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# Low flow regionalization by regression and hybrid methods

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# INTRODUCTION



## ➤ *Low Flow*

Knowledge of the magnitude of low flow events is required in order to drought management.

The 7-day, 10-year low flow ( $Q_{7,10}$ ) is a commonly used low flow statistic. The ( $Q_{7,10}$ ) is the average annual 7-day minimum flow that is expected to be average every 10 years.

# ...INTRODUCTION



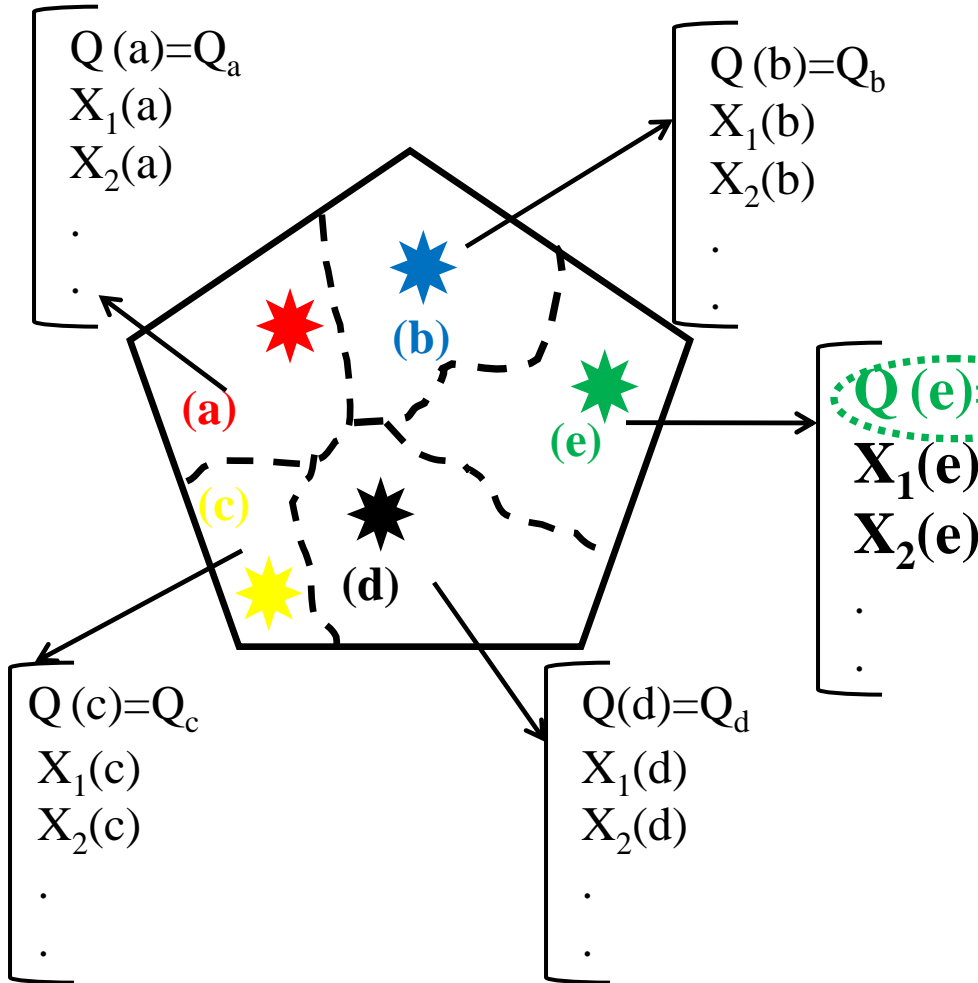
In this study frequency of the 7-day annual low flow is specified and developed regional regression equations to estimate the low-flow frequency at ungaged sites.

# ...MATERIAL & METHODS

- *Low flow models*
  - Regression method
  - Low flow index
  - Regionalizing the frequency formula parameters

# ...MATERIAL & METHODS

## - Regression method



<u>Q</u>	<u>X<sub>1</sub></u>	<u>X<sub>2</sub></u>	·	·
Q <sub>a</sub>	X <sub>1</sub> (a)	X <sub>2</sub> (a)	·	·
Q <sub>b</sub>	X <sub>1</sub> (b)	X <sub>2</sub> (b)	·	·
Q <sub>c</sub>	X <sub>1</sub> (c)	X <sub>2</sub> (c)	·	·
Q <sub>d</sub>	X <sub>1</sub> (d)	X <sub>2</sub> (d)	·	·

