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Hydrographs’ attenuation in sewers. Effects on stormwater source control regulation

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ABSTRACT
In this paper we propose an empirical approach to quantify the hydrographs’ attenuation in sewers. The purpose is to evaluate attenuation’s consequences for the scale-transfer between the parcel and the catchment in urban settings, and in particular for stormwater source control (SC) regulation. In fact, SC regulation practices often consider this scale-transfer by a linear approach. We draw a formulation for the attenuation of peaks in partially full pipes, and we assess, by a sensitivity analysis, the respective weights of the different pipes’ and hydrographs’ characteristics on attenuation. We found that the driving factors of the peak attenuation vary according to the distance from the outlet, and we draw possible guidelines for SC regulation.

KEYWORDS
Attenuation; BMP; peak flow-rate; source control; stormwater sewer; urban runoff

INTRODUCTION
In urban hydrology, a scale-gap exists between the parcel-scale (~10³ m²), where runoff is produced, and the city-scale (~10 km² = 10⁷ m²), where global hydrological effects are observed. As an example, total imperviousness area of the catchment (TIA, a city-scale data) has been considered for years as a driver of urban impact on water bodies (a city-scale effect). Now, many researchers consider that the urban impact is not directly linked to the total imperviousness, but to the effective one (EIA, e.g. Brabec et al., 2002). “Effectiveness” of an area is defined by the connection to the sewer system, which is a parcel-scale characteristic and should be measured at that scale (Lee and Heaney, 2003).

More in general, a scale-transition is necessary when catchment-wide hydrographs characteristics (in this paper, peak flow-rate and volume but also, for example, runoff pollutant load) are to be traced back to elementary runoff production.

The regulation of stormwater source control (SC) is a typical application “making the jump”: SC consists in managing stormwater runoff at the city-scale by using small facilities diffused in all the catchment, typically at the parcel-scale. These parcel-scale facilities are usually called Best Management Practices (BMPs) in the US, or Sustainable Urban Drainage Systems (SUDS) in the UK. Terms like Low Impacts Development (LID) or Water Sensitive Urban Design (WSUD) are used, today, to identify the application of this principle to the design of new developments, which is typical of an intermediate scale (~10⁴-10⁵ m²). (Revitt et al., 2008; Morison and Brown, 2011).
Often, in SC regulation practice, the transfers between scales are supposed to be linear (as described in next section or, for example, by Cantone and Schmidt, 2009). This approach, oversimplifying physical processes, can bring to unsuitable regulations of SC. Consequences will be a lack of efficiency of SC at the city-scale (i.e. more BMPs, and with higher capacities, will be necessary to obtain a same global effect) and, in some cases, even a worsening of the catchment hydrologic behaviour (Goff and Gentry, 2006).

In this paper we analyse the transport process in the sewer system, and in particular hydrographs’ attenuation. The purpose of the analysis is to understand how this physical process intervenes on the scale-transition and how, taking it into account, SC regulation could be improved.

We focus our study on the sewer system, considering that sewers provide the link between scales in most urban settings, particularly those urbanised typically during the XX\textsuperscript{th} century. These networks are in charge of collecting surface runoff at the small-scale and draining it to the catchment outfall. Recent approaches may lead in the future to the disappearance of the stormwater sewer but today most of the stormwater flowpath, in many urban catchments, occurs in the sewers.

In this perspective, after an overview of current practices in SC regulations, we present and validate a formulation of hydrographs attenuation in sewers. We base our formulation on empirical results obtained by Ackers and Harrison (1964) and we evaluate the implications for SC through an analysis of attenuation’s sensitivity to sewer’s and hydrograph’s characteristics.

**Stormwater source control regulation and the linear approach.**
In the last four decades, SC has gained relevance in many countries (e.g. France, UK, USA, Brazil, Australia), mainly for its potential to reduce negative impacts of fast urbanization and imperviousness increase.

In hydrologic literature, increases of runoff volumes and peak flow-rates due to urbanization, are well documented (e.g. Dunne and Leopold, 1978). So it is the capacity of BMPs to reduce these effects at the local scale (e.g. Chocat, 1997). What is less known is the effect, at the catchment-scale, of a large number of BMPs (i.e. SC implementation). Although some recent studies begin to provide field data-based analyses on the topic at various catchment sizes (Petrucci et al., submitted; Meierdiercks et al., 2010), they are still too scarce to provide practical guidance for SC regulation.

Today, the main instruments in SC strategy are regulation policies. They mainly belong to two categories:

- A constraint for new developments to maintain a (or to reduce to a given fraction of) predevelopment peak runoff for a series of rain events (for US examples, see Balascio and Lucas (2009); for UK ones, see Faulkner, 1999).
- A constraint to limit area-specific peak runoff from a new development, for a given rain event, to a given value. For example 5 l/s/ha for a 10 year return period rain event. UK examples are presented by Faulkner (1999), while in France most local regulation follow this scheme, in some cases with extremely low limits (e.g. 1 l/s/ha). The limit value is based on sewer capacity (e.g. Vuathier et al., 2004) or on a predevelopment runoff regime (e.g. Régions Aquitaine et Poitou-Charentes, 2007).
Both these categories represent regulations based on runoff flow-rate. In some cases, volume or infiltration specifications are joined to the flow-rate regulations in order to improve runoff quality. In facts, small runoff events contribute for most of the annual pollution load, and are virtually unaffected by regulations concerning the runoff peak (Pitt and Clark, 2008).

Both the two forms of regulation are based on an implicitly linear approach to scale-transition. The rationale behind the first form of regulation can be summarized as: "if every parcel of the catchment does not produce a runoff peak higher than the pre-development one, the whole catchment too will not produce a runoff peak higher than the pre-development one". But, also if every parcel of a catchment produces a runoff limited to the pre-development peak level, it will anyway produce a higher runoff volume because of its increased imperviousness. The pre-development peak level from every parcel will then be maintained for an increased time (Booth and Jackson, 1997). Thus, it becomes more and more probable, with increasing imperviousness and despite the regulation, that contributions arriving from different parcels could superpose, producing at the catchment outlet a runoff peak higher than the pre-development one (as showed by Goff and Gentry, 2006). For the second form of regulation, linear addition of flow-rates is also implicit, but the reasoning is inversed: an admissible catchment flow-rate is determined, and the parcel-scale limit is obtained dividing this catchment flow-rate by the catchment’s area.

A first consequence of this linear approach is that to limit catchment-scale peak flow-rate all authorities use parcel-scale flow-rate-based regulations. Booth and Jackson (1997) and Goff and Gentry (2006) suggested that limiting parcel-scale runoff volume could be more efficient.

A second consequence is that all parts of a catchment are considered equivalent by the regulations: uniform regulations are in fact the standard. By an hydrologic point of view, the contribution to catchment-scale runoff from, for example, upstream and downstream parts of a catchment are not the same. The differences are generated by the catchment physical processes that participate to the scale-transition.

In this perspective, we analyse hydrographs’ attenuation in sewers as a major process involved in the scale-transition in urban settings. This allows to determine how catchments’ physics introduces actual distinction among catchments’ parts, and to provide useful guidance to improve SC regulations.

**METHODOLOGY**

An hydrograph peak flowing through a channel or pipe is attenuated (Chow, 1959). De Saint Venant’s equations and their approximations describe this phenomenon (Hingray et al., 2009). In particular, the diffusive wave model presents a diffusion coