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An empirical evaluation of the EOQ model of choice of shipment size in freight transport

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Abstract

Deciding shipment size is important in freight transport: it depends on the logistical imperatives of shippers and the technical possibilities of carriers. Shipment size choice is also closely related to transportation mode; it is therefore important from a public policy perspective.

The theory of optimal shipment size and mode choice is robust. There are many inventory-theoretical models of optimal shipment size, applied by shippers in operational contexts. However, none of them has been validated empirically over a large and heterogeneous population of shipments, so that they are virtually useless for modeling freight transportation demand. This is due in particular to the lack of adequate data.

In this paper, the simple Economic Order Quantity (EOQ) model is assessed empirically on a national scale, over a heterogeneous population of shipments. Using the French ECHO database, which notably observes commodity flow rates between shippers and receivers, the EOQ shipment size specification is estimated. The validity of the EOQ model is confirmed. In addition, the dominant role of the commodity flow rate between the shipper and the receiver, and of commodity value density is revealed. The relationship between mode choice and shipment size is also highlighted.

Keywords

Optimal shipment size; freight transportation economics; inventory theory; empirical analysis; EOQ model

1. INTRODUCTION

Freight transportation demand models are largely inspired by passenger transportation demand models (*1*). Their architecture is generally based closely on the classic four stages of passenger transportation demand modeling (*2*) i.e. generation, distribution, mode choice, and assignment. In particular, classical passenger mode choice models are generally discrete choice models, where the value of a mode alternative to a passenger is represented by additive random utility functions (*3*; *4*; *5*). These utility functions consist of variables associated to the individuals and of variables associated to the transport alternatives (typically their rates, travel time, and reliability). Freight mode choice models generally follow the same principles.

These models are relatively efficient empirically, but they have shortcomings. Probably one of the most significant is the seemingly total neglect of the role of logistics. This is an issue for at least three reasons. First because it suggests a theoretical flaw: for example, it is impossible to establish a formal relationship between the preferences of a shipper regarding freight transport and its own logistical environment (customer preferences, position in the supply chain, etc.) Second, many transport policy instruments fall outside the scope of these models (warehouse location policies, taxes on warehousing, etc.) Third, an important variable lacks: shipment size.

Shipment size plays an important role in freight transportation. For shippers, sending ten shipments of a thousand tons a year is entirely different from sending ten thousands one-ton shipments. For carriers, different shipment sizes mean different vehicles and organizations. This applies especially to the choice of transportation mode.

Theoretical models of optimal shipment size and mode choice have been around for a long time. They are most often based on the century-old Economic Order Quantity model. Initially designed to optimize production in certain contexts, it applies easily to the context of freight transport. However, its validity for a large and heterogeneous population of shippers has never yet been assessed.

This is mainly due to the lack of an adequate database. Indeed, an important explanatory variable in the EOQ model is the commodity flow rate between the shipper and the receiver. This variable is not usually recorded. The French shipment database ECHO is one exception. This paper uses that database in order to reach an empirical assessment of the Economic Order Quantity model for shipment size choice in freight transport.

The discussion proceeds as follows. First, Section 2 presents a review of the literature mainly focused on the choice of shipment size in freight transport. Section 3 recalls the theoretical underpinning and formulas of the EOQ model. Section 4 then presents the ECHO database and the estimation methodology. Section 5 discusses the results of that estimation. Section 6 presents the estimation of an extended EOQ model, which gives additional insight into the structure of transportation costs and their impact on the choice of shipment size. The paper is concluded in Section 6.

2. LITERATURE REVIEW

Freight mode choice models are generally based on microeconomic, often disaggregate models of the interaction of shippers and carriers. Initial investigations on freight mode choice were focused on direct transport costs (*6*; *7*). Later studies concluded that indirect costs played an important role (*8*; *9*; and also *10*). Some models were based on total cost minimization and linear programming (*11*). With the parallel development of discrete choice modeling techniques (*12*), this led to a now dominant model architecture where each modal alternative is represented by a utility (or generalized cost) and the choice between alternatives by a probabilistic model, such as the multinomial logit and its refinements (*13*, *14*; for a comprehensive review of recent freight models, refer to *15*; for a review of some recent discrete choice model specifications in freight transport, refer to *16*). All these models have in common that shipment size is absent.