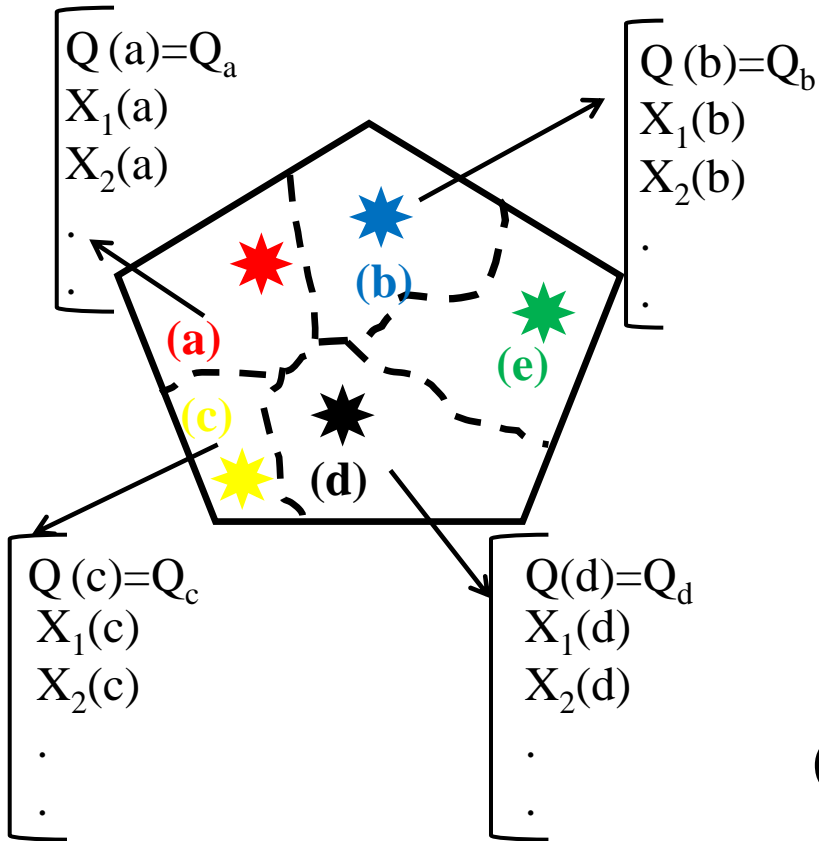
$$Q_T = f[X_1^{b_1}, X_2^{b_2}, \dots]$$

