-carbon capacity is not optimal. Therefore the latter stops before LCT production drops below installed capacity:

$$T_\ell^e \leq T_\gamma \tag{39}$$

Lemma 5. *ZCT investment starts before LCT investment ends ($T_z \leq T_\ell^e$)*

Proof. At any given time, existing capacities are used in the merit order, i.e. capacities with the lowest variable cost are used first. This means that LCT production is never replaced by HCT production (9), or in other terms total instantaneous emissions never increase.

If LCT investment stops before investment in the ZCT starts, i.e. before ZCT production starts replacing HCT and LCT production, LCT production necessarily decreases at least with the depreciation rate of LCT capacity, and hence is replaced by HCT production to comply with the demand constraint, which is in contradiction with the previous statement. □

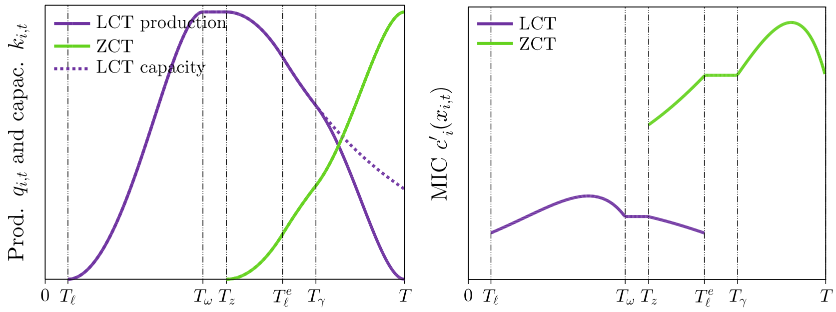
Several ordering possibilities remain, summarized in the following proposition:

Proposition 3. *Investment phases may be ordered in following ways (Fig. 2):*

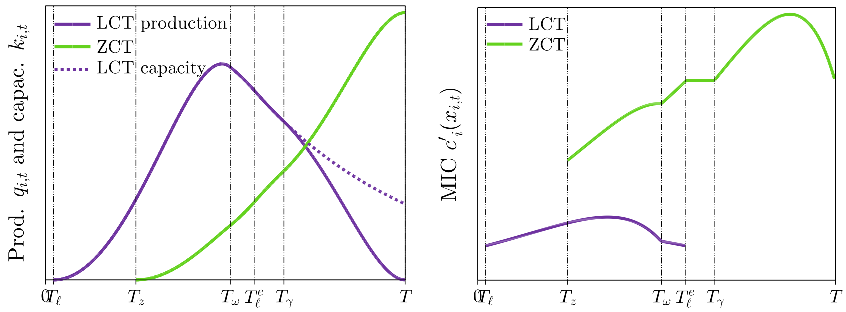
1. *Two successive transitions, starting with LCT investment. The LCT completely replaces the HCT first, then the ZCT replaces the LCT (Fig. 2a).*
2. *Two overlapping transitions, with a phase of simultaneous investment in the LCT and the ZCT. Investment in the LCT start first, and investment in the ZCT start before the HCT has been completely replaced (Fig. 2b). Investment in the LCT can stop before or after the HCT has been completely replaced.*
3. *Two overlapping transitions, with a phase of simultaneous investment in the LCT and the ZCT. Investment in the more expensive ZCT start first, and investment in the LCT start before the HCT has been completely replaced. Investment in the LCT can stop before or after the HCT has been completely replaced (Fig. 2c).*

Prop. 3 is similar to the finding by Chakravorty et al. (2008) that the optimal extraction of several polluting nonrenewable resources may follow several unintuitive orderings. In their work, however, the dynamics comes from the interaction of several scarcity rents; in ours, it comes from the convexity on investment costs.