The importance of shipment size was identified long ago. It was soon found that shipment size is an empirically important explanatory variable in mode choice (*17*; *18*). (*19*) designed and estimated a behavioral model where the transportation mode is assumed to be chosen by shippers (or receivers) for given, exogenous shipment sizes. Another example is (*20*), where the authors estimated a disaggregate model of mode choice using shipment characteristics as explanatory variables, on the basis of the 1988 French shipment database *Enquête Chargeurs*.

However, it was soon understood that shipment size depends as much on mode choice as mode choice depends on shipment size. This was confirmed empirically using joint estimation techniques (*21*). In this study, a switching regression model was estimated. The shipment sizes conditional to each mode were specified as linear combinations of exogenous parameters. Then, a latent variable, specified as a linear combination of these parameters and of the two conditional shipment sizes, was used itself to predict the transportation mode, on the basis of its sign. A similar approach, yielding similar qualitative results, is presented in (*22*). In (*23*), shipment size was modeled as a discrete variable, so that the discrete choice modeling toolbox could be applied straightforwardly. The interdependency of mode and shipment size was confirmed again. All those approaches led to the same empirical conclusion. Nevertheless, they gave limited insight on its underlying microeconomic drivers.

Theoretical models of optimal shipment size are more than a century old. They are part of inventory theory. The first one is the EOQ – Economic Order Quantity – model (*24*; *25*). It was initially derived to provide the optimal batch size for a production chain, but it applies straightforwardly to freight transport. According to this model (described in more detail in the next section) the optimal shipment size is the result of a trade-off between transportation costs and inventory costs. It can be extended to take mode choice into account as well, and even other variables. This is the approach chosen in the very important paper (*26*), which derived a theoretical, disaggregate model of a combined decision on transportation mode, shipment size, and safety inventory at destination.

This paper was followed by a series of contributions, of which we will mention a small, significant sample. The influence of the shape of freight rates on choice of shipment size and transportation mode was investigated in (*27*). The model introduced in (*26*) was simplified to represent the choice of shipment size and type of transportation operation by road, between parcel, less-than-truckload and truckload shipments (*28*). A dynamic inventory-theoretical model is derived and estimated in (*29*). However, the econometric specification was only loosely based on the theoretical developments from which it purportedly derives. In (*30*), the EOQ model formed the basis of a theoretical model of optimal ship size. A significant current goal of research in freight transportation modeling is to build a fully-fledged freight transportation demand model with an explicit representation of shipment size. Such an attempt is described in (*31*). Finally, the overall status of inventory theory in freight transportation modeling is discussed in (*32*; *33*; *34*).

Despite their theoretical appeal, these models all lack solid empirical validation. This is mainly due to the absence of adequate data. For example, to estimate the EOQ model, one needs to observe shipment sizes as well as shipper-receiver commodity flow rates. While there are databases where shipment size is available – such as the US or Swedish Commodity Flow Surveys (*35*; *36*), the commodity flow rate between shipper and receiver is overwhelmingly ignored. Some authors have been able to access relevant datasets, but these are restricted to very small samples (no more than two firms). In such cases, estimating an inventory-theoretical model may determine whether a particular logistical doctrine holds for those firms, but this does not constitute evidence that it is valid at an aggregate scale (*4*; *37*; *38*). Experimental economics can partially address this issue, but not as satisfyingly as real world data (*39*). Attempts to replace the shipper-receiver commodity flow with proxies in a large-scale spatialized model have encountered difficulties (*31*). In consequence, no functional spatialized freight transportation model currently includes shipment size as an endogenous variable (*40*).

An adequate disaggregate shipment database is required to assess the empirical validity of the EOQ model. In particular, it should describe not only the characteristics of shipments, and of the way they are transported, but also the logistical contexts in which these transportation operations take place. One database provides such information: the French shipper survey ECHO (*Enquête CHargeurs et Opérateurs de transport*, shippers and transport operators survey), (*41*; *42*). This is the database used in this paper.