$$X_1(e) \quad X_2(e)$$

$$Q_e$$

# ...MATERIAL & METHODS

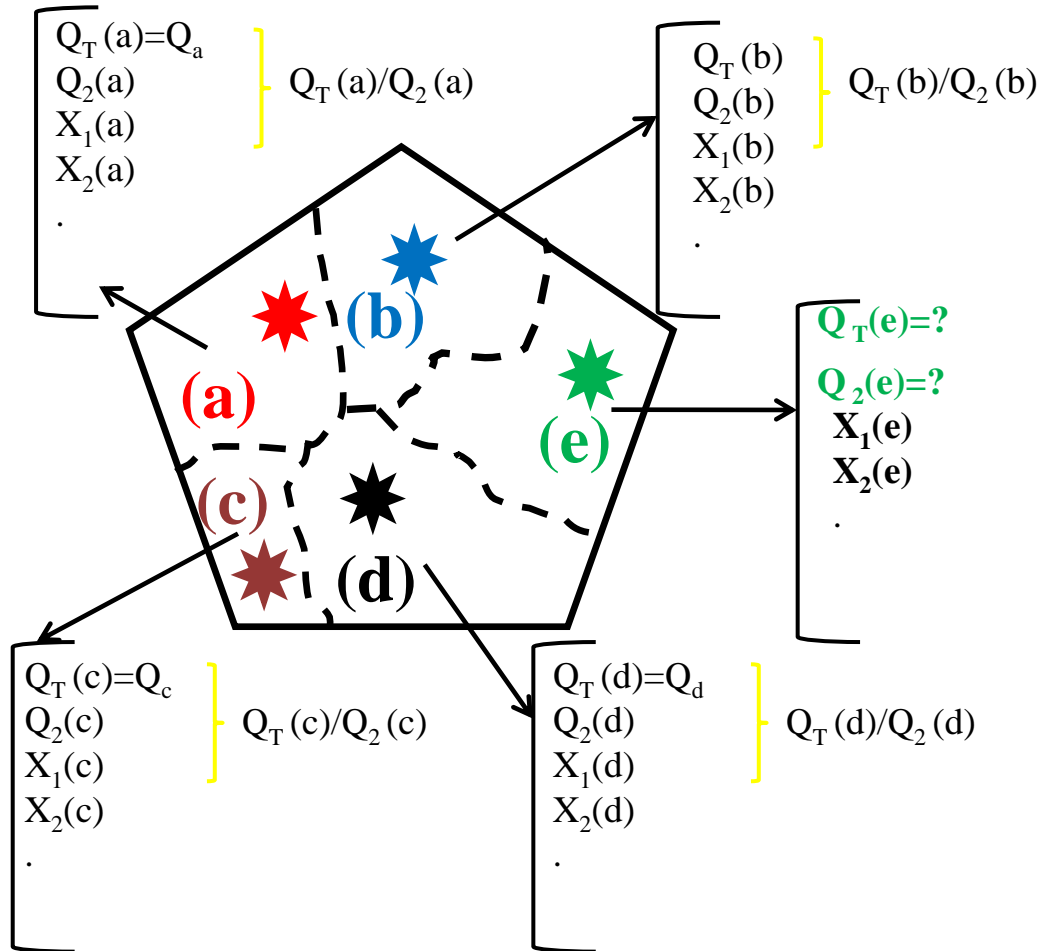
Select the best regression model



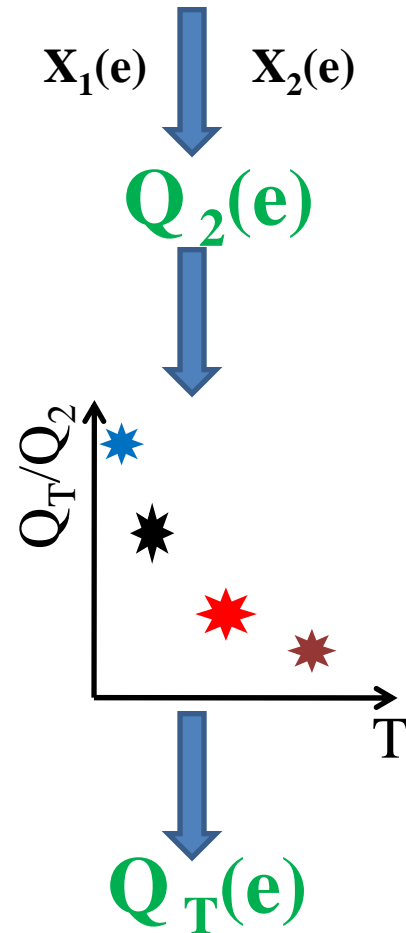
<u>Q</u>	<u>X<sub>1</sub></u>	<u>X<sub>2</sub></u>	<u>X<sub>3</sub></u>	...
Q <sub>a</sub>	X <sub>1</sub> (a)	X <sub>2</sub> (a)	X <sub>3</sub> (a)	·
Q <sub>b</sub>	X <sub>1</sub> (b)	X <sub>2</sub> (b)	X <sub>3</sub> (b)	·
Q <sub>c</sub>	X <sub>1</sub> (c)	X <sub>2</sub> (c)	X <sub>3</sub> (c)	·
Q <sub>d</sub>	X <sub>1</sub> (d)	X <sub>2</sub> (d)	X <sub>3</sub> (d)	·
$Q_1 = f[X_1^{b_1}, X_2^{b_2}, X_3^{b_3}, \dots]$ $Q_2 = f[X_1^{b_1}, X_2^{b_2}, \dots]$ $Q_3 = f[X_1^{b_1}, X_3^{b_2}, \dots]$				
$(e_i = Q_i - \hat{Q}_i) \quad e_i \downarrow$				
$VIF_i > 5 \quad (VIF_j = \frac{1}{1 - R_j^2})$				
$(R_j \text{ is the multiple correlation coefficient})$				
$Q_i$				

# ...MATERIAL & METHODS

## - Low flow index method



$$Q_2 = f[X_1^{b_1}, X_2^{b_2}, \dots]$$

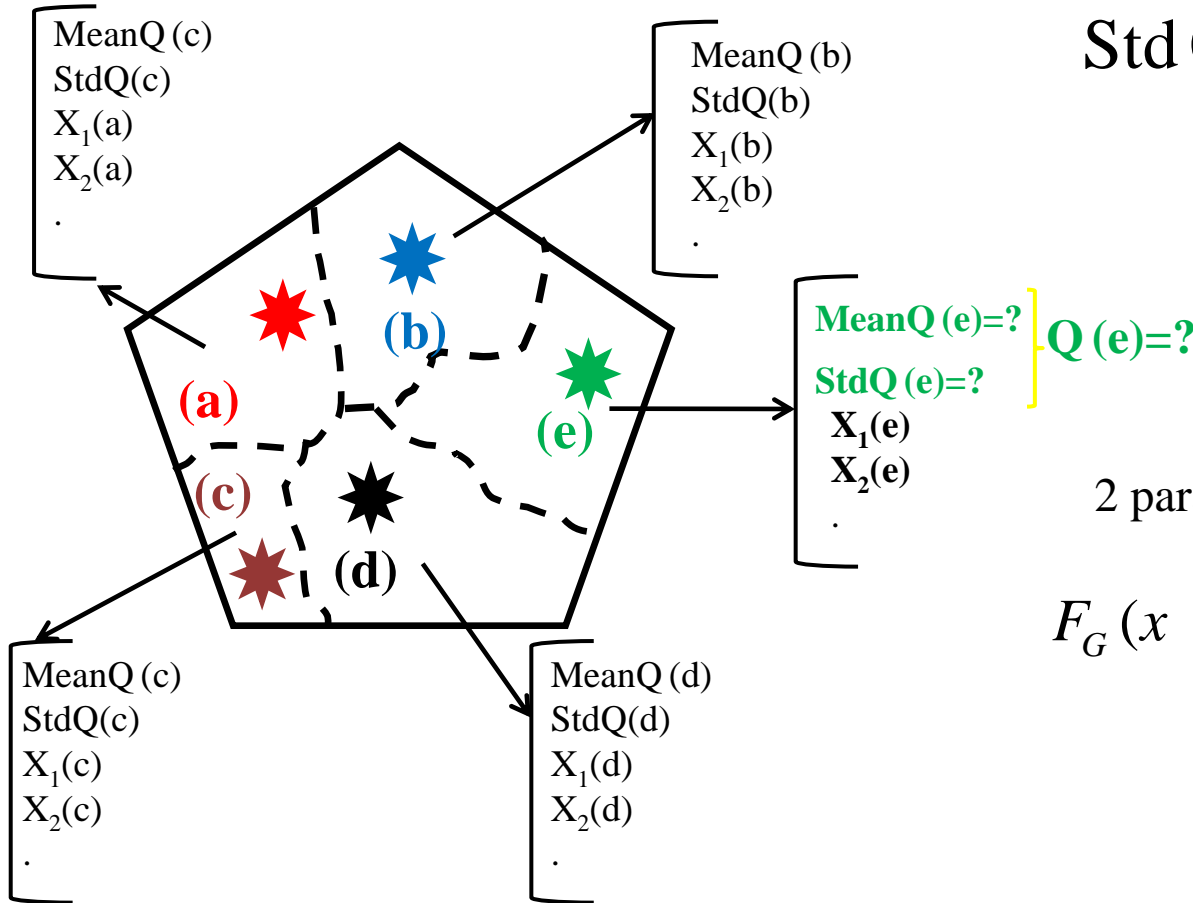


# ...MATERIAL & METHODS

- Regionalizing the frequency formula parameters method

$$\text{Mean Q} = f[X_1^{b_1}, X_2^{b_2}, \dots]$$

$$\text{Std Q} = f[X_1^{b_1}, X_2^{b_2}, \dots]$$



MeanQ (e)

StdQ (e)

2 parameters gamma distribution

$$F_G(x : \alpha, \beta) = \int_0^{\infty} \frac{\alpha^\beta t^{\beta-1} e^{-\alpha t}}{t \Gamma(\beta)} dt$$