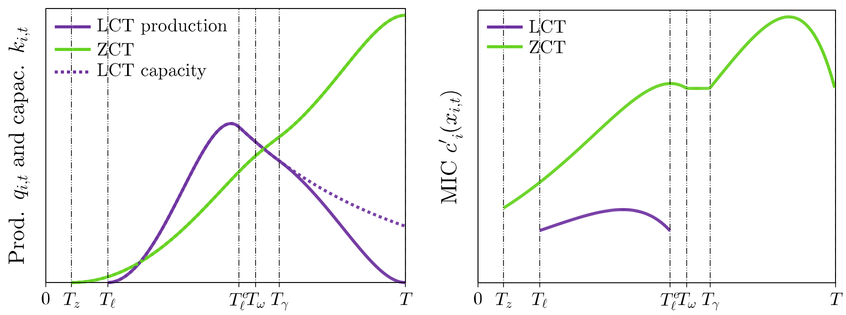
[12] We do not need an equivalent definition for the ZCT as it used in the steady state and investment never stop.



(a) Two successive transitions. HCT is phased out by LCT, then LCT is phased out by ZCT: $T_\ell < T_\omega < T_z < T_\ell^e$



(b) Two overlapping transitions, starting with the cheapest substitute (LCT): $T_\ell < T_z < T_\omega < T_\ell^e$



(c) Two overlapping transitions, starting with the most expensive substitute (ZCT): $T_z < T_\ell < T_\ell^e < T_\omega$

Figure 2: Numerical simulations displaying three possible transition profiles. Figures on the left display capacities and productions, figures on the right display optimal MICs. The parameters used to produce these figures are gathered in Appendix B.

4.3. Explicit optimal marginal investment costs

Using previous results, the optimal MIC for the LCT and the ZCT can be expressed as a function of the carbon price and the resource costs during the different phases, refining the general expression given by Eq. 29:

$$\begin{aligned}
\forall t \geq T_z, \quad e^{-rt}\, c_z'(x_{z,t}) &= \int_t^{T_\omega} e^{-\delta(t-\theta)}(\mu\, R_h + \alpha_h)\, d\theta + \int_{T_\omega}^{T_\gamma} e^{-\delta(t-\theta)}\omega_\theta\, d\theta \\
&\quad + \int_{T_\gamma}^{T} e^{-\delta(t-\theta)}(\mu\, R_l + \alpha_\ell)\, d\theta + \int_T^{\infty} e^{-\delta(t-\theta)}\omega_\theta\, d\theta \\
\forall t \in [T_\ell, T_\ell^e], \quad e^{-rt}\, c_\ell'(x_{\ell,t}) &= \int_t^{T_\omega} e^{-\delta(t-\theta)}(\mu\,(R_h - R_\ell) + \alpha_h - \alpha_\ell)\, d\theta \\
&\quad + \int_{T_\omega}^{T_\ell^e} e^{-\delta(t-\theta)}(\omega_\theta - \mu\, R_\ell - \alpha_\ell)\, d\theta + c_\ell'(0)\, e^{(r+\delta)(t-T_\ell^e)}
\end{aligned} \tag{40}$$

Proposition 4. *When the social planner invests in both the ZCT and the LCT, it builds zero-carbon capacity at a higher cost than low-carbon capacity.*

Proof. From (40):

$$\forall t \in [\max_i (T_i)\,,\ T_\ell^e],\ c_z'(x_{z,t}) - c_\ell'(x_{\ell,t}) =$$
$$\underbrace{(\mu\, R_\ell + \alpha_\ell)\, e^{rt}\int_t^{T_\ell^e} e^{-\delta(\theta-t)} d\theta}_{\Delta p} + \underbrace{\left(c_z'(x_{z,T_\ell^e}) - c_\ell'(0)\right) e^{(r+\delta)(t-T_\ell^e)}}_{\Delta c'} \tag{41}$$

Δp is the discounted value of emissions and fossil fuels that the marginal zero-carbon capacity built at time t allows to save before T_ℓ^e when compared to the marginal low-carbon capacity built at time t.
$\Delta c'$ is the difference between the values of the marginal capacities built at T_ℓ^e discounted to t. It is strictly positive, as $c_z'(x_{z,T_\ell^e}) > c_z'(0)$ as c_z' is growing by assumption and $c_z'(0) > c_\ell'(0)$ (10). □

The left column of Fig. 2 illustrates Prop. 4. In particular, Fig. 2a displays a case where it is optimal to start with the most expensive option, similarly to the previous result by Vogt-Schilb and Hallegatte (2011).