3. THE ECONOMIC ORDER QUANTITY MODEL

The recent discussions on freight transportation modeling insist that supply and demand should be clearly distinguished (*43*; *44*). In this paper, we follow (*41*), where a shipment is defined as a "quantity of freight that is made available at a given time in order to be transported during a single transportation operation from a given shipper to a given consignee", and consider it is as the relevant measurement unit with regard to the economics of freight transportation.

The main elements of the Economic Order Quantity model are now briefly recalled. Consider a firm sending a regular commodity flow of constant rate Q, from a given location to another by a given transportation mode. Freight transportation operations being discrete by nature, commodities are carried in shipments. We assume that all the shipments are of the same size s, and that each shipment is dispatched as soon as there are enough commodities at the origin. Then, the average origin stock level is $s/2$. It is the same at destination, provided that the commodity is sold or consumed at the rate it is produced. The average total inventory level is then s.

The shipper decides shipment size on the basis of all its costs. This concerns the freight rates paid to carriers, and the potential effort needed to carry out the transport operation, order costs, etc. These are assumed to consist of a fixed cost b independent of shipment size, and a variable cost $K.s$ proportional to shipment size. Over a time period, Q/s shipments are sent by the shipper. Then, the total freight rates per time period amount to:

$$KQ + \frac{Qb}{s}. \qquad (1)$$

The willingness of the shipper to pay for a reduction in inventory is assumed proportional to the inventory level and to the time that elapses between the moment a commodity unit is produced and the moment it is sold, up to a coefficient a. The travel time is denoted by t. The total inventory is on average equal to $s + Qt$, so that the inventory cost per time period is $a(s+Qt)$.

The total logistical cost function, denoted by g, is the sum of these two components:

$$g(s) = as + (at + K)Q + \frac{Qb}{s}. \qquad (2)$$

The optimal shipment size is obtained by minimizing this convex cost function; it is the unique root of its first derivative:

$$g'(s^*) = a - \frac{Qb}{(s^*)^2}. \qquad (3)$$

It depends only on a, Q, and b, and has the familiar "square-root" shape:

$$s^* = \sqrt{\frac{Qb}{a}}. \qquad (4)$$

s^* does not depend on costs that are proportional to shipment size, such as the pipeline inventory cost (atQ) or the proportional component of the transportation cost (K). However, the total logistical cost does depend on such costs, and this has a direct effect and substantial effect on mode choice.

This model can easily be estimated by linear regression. Indeed, by taking the logarithm of both sides of Equation (4), we obtain:

$$\ln s^* = \frac{1}{2}\ln Q + \frac{1}{2}\ln b - \frac{1}{2}\ln a \qquad (5)$$

This equation is valid for each transport mode. As explained in the next section, Q and a are observed, and $(1/2)\ln b$ will be estimated as a mode-specific constant. From now on the optimal shipment size will simply be denoted by s, instead of s^*.

4. DATA PRESENTATION AND MODEL SPECIFICATION

The ECHO database describes 10,462 shipments sent by some 3,000 shippers. These shipments are of all natures, sizes, origins and destinations (either the origin or destination is in France). All transportation modes are represented. Heavy modes, where the relative number of shipments is very low, were oversampled. There are much less observations than in other shipment databases, but each shipment is described with a large quantity of detail, which regards: the shipping and receiving firms; their relationships; the shipment itself; the transportation operation. In each of these categories, variables relating to economic, logistical or transport-related factors are available.

The equation that will be estimated is closely inspired by Equation (5). $\ln s$ is the dependent variable. Note that shipment size is measured in both weight and in volume in the ECHO database. The weight measurement is preferred in this paper; mainly because the volume measurement is much less accurate, insofar as it is given in m^3, without decimals. Summary statistics of s and $\ln s$ can be found in Table 1.

In Equation (5), the explanatory variables are Q, a and b. However, they are not observed directly in the ECHO database. Let us now examine how the variables available in the ECHO database can be used instead. For that purpose, variables relating to transportation demand and to transportation supply are addressed separately.