$Q_T(e)$



# ...MATERIAL & METHODS

## *-Hybrid method*

1)The study area divide into some homogeneity classes

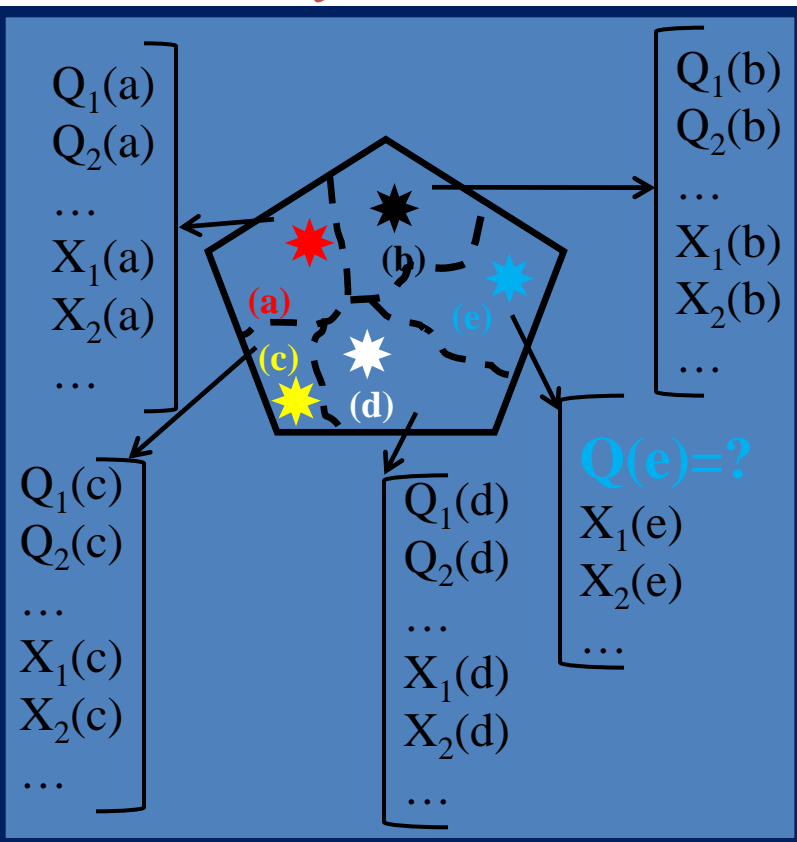
2)The parameters of Hybrid equation have been calculated:

$$Q_T = kX_1^l X_2^m \dots$$

$Q_T$  is discharge with T-year return period,  $X_1$  and  $X_2$  are independent hydrologic parameters and  $k$  is the constant component and  $l$  and  $m, \dots$  are the coefficients of the regression model.

# ...MATERIAL & METHODS

## ...Hybrid method



Grouping base  
 on  $X_1, X_2, \dots$   
 (a,d) and (c,b)

a, d	c, b
-----	-----
$Q_1(a)$	$Q_1(c)$
$Q_2(a)$	$Q_2(c)$
...	...
$Q_1(d)$	$Q_1(b)$
$Q_2(d)$	$Q_2(b)$
...	...
-----	-----
$Q_T(1)$	$Q_T(2)$

Calculate discharge with different return periods

Calculate  $k, l, m, \dots$  for hybrid model:

$$Q_T = kX_1^l X_2^m \dots$$

Calculate  $l, m, \dots$  and  $k$  coefficients

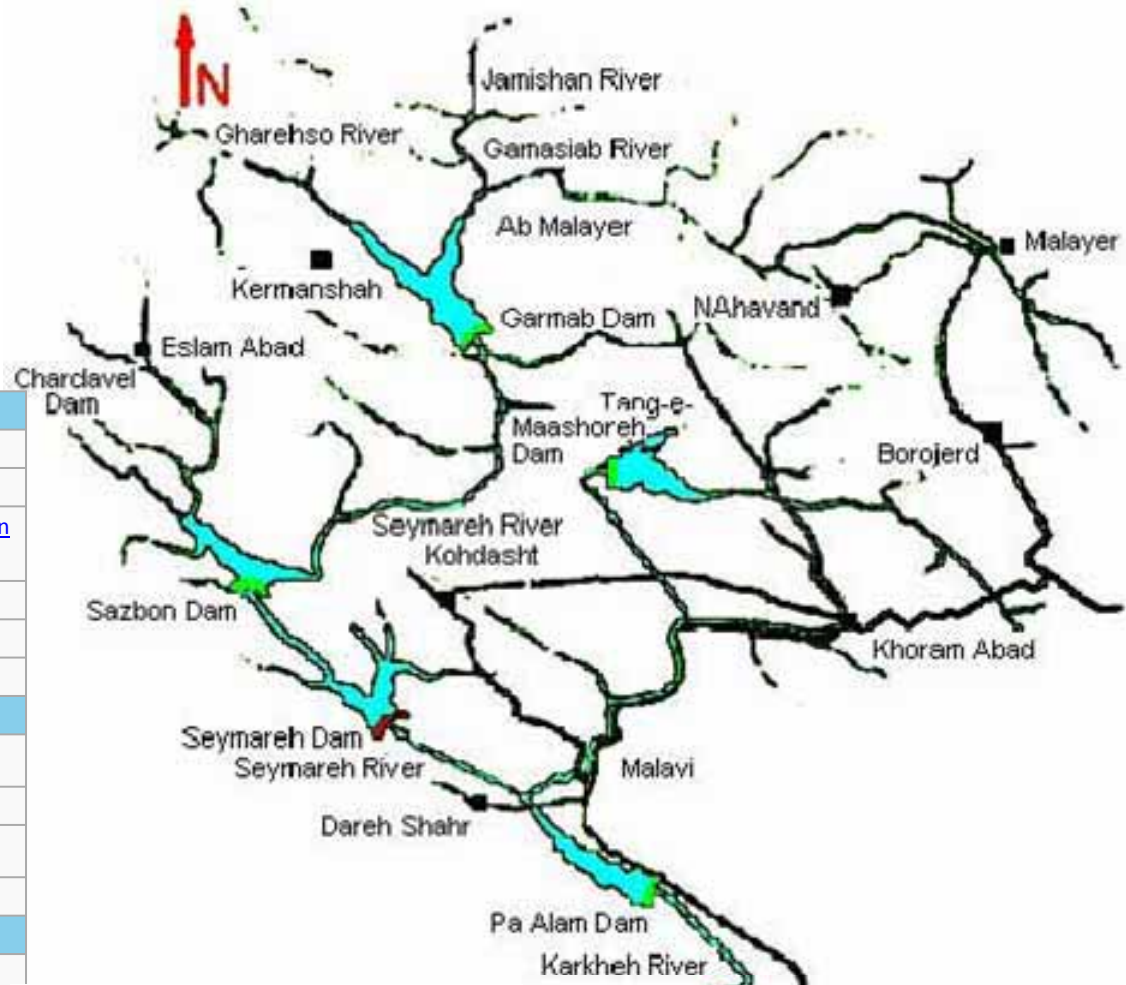
Put  $X_1(e), X_2(e)$  in Hybrid model

$Q_T(e)$

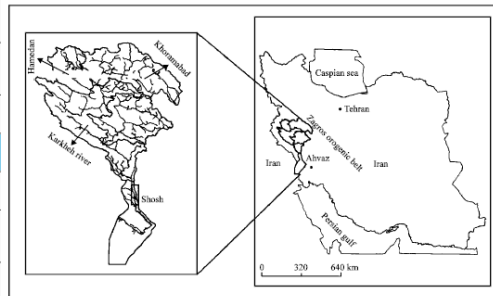
$$l_T = \frac{\sum_{i=1}^f \bar{A}_i Q_{Ti} - \frac{\left[ \sum_{i=1}^f \bar{A}_i \sum_{i=1}^f Q_{Ti} \right]}{f}}{\sum_{i=1}^f \bar{A}_i^2 - \frac{\left[ \sum_{i=1}^f \bar{A}_i \right]^2}{f}}$$

# ...MATERIAL & METHODS

## Study area



Karkheh Dam	
Official name	Karkheh Dam
Location	<a href="#">Khūzestān-Iran</a>
Coordinates	<a href="#">32°29′21″N48°07′36″E</a> Coordinates: <a href="#">32°29′21″N 48°07′36″E</a>
Construction began	1992
Opening date	2001
Construction cost	\$700 million
Dam and spillways	
Height	127m from foundation
Length	3030 m
Base width	1100 m (12m at crest)
Impounds	<a href="#">Karkheh River</a>
Reservoir	
Capacity	5,900,000,000 m <sup>3</sup> (4,800,000 acre-ft)
<a href="#">Catchment area</a>	42,000 km <sup>2</sup> (16,000 sq mi)
Surface area	162 km <sup>2</sup>
Power station	
Turbines	3
Installed capacity	420 MW (3x140 MW) <sup>[1]</sup>
Annual generation	934 GWh



Karkheh Dam



# RESULTS AND DISCUSSION

- *Low flow models*

- Multivariate regression*

- 1) 7-day low flows with different return periods (5, 10, 20, 25, 50 and 100 years) used

- 2) The most important physiographic characteristics that are effective on the low flow estimates have been selected.