Eq. 40 expresses the optimal MIC in a more detailed fashion than (29). However, Eq. 40 still does not give a direct assessment of the optimal investment cost in any of the two technologies. In particular, the output price when the LCT is fully used and during the steady state is unknown ($\omega_t,\ t \in [T_\omega,\ T_\gamma] \cup [T,\ \infty)$), as well as the amount of investment in the ZCT when investment in the LCT stops ($c_z'(x_{z,T_\ell^e})$), and the dates when the different phases begin and end.

In order to investigate whether the levelized cost of low-carbon capital can provide an accurate rule of thumb to assess investment in different types of low-carbon capital, the next section uses a numerical version of the model calibrated on the European electricity sector.

5. Numerical application: the case of the European electricity sector

5.1. Modeling framework, data, calibration

Let us calibrate a modified version of our model with data from the European power sector. In this numerical application, efficient gas power plants (the LCT)

Table 2: Technology sets considered in the numerical model

Set	Acronym	Description	Composition
High carbon technology set	HCT	Average current thermal production mix in 2008	Gas (approx. 40 %), coal (approx. 50 %), oil (approx. 10 %), source ENERDATA (2012)
Low carbon technology set	LCT	Efficient peak-load technologies	Combined cycle gas turbine, pulverized or super-critical coal power stations
Zero carbon technology set	ZCT	New generation renewable technologies	Onshore wind, biomass

and renewable power (the ZCT) are used to phase out the existing emitting capacities represented as the average current thermal production mix. Table 2 gives the aggregation of technology sets used in the numerical simulation.

To better fit the data, we express installed capacity $k_{i,t}$ in peak capacity (GW), and production $q_{i,t}$ in GWh/yr. Production is constrained by a maximum number of operating hours H_i (lower for renewables to capture intermittency issues). This constraint captures the imperfect substitutability between different green technologies. For instance, a given windmill will produce power only at the moments where it is windy, which expectedly happens a given number of hours per year.[13]

We define the utilization rate $u_{i,t}$ of installed technology i at time t as:

$$u_{i,t} = \frac{q_{i,t}}{H_i\, k_{i,t}} \tag{42}$$

We also model different lifetimes for different technologies, hence different depreciation rates δ_i (renewable plants have shorter lifetimes than fossil-fueled plants).

As resource depletion happens at the global scale, we consider that Europe is price-taker for exhaustible resources (coal and gas), which costs are included in the form of fuel costs α_i (constant in present value).

The model becomes (omitting the positivity constraints):

$$\begin{aligned}
&\min_{x_{i,t},q_{i,t}} \int_0^\infty \sum_i (e^{-r\,t} c_i(x_{i,t}) + \alpha_i\, q_{i,t}) dt \qquad (43)\\
\text{s.t.}\quad & \dot{k}_{i,t} = x_{i,t} - \delta_i k_{i,t}\\
& q_{i,t} \le H_i \cdot k_{i,t}\\
& \sum_i q_{i,t} = D\\
& \dot{m}_t = \sum_i R_i\, q_{i,t}\\
& m_t \le \bar{M}
\end{aligned}$$

[13] A better representation of the power generation sector would model windy periods as a stochastic process. This refinement is out of the scope of this paper.

Table 3: Technology-specific data used in the numerical application.