4.1. Transport demand related variables

Strictly speaking, the rate Q of the commodity flow between the shipper and the receiver is not available in the ECHO database. The closest available variable is the total commodity flow denoted by Q_{tot}, without consideration of commodity type. This variable is available for 81.5% of the observations. Using Q_{tot} instead of Q without care can lead to an underestimation of the influence of Q on s.

Nevertheless, there is a possibility that this bias may be limited. Indeed, in the ECHO survey, the shippers and receivers are identified at the level of facilities, or premises, i.e. physically well-defined components of firms. In addition, if a given facility is the origin (or destination) of a large number of distinct commodity flows, it is probable that these flows are being sent to (or coming from) distinct locations. In any case, it is probable that Q and Q_{tot} often coincide, although no quantitative evidence can be drawn from the ECHO survey to support this statement. From now on Q_{tot} will be used as a proxy for Q.

The commodity value of time a is not directly available either. Fortunately, in this case, there is a good candidate to replace it. Indeed, for 64.5 % of shipments, the market value in euros (before tax) is specified. By combining this with the shipment weight, it is possible to calculate the value density of these shipments. The value density is denoted by a_{dens}, and is measured in euro per kilogram.

Using a_{dens} instead of a is a strong hypothesis. The commodity value of time, considered from the shipper's perspective, is certainly closely related to the value density. However, it is also expected to depend on many other parameters, such as the commodity's depreciation, the opportunity cost of capital for the shipper, the organization of the supply chain between the shipper and the receiver, and so on. Assume this can be represented by an interest rate applied to the value of the commodity. Then, due to the logarithmic specification of the model, this interest rate goes in the intercept of the model, and its variations in the residuals. Assuming this rate is not too strongly correlated with other exogenous variables in the model, this should not introduce a bias in the outcome of the estimation. Within the framework of this study, this assumption is made, and a_{dens} is used as a proxy for a. The summary statistics of Q_{tot}, a_{dens}, $\ln Q_{tot}$ and $\ln a_{dens}$ are all shown in Table 1 (where Q1 and Q3 refer to the first and third quartiles).

[TABLE 1]

4.2. Transport supply related variables

The vast quantity of variables relating to transportation operations can be classified into two groups: technical variables, and freight rates. Technical variables describe the transportation technique and the availability and compatibility of other techniques. For example, live animals and chemicals cannot be carried in the same kind of trucks or rail cars. The technical variable used in the estimation is the main mode, that is to say the mode of the longest leg in the transportation operation (Table 2).

[TABLE 2]

Freight rates play a central role in the choice of shipment size. They are available in the ECHO database. However, the choice of shipment size actually depends on the generalized transportation cost from the perspective of shippers. The relation between freight rates and generalized transportation costs may be far from trivial. As a consequence, it has been decided not to use freight related variables in this study.. A simpler approach is preferred: on the assumption that b does not depend on origin-destination distance (given that b represents such costs as maneuvering, loading and unloading, and administrative operations that have no clear link with distance), it is sufficient to consider it as a mode-specific constant (note that b can be assumed to depend on other variables, such as the number of legs in the transport operation and others; this is done in Section 6). Subsequently, the following specification is estimated:

$$\ln s = \beta_Q \ln Q_{tot} + \beta_a a_{dens} + \sum_{mode} \beta_{mode} X_{mode} + u \,. \tag{6}$$

where u is the error term and $\{X_{mode}\}_{mode}$ are dummy variables indicating the transportation mode used.

5. ESTIMATION OF THE EOQ MODEL

The model is estimated using ordinary least square regression. The results are given in Table 3. The coefficients are highly significant. The R^2 coefficient is close to 0.8. Additional analyses of the residuals, available from the author or in the appendices of (*45*), do not invalidate the ordinary least square specification. Overall, specification (6) seems adequate. The estimated model is:

$$\begin{aligned} \ln s = {} & 0.50 \ln Q_{tot} - 0.44 \ln a_{dens} + 1.05 X_{commoncarrier} \\ & + 1.46 X_{privatecarrier} + 3.42 X_{rail} + 2.09 X_{combined} \\ & + 4.37 X_{waterway} + 2.89 X_{sea} + 1.47 X_{air} \end{aligned} \tag{7}$$

β_Q is close to 0.5, and β_a relatively close to -0.5, as predicted by the theory. The basic EOQ model proves empirically effective in explaining the choice of shipment size by a large and heterogeneous population of shippers. It explains about 80% of the variance in the shipment size for which the explanatory variables are available (i.e. about 55% of the observations). The high importance of Q_{tot} as an explanatory variable of shipment size is also confirmed. This tends to indicate that Q_{tot} is an important explanatory determinant of mode choice in freight transport.