## ...RESULTS AND DISCUSSION

3) Estimated regression models are as follows:

$$Q_5 = 10^{-0.44BR - 5.01 \times 10^{-4} E + 1.38 \times 10^{-4} A + 0.54}$$

$$Q_{10} = 10^{-0.46BR - 5.44 \times 10^{-4} E + 1.41 \times 10^{-4} A + 0.45}$$

$$Q_{20} = 10^{-0.48BR - 5.94 \times 10^{-4} E + 1.411 \times 10^{-4} A + 0.39}$$

$$Q_{25} = 2.54 \times 10^{-4} A + 0.12S - 1.77$$

$$Q_{50} = 2.21 \times 10^{-4} A + 0.11S - 1.59$$

$$Q_{100} = 1.95 \times 10^{-4} A + 0.10S - 1.44$$

*A: Area (km<sup>2</sup>), S: Slope (%), BR: Bifurcation Ratio (the ratio of the number of streams of any given order to the number of streams in next higher order (Schumn, 1956)), E: Elevation (m) and Q<sub>T</sub> 7-day low flow with different return periods for a specific basin*

# ...RESULTS AND DISCUSSION

## - *Low flow index method*

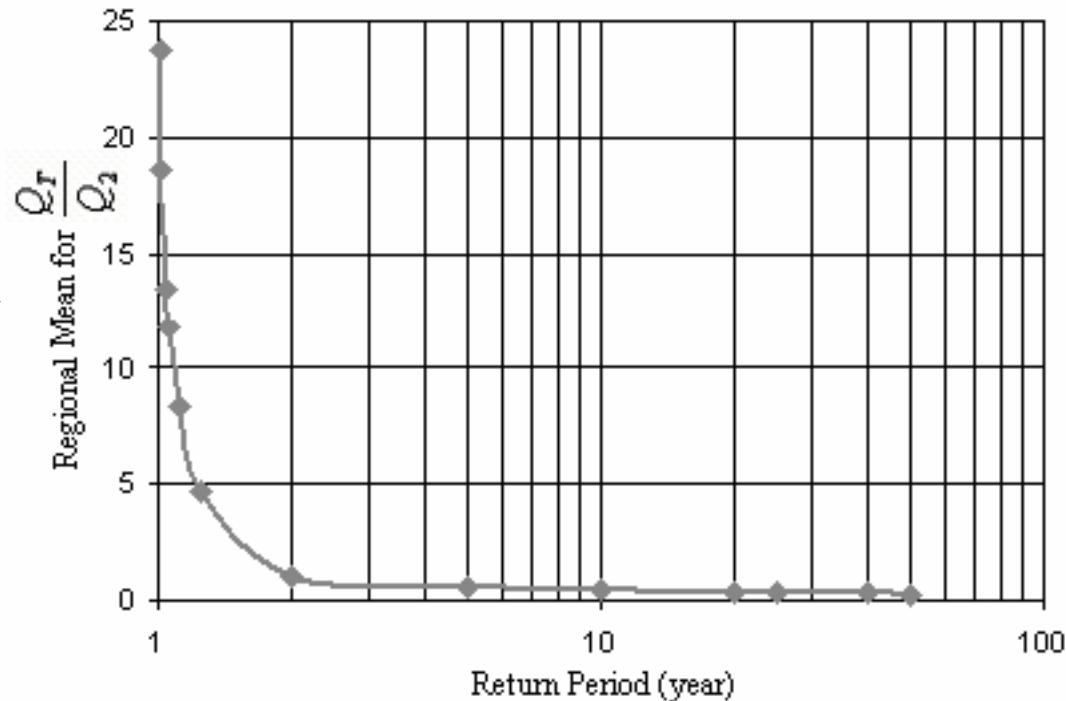
$$1) \quad Q_2 = 10^{-0.41BR - 3.65 \times 10^{-1} E + 1.38 \times 10^{-4} A + 0.63}$$

*A: Area (km<sup>2</sup>), BR: Bifurcation Ratio, E: Elevation (m) and Q<sub>2</sub> 7-day low flow with 2-year return period*

2) regional frequency curve



3) calculate Q<sub>T</sub> for a specific basin



# ...RESULTS AND DISCUSSION

## *- Regionalizing the frequency formula parameters method*

1) The relation between average and standard deviation of 7-day low flows and characteristics of the basin are:

$$\text{Mean}(Q) = 10^{1.44 \times 10^{-4} A - 2.68 \text{Log} BR - 2.8 \times 10^{-4} E + 0.59}$$

$$\text{Std}(Q) = 10^{6.15 \times 10^{-3} \text{MsL} - 0.26 \text{Log} E - 0.20 BR + 4.62 \times 10^{-2}}$$

*Mean(Q) is the average of 7-day low flows (m<sup>3</sup>/s), A is the area of basin (km<sup>2</sup>), BR is bifurcation ratio, E is basin elevation (m), MsL is the mainstream length (km) and Std(Q) is the standard deviation of 7-day low flows series.*

2) Put Mean(Q) and Std(Q) in cumulative distribution function of 2 parameters gamma distribution for calculating 7-day low flow values with different return periods in a specific basin.