	Description	Unit	HCT	LCT	ZCT	Source
α_i	Fuel costs	\$/MWh	55	60	0	IEA (2010)
C_i^m	Nominal investment costs	\$/kW	1 800	1 200	2 000	IEA (2010)
X_i	Average annual new capacity in Europe	GW/y	4.2	11	10	ENERDATA (2012)
H_i	Average annual operating hours	h/y	7 500	7 500	2 000	IEA (2010)
L_i	Learning rate	%/yr	0	0	2	EWEA (2012)
δ_i	Deprecation rate	%/yr	2.5	3.33	4	EWEA (2012)
R_i	Carbon intensity	gCO_2/kWh	530	330	0	ENERDATA (2012); Trotignon and Delbosc (2008)

Table 4: General parameter values used in the numerical application.

	Description	Unit	Value	Source
r	Discount rate	%/y	5	
$\bar{M}$	Carbon budget	$GtCO_2$	17	UE (2011); Trotignon and Delbosc (2008)
D	Power demand	TWh/y	1 940	ENERDATA (2012)
A	Convexity parameter	·	0.1	

We assume quadratic investment costs. We complete the cost function with an exogenous learning rate L_i.[14] To calibrate the cost functions, we assume that when investment equals the average annual investment flow in Europe between 2009 and 2011 (X_i), the marginal investment cost C_i^m is equal to the OECD median value for 2010 (as found in IEA (2010)). We write the cost function as:

$$c_i(x_{i,t}) = C_i^m \cdot X_i \cdot \left(A \, \frac{x_{i,t}}{X_i} + \frac{1 - A}{2} \left(\frac{x_{i,t}}{X_i} \right)^2 \right) \cdot e^{-L_i \, t} \tag{44}$$

$$t = 0 \implies c_i'(X_i) = C_i^m \tag{45}$$

A is a convexity parameter, assumed equal across technologies. If $A = 1$, the marginal investment cost is constant (the cost of new capacity does not depend on the investment pace), and optimal investment pathways would exhibit jumps: there would be no economic inertia (Vogt-Schilb et al., 2012). If $A = 0$ the marginal cost curves become linear (the cost of new capacity doubles when the investment pace doubles) and capacity accumulated at very low speed is almost

[14] The numerical simulations show that adding this feature does not change the qualitative form of the solution: the transition still displays the phases described in the analytical section.

free ($\lim_{x_{i,t}\to 0; A=0}\ c_i'(x_{i,t}) = 0$). An intermediate value $A \in (0,1)$ means that new capacity is always costly, and that its cost grows with the investment pace.

The illustrations in this section are obtained with $A = 0.1$, i.e. with a relatively low convexity (investment cost doubles at 2.11 times the nominal pace). For instance, in the base year (2008), building new renewable at the pace of 10 GW/yr costs 2\$/W. At 20 GW/yr, it would cost 4.2\$/W.

The emission allowances allocated to the power sector amounted to $E_{\text{ref}} =$ 1.03 GtCO_2/yr in 2008 (Trotignon and Delbosc, 2008). The reference fossil energy production (from coal, oil and gas) was $D = 1\,940$ TWh/yr that year (ENERDATA, 2012), leading to a reference emission rate of 530 tCO_2/GWh. We take a carbon budget corresponding to roughly half of the BAU cumulative emissions, i.e. 17 GtCO_2.

We calibrate the depreciation rate as $\delta_i = 1/\text{lifetime}$ and assume a lifetime of 30 years for the existing capacity and new gas and 25 years for wind (IEA, 2010). We use $r = 5$ %/yr for the social discount rate.