The absolute values of the mode-specific constants provide little information in themselves. However, their rankings are interesting. On average, the ranking of the mode-specific constants should be the same as the shipment-independent component of generalized transportation costs b for each mode. Indeed, as shown in Table 3, the higher the mode capacity, the larger the mode-specific constant. It confirms the fact that the lower the unit transportation cost, the larger the fixed per-shipment cost, and the hierarchy which goes from waterway to rail, sea, combined transport, and then road and air transport, is intuitively correct. The apparently counter-intuitive gap between common carrier and private carrier is explained in the next section.

[TABLE 3]

6. EXPLORATORY ESTIMATION OF AN EXTENDED EOQ MODEL

Other variables may be expected to influence shipment size. Some of them are available in the ECHO database, and their influence can be assessed by a small additional effort. In this section, the respective roles of the following variables are assessed: the origin-destination distance variable d, the number of agents physically or administratively intervening in the transportation operation N_{interv}, the number of legs in the transportation operation N_{trips}, and the organization of the transportation operation O (three possible organizations are distinguished: "isolated shipment", when the shipment is carried alone; "bundle", when the shipment is carried with other shipments; "part of a round", when the vehicle delivering or picking the shipment up reaches several distinct locations during the same movement). The latter two variables, when available, only concern the part of the transportation operation which has been carried out in the UE15 region. Summary statistics on these variables are given in Table 4. Variable O is summarized in Table 5.

[TABLE 4]

[TABLE 5]

In order to keep the estimation as simple as possible, and since there is no convincing microeconomic model relating b to these variables, a simple specification without any interactions is estimated:

$$\begin{aligned} \ln s = & \beta_Q \ln Q_{tot} - \beta_a \ln a_{dens} + \sum_{mode} \beta_{mode} X_{mode} \\ & + \beta_d \ln d + \beta_{interv} N_{interv} + \beta_{trips} N_{trips} \\ & + \beta_{bundle} X^O_{bundle} + \beta_{round} X^O_{round} + u \end{aligned} \qquad (8)$$

where X^O_i equals 1 if $O = i$, or otherwise 0; and u is the error term. Note that $\beta_{isolated}$, the parameter corresponding to the case of isolated shipments, is set to zero by convention.

The model is once again estimated using ordinary least square regression. The results of the estimation are given in Table 6. A basic analysis of the residuals, available from the author or in (45), does not invalidate the ordinary least square hypotheses.

[TABLE 6]

All the coefficients are significant. Furthermore, as confirmed by the analysis of variance in Table 7, all the additional variables significantly improve the model. However, it remains true that the explanatory power of the model comes predominantly from $\ln Q_{tot}$ and $\ln a$, i.e. the core of the EOQ model.

The estimated coefficients of $\ln Q_{tot}$ and $\ln a$ remain reasonably close to (although not exactly the same as) the theoretical values. The hierarchy of the main transportation modes does not change substantially. It may be noted that the mode specific constants associated with private carriers and common carriers are not significantly different contrary to the previous model. The difference was due to the number of trips: the average number of trips when the main transportation mode is private carrier is 1.02, whereas it is 2.12 when the main transportation mode is common carrier.

Let us now briefly interpret the results obtained for the new variables. N_{interv} has the expected positive effect: a larger number of agents intervening on the transportation chain seems to imply a larger $\ln b$, thus a larger shipment size.