## ...RESULTS AND DISCUSSION

### - *Hybrid method*

The regional models for estimating low flows by this method are:

$$Q_5 = 4.87 A^{-0.16}$$

$$Q_{10} = 14.10 A^{-0.03} S^{-0.81}$$

$$Q_{20} = 2.35 A^{-0.01} S^{-0.27}$$

$$Q_{25} = 1.77 A^{-0.01} S^{-0.20}$$

$$Q_{50} = 1.03 A^{-0.00} S^{-0.06}$$

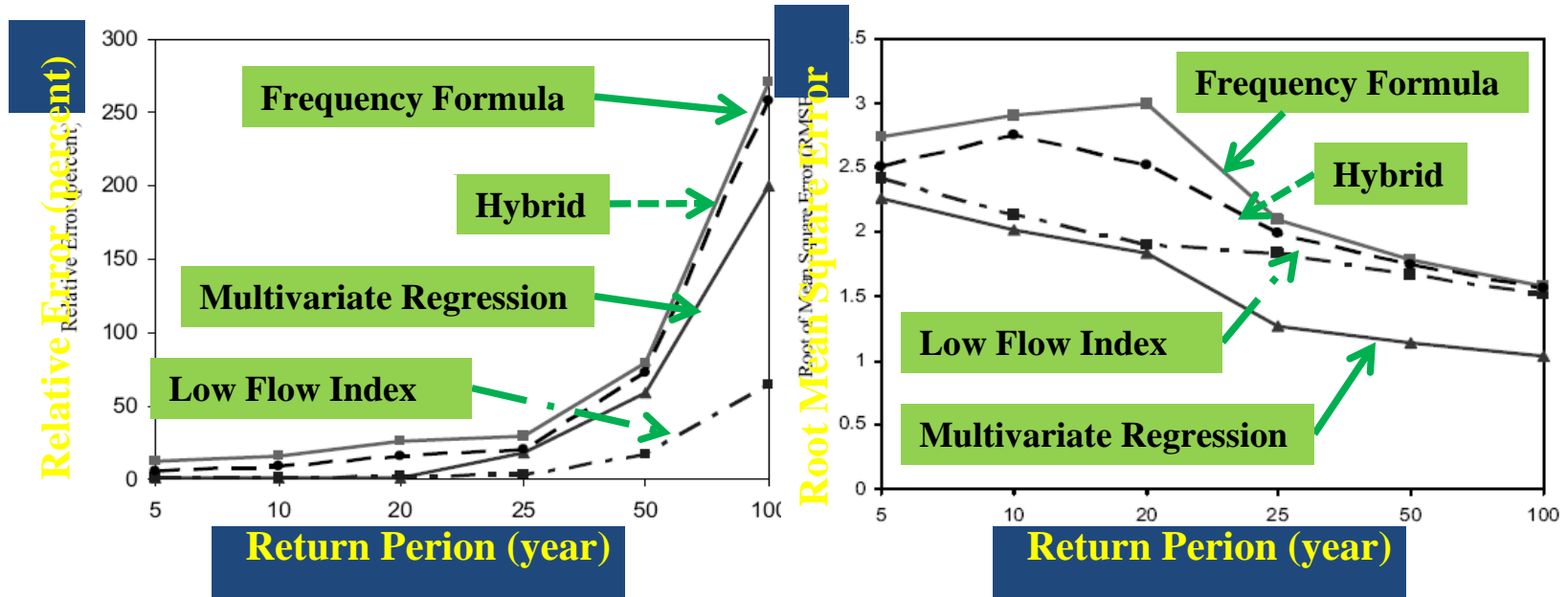
$$Q_{100} = 0.78 A^{-0.00} S^{-0.02}$$

*A: Area (km<sup>2</sup>), S: Slope (%)*



# ...RESULTS AND DISCUSSION

- *Validation step*



Relative Error (percent) and Root of Mean Square Error (RMSE) for four methods

➤ For all of return periods, the multivariate regression and low flow index methods have more accuracy in comparison with regionalizing the frequency formula parameters and Hybrid methods.

# CONCLUSIONS

- Multivariate regression and low flow index methods are more suitable than Hybrid model.
- Using climatic data (for example Precipitation, Temperature and ..., that are important in low flows) instead of physiographic data (Area and Slope) in Hybrid method, maybe have more accuracy for applying this model (it can consider in future researches)

# THANKS FOR YOUR ATTENTION



Karkheh Dam