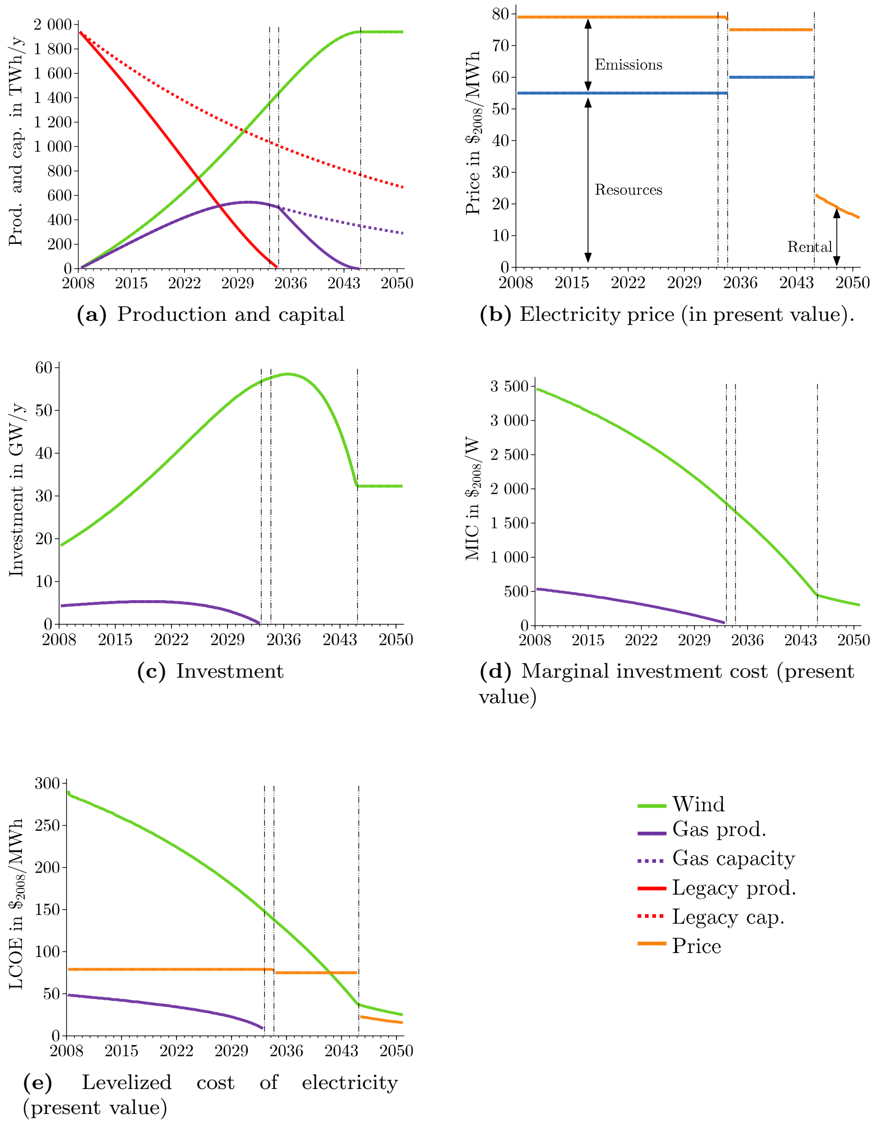
5.2. Results

Fig. 3 shows various variables of the numerical application to the European electricity sector. First of all, the investment profile goes through the phases discussed in the theoretical section. Results from Section 4 are robust to the particular extensions made in this section (the social planner can invest in dirty capital, depreciation rates are different for wind, gas and legacy capacity, there is exogenous learning for wind). In particular, the social planner does not invest in the legacy capacity, which is entirely phased out in 2035 (Fig. 3a). There is unused gas capacity as soon as the dirty technology is phased out ($T_\gamma = T_\omega =$ 2035), and investment in gas stops a couple of years earlier ($T_\ell^e = 2033$).

Investment in both efficient gas and renewable power starts from the beginning of the simulation (Fig. 3c). The exogenous technical progress on the windmills is not sufficient to postpone investment in windmills in the optimal solution. Until 2038, investment in windmills grows over time. Investment starts at 18 GW/yr in 2008, almost twice the actual average investment rate X_i, and reach 60 GW/yr in 2038. It decreases after 2040 as most of the power plants have already been replaced (Fig. 3a), and stay constant after 2045 to maintain the wind capacity constant.

Fig. 3d displays the resulting marginal costs for new capacity (MICs) along the period, expressed in present value. They decrease over time, as the average power plants becomes less and less carbon-intensive, making investment in low carbon capacity less and less profitable. Investment in gas remains relatively low by contrast. Prop. 4 holds: the MIC is always higher for the renewables, despite renewable being subject to exogenous technical progress and having a higher depreciation rate (both reasons to invest less in renewables in the short term).

Electricity prices are displayed in Fig. 3b. While production comes from fossil resources the price decomposes as resource cost and emission cost (Lemma 1 and Lemma 2). In a first phase (before 2035), the marginal capacity is the legacy dirty technology, and the electricity price is high. After the dirty technology has been phased out, from 2035 to 2045, gas becomes the marginal technology and the price drops. The endogenous carbon price is 46 \$/tCO_2, and the lower carbon intensity of gas compared to coal more than compensates the higher



(a) Production and capital

(b) Electricity price (in present value).

(c) Investment

(d) Marginal investment cost (present value)

(e) Levelized cost of electricity (present value)

Figure 3: Outputs from the numerical application to the European electricity sector

resource cost. In the last phase, all the electricity comes from renewable power, and electricity price equals the rental cost of renewable power plants (Lemma 3). This rental cost decreases because we assumed exogenous technical progress in the renewable power sector (see L_i in equation B.1 and Table 3).

5.3. Levelized Cost Of Electricity

Levelized costs of electricity are frequently used to compare different technologies in the power sector, sometimes with the underlying idea that technologies with lower LCOEs are cheaper, hence superior, to technologies with higher LCOEs (e.g. IPCC, 2007; Alok, 2011; Kost et al., 2012; EIA, 2013).

As the cost-efficiency of investment in low-carbon capital is not easy to assess precisely (Section 4.3), a question is whether the LCOE may be used as a good proxy.

Definition 7. *The* Levelized Cost of Electricity *(LCOE), denoted $\mathcal{L}_{i,t}$, is the ratio of discounted costs to discounted production of the marginal capacity (we express them in present value):*

$$\mathcal{L}_{i,t} = \frac{e^{-rt} c_i'(x_{i,t}) + \int_t^\infty (\mu R_i + \alpha_{i,\theta})\, u_{i,\theta}\, e^{-\delta_i(\theta - t)}\, d\theta}{\int_t^\infty u_{i,\theta}\, e^{-(r+\delta_i)(\theta - t)}\, d\theta} \tag{46}$$

The total costs from the marginal capacity built at t express as the investment cost $c_i'(x_{i,t})$, plus the variable costs $(\mu R_i + \alpha_{i,\theta})$ associated with the marginal capacity along its lifetime (during which it will depreciate at the rate δ_i and will be used at a rate $u_{i,\theta}$ (42)). The denominator is the discounted production of the depreciating marginal unit of capacity over time.

Fig. 3e shows the levelized costs of electricity along the optimal pathway simulated for the European Union, and compare them with the corresponding electricity price. Optimal LCOEs differ among technologies, and differ from the electricity price. The optimal LCOEs of wind are found higher than electricity prices, and electricity prices are themselves higher than the LCOE generated from gas.

Indeed, investment costs should be higher in renewable power for four reasons: (i) renewable power saves more GHG than gas; (ii) renewable power saves fossil energy, compared to gas; (iii) renewable power has a higher share than gas in the optimal long-term technology mix, and (iv) investment costs are convex. Levelized cost of electricity account for fixed investment costs assume constant, GHG emissions and variable energy costs. They leave aside the third and four reasons (in other terms, they "forget" the $\Delta c'$ term from Prop. 4). This term accounts for the fact that renewable power will be used forever (during the steady state), while at some point investment in gas power must stop as gas plants built to phase the coal out shall themselves be eventually replaced by renewable power. Because investment costs are convex, investing early reduces the discounted cost of the transition, and the optimal long-term technology mix has consequences on optimal short-term efforts.

Vogt-Schilb et al. (2013) demonstrate that a similar criteria (the levelized abatement cost) is accurate only if capacity costs are constant in time and do not depend on the investment pace. In our numerical simulations, even if the capacity cost slowly increases with the investment pace (the convexity of the cost function is low $A = 0.1$), the optimal levelized cost of electricity produced from renewable sources is almost six times greater than the levelized cost of electricity produced from gas. This suggests that LCOEs should not be used as a rule-of-thumb metrics to assess investment.[15]

In our simulation, if decision makers decided investment in new capacity by comparing LCOEs to electricity price, they would build too much low-carbon capacity, and not enough zero-carbon capacity.

6. Conclusion

We investigate in an analytical model the optimal timing of investment in low-carbon (e.g. gas power plant, efficient thermal vehicles) and zero-carbon (e.g. renewable power, electric vehicles) capital to phase out preexisting high-carbon capital within a sector facing an inelastic demand (electricity or private mobility) and a carbon budget. We then run numerical simulations calibrated on the European power sector.

We find that in a first phase, new gas and renewable power plants should progressively replace the preexisting higher-carbon plants. During this phase, the electricity price equals the variable costs of producing electricity from coal (i.e. energy costs and the cost of carbon emissions). Then, electricity is produced by both types of greener capital used at full capacity. The renewable power plants continue to accumulate to increase current abatement, and the social planner allows the natural depreciation process to decrease available gas power capacities. In a third period, gas power plants may be underused, to allow even more abatement to be performed by production from additional renewable power plants. In this period, the market price of electricity equals the cost of buying gas and paying for induced GHG emissions. Finally, a steady state is reached where all the production comes from renewable carbon-free power.[16]

Within our modeling framework, the availability of gas resources does not modify qualitatively these phases. This finding contrasts with previous results from the resource economics literature. Further research could investigate whether it comes from assumptions made for analytical simplification (namely constant demand and no proportional natural dilution of atmospheric pollution) or from assumptions made for realism (consuming resources requires to accumulate adequate capital at a convex cost).

Another finding is that the ordering of investment does not follow any easily predetermined order; in particular, investment in the expensive carbon-free capital (renewable power, electric vehicles) may begin at the same time, or even before, investment in the lower-cost low-carbon capital (e.g gas plants, efficient thermal engines).

Assessing the optimal cost of investment in low-carbon capital turns out to be tricky. In our model, the equimarginal principle gives information on the optimal price at which capacities should be rented once constructed. In the power sector, for instance, this results in equalizing the rental cost of a particular plant plus the costs of buying fuel and paying for the GHG emissions to the price of electricity. It does not give a direct information on the social cost at which new capacities

[15] Further research should carry out a sensitivity analysis on the convexity parameter A and the climate policy stringency $\bar{M}$.

[16] The private mobility sector would go through the same phases.

should be built in the first place. In theory, the optimal investment cost simply expresses as the discounted sum of all future revenues derived from renting the capital (Section 3.2). However, actually calculating optimal investment costs requires to solve the model backward, and in particular to know in advance the output price along the various investment and production phases.

The only analytical result concerning optimal investment costs is that, when capacities in both gas and renewable power are being built, we should always invests more dollars per installed capacity in renewable power. This is not explained only by cheaper operation costs of renewable power coming from both the carbon price and nil fossil energy requirements. Renewable power may be used forever, while the exhaustible and polluting low-carbon capacity built to phase out the preexisting dirtier plants will eventually be phased out itself by the renewable power. The same conclusion applies in the private transportation sector: if electric vehicles and efficient thermal vehicles are built at the same time, electric vehicles should be built at a higher cost.