What might be perceived as counter-intuitive is the negative sign of β_{trips}. In fact, it confirms the economic rationale of hub-and-spokes transport networks: such networks are especially designed to handle small shipments efficiently. As a consequence, the transshipment cost is smaller in a hub-and-spokes transportation network than in more direct transportation organizations. Note that this is not in contradiction with the assessment that β_{interv} is positive; these two variables are scarcely related (their correlation is only 0.26).

[TABLE 7]

The influence of the transportation organization variable O on the shipment size is intuitive: shipments sent in bundles and in routes certainly share some fixed transshipment and handling costs, hence the negative signs in β_{bundle} and β_{round}.

The positive influence of $\ln d$ on shipment size is perhaps the most difficult to understand. Normally, the distance should not influence the shipment size (subject to the use of a given mode). Here are three possible causes. The first is a frontier effect: shipments arriving from or going abroad entail more administrative work, which positively influences transportation cost per shipment. Second, unused vehicle capacity is more expensive to carriers on longer distances. It is possible that, through the freight rates, and particularly through surcharges and discounts, carriers induce shippers to send bigger shipments (*46*). Third, it may arise from the structure of logistics networks: if large flows travel from plants to regional distribution centers where they are then stocked for subsequent dispatch, there is a sort of logistical disconnect between the two operations (shipments are not targeted to a given retail center when they leave the plant, but only when they leave the regional distribution center), which results in the observation that bigger shipments move over longer distances.

7. CONCLUSION

In this paper, the Economic Order Quantity (EOQ) for shipment size choice in freight transportation is estimated using the French disaggregate shipment database ECHO. After a brief recap of the theoretical EOQ model, it is estimated using ordinary least squares. The estimation performs correctly, as about 80% of the variance in the sample is explained by the EOQ model. The theoretical model itself is clearly confirmed empirically.

From a microeconomic perspective, a number of conclusions can be drawn. First, inventory theory models, although (and perhaps because) they have been designed to optimize daily decision making for firms in well-defined logistical contexts, also prove valuable from the aggregate perspective of freight travel demand modeling. They offer theoretical insights into the behavior of shippers. In addition, it is reassuring to observe that a very simple model performs well at the aggregate scale of a large population of shippers carrying all kinds of commodities by all kinds of transportation modes. The intuition in (*26*) was relevant.

Second, shipment size is clearly dependent on transportation mode, and freight mode choice depends on shipment size as well. This paper brings an additional confirmation to the previous studies according to which shipment size choice and freight mode choice are simultaneous decisions.

Third, the significant role of a seemingly unimportant variable is highlighted in this paper: the rate of the commodity flow to which the shipment belongs. It plays an important role in determining shipment size and therefore most certainly also in determining mode choice. If this is true (further research is needed to confirm this statement, but it is consistent with the EOQ model, which fits very well in a mode choice framework) then – from the perspective of freight transportation demand – shippers should not only be characterized by their value of time and commodity type, but also by the rate of the commodity flows that they send to receivers.

Fourth, the availability of adequate data is crucial. It is widely agreed that logistics have a very powerful influence on freight transport, and need to be studied in detail. However, to do this, data are required, particularly on the logistical contexts in which transportation operations take place. Fortunately, the ECHO database contains such information, in addition to the characteristics of shipments themselves, and the related transportation operations, which are described with a lot of useful detail. It could be improved so that variables, such as the shipment volume, or the commodity flow between the shipper and the receiver concerning a particular commodity, be available.

This work offers many avenues for future research. The model used to explore shipment size choice could be improved in many ways; one of the most straightforward would be to account for the capacity constraints of vehicles, or to examine more closely how transportation prices, which are observed in the ECHO database, could be used in the estimation. Another natural direction for research would be to design a freight mode choice model, based on the EOQ model along the lines suggested in provided in (*26*) and (*31*). This could lead to the development of a new type of freight transportation demand model, with an explicit (although still partial) representation of logistics. Other decisions, such as the choice of safety stock, could be introduced. This would entail significant efforts of both theoretical and empirical natures, as well as better surveys.