In practice, a tempting approach to assess the optimal cost of investment in low- and zero-carbon capital could be to use a rule based on the levelized cost. The investment criteria would be to build new capacities that appear competitive, e.g. would produce electricity at a levelized costs lower or equal to the market price (a hasty equimarginal principle). In our numerical simulations, along the optimal path, the levelized cost of electricity produced from renewable sources is almost six times greater than the levelized cost of electricity produced from gas. This suggests that ranking technologies according to their levelized cost of electricity would lead to too much investment in intermediate technologies (such as gas or efficient thermal vehicles), and too little in more expensive zero-carbon capital (as renewable power or electric vehicles). The *levelized cost* does not provide enough information to assess and rank investment in polluting fossil-fueled and zero-carbon capital.

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Appendix A. Solving for optimal MICs (proof of proposition 1)

We use the generic algorithm to solve the following first-order linear differential equation:

$$\frac{d}{dt} c_i'(x_{i,t}) = (\delta + r)\, c_i'(x_{i,t}) - e^{rt}\, \left(\omega_t - \mu\, R_i - \alpha_i\,\right) \tag{A.1}$$

The general theory[17] ensures that if $z_{i,t}$ satisfies:

$$\dot{z}_{i,t} = -e^{-(\delta+r)t} \left(e^{rt} \left(\omega_t - \mu\, R_i - \alpha_i\,\right)\right) \tag{A.2}$$

Then $c_i'(x_{i,t}) = e^{(\delta+r)t}\, z_{i,t}$ is a solution of (A.1). The general solution of (A.2) on an interval (σ_i, τ_i) reads:

$$z_{i,t} = z_{i,\tau_i} + \int_t^{\tau_i} e^{-(\delta+r)\theta}\, e^{r\theta} \left(\omega_\theta - \mu\, R_i - \alpha_i\,\right) d\theta \tag{A.3}$$

Leading to:

$$c_i'(x_{i,t}) = e^{(\delta+r)t} z_{i,\tau_i} + e^{(\delta+r)t} \int_t^{\tau_i} e^{-(\delta+r)\theta}\, e^{r\theta} \left(\omega_\theta - \mu\, R_i - \alpha_i\,\right) d\theta \tag{A.4}$$

[17] See for instance Wikibooks contributors (2013)

$$= e^{(\delta+r)t} z_{i,\tau_i} + e^{rt} \int_t^{\tau_i} e^{-\delta(t-\theta)} (\omega_\theta - \mu\, R_i - \alpha_i) d\theta \tag{A.5}$$

The constant z_{i,τ_i} may be determined by evaluating the RHS at $t = \tau_i$, leading to:

$$c_i'(x_{i,t}) = e^{(r+\delta)(t-\tau_i)} c_i'(x_{i,\tau_i}) + e^{rt} \int_t^{\tau_i} e^{-\delta(t-\theta)} (\omega_\theta - \mu\, R_i - \alpha_i) d\theta \tag{A.6}$$

Appendix B. Numerical values used to produce Fig. 2

The simulations displayed in Fig. 2 were produced using the following quadratic cost functions, and parameters described in Table B.5:

$$c_i(x_{i,t}) = A_i\, x_{i,t} + \frac{B_i}{2} x_{i,t}^2 \tag{B.1}$$

	Fig. 2c & 2c	Fig. 2b & 2b	Fig. 2a & 2a
δ	0.03	0.03	0.03
$\bar{M}$	60	60	60
D	3600	3600	3600
r	0.05	0.05	0.05
R_z	0	0	0
R_h	0.0005	0.0005	0.0005
R_ℓ	0.0002	0.0002	0.0001
A_z	6.4	8	10
A_ℓ	4.2	2.3	3
Bz	0.09	0.05	0.03
B_ℓ	0.01	0.01	0.01

Table B.5: Parameters used to produce the figures