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LIST OF TABLES

TABLE 1 Basic EOQ continuous variables summary statistics

Variable	Min.	Q1	Med.	Mean	Q3	Max	NAs
s (t)	0.00	0.05	0.65	19.58	7.8	10,800	0
Q_{tot} (kt/y)	0.00	1.00	18.00	2.12	350.00	63,000	1,934
a_{dens} (k€/t)	0.00	1.07	4.56	59.37	20.00	10,400	3,715
ln s	-6.91	-3.00	-0.43	-0.65	2.05	9	0
ln Q_{tot}	-6.90	0.18	2.89	3.02	5.86	13	1,934
ln a_{dens}	-0.94	6.97	8.43	8.49	9.90	16	3,715

TABLE 2 Main transportation mode summary statistics

M	Number	%
Private carrier	1727	17
Common carrier	6648	64
Rail	224	2
Combined transport	133	1
Inland waterway	44	0
Sea	825	8
Air	859	8
NAs	2	0

TABLE 3 Estimation of the EOQ model

Coefficients	Estimate	Std. Error	*t*-stat	
β_Q	0.50	0.01	73.27	***
β_a	-0.44	0.01	-37.57	***
$\beta_{common\ carrier}$	1.05	0.11	9.47	***
$\beta_{private\ carrier}$	1.46	0.11	12.87	***
β_{rail}	3.42	0.18	19.30	***
$\beta_{combined}$	2.09	0.20	10.31	***
$\beta_{waterway}$	4.37	0.33	13.05	***
β_{sea}	2.89	0.13	21.49	***
β_{air}	1.47	0.14	10.29	***
N	10,462			
NAs	4,741			
R^2	0.795			
Adjusted R^2	0.795			

Significance levels: '.' at 10%; '*' at 5 %; '**' at 1%; '***' at 0.1 %

TABLE 4 Extended EOQ continuous variables summary statistics

Variable	Min.	Q1	Med.	Mean	Q3	Max	NAs
d (km)	1.00	74.00	278.00	1,253.00	611.00	18,840.00	8
Ln *d*	0.00	4.30	5.63	5.44	6.42	9.84	8
N_{interv}	0.00	1.00	2.00	2.76	3.00	12.00	720
N_{trips}	1.00	2.00	2.00	2.06	3.00	8.00	720

TABLE 5 Shipment organization summary statistics

O	Number	%
Isolated shipment	7647	73
Part of a bundle	989	9
Part of a round	1767	17
NA's	59	1

TABLE 6 Estimation of the extended EOQ model

Coefficients	Estimate	Std. Error	*t*-stat	
β_Q	0.44	0.01	61.94	***
β_a	-0.43	0.01	-37.57	***
$\beta_{common\ carrier}$	0.95	0.12	7.57	***
$\beta_{private\ carrier}$	0.97	0.13	7.26	***
β_{rail}	2.78	0.19	14.56	***
$\beta_{combined}$	1.60	0.23	7.04	***
$\beta_{waterway}$	3.91	0.38	10.42	***
β_{sea}	1.59	0.19	8.57	***
β_{air}	0.34	0.18	1.85	.
β_d	0.21	0.01	14.60	***
β_{interv}	0.16	0.02	8.92	***
β_{trips}	-0.40	0.02	-21.12	***
β_{bundle}	-0.62	0.07	-9.38	***
β_{round}	-0.69	0.06	-12.18	***
N	10,462			
NAs	5,134			
R^2	0.827			
Adjusted R^2	0.827			

Significance levels: ‘.’ at 10%; ‘*’ at 5 %; ‘**’ at 1%; ‘***’ at 0.1 %

TABLE 7 Analysis of variance in the extended EOQ model

Variable	Df	Sum Sq.	Mean Sq.	F-value	
$\ln Q_{tot}$	1	13,263.7	13,263.7	7452.8	***
$\ln a_{dens}$	1	28,915.7	28,915.7	16,247.5	***
M	7	1,563.8	223.4	125.5	***
$\ln d$	1	345.7	345.7	194.3	***
N_{interv}	1	95.8	95.8	53.8	***
N_{trips}	1	788.6	788.6	443.1	***
O	2	372.3	372.3	104.6	***
Residuals	5,314	9,457.3	1.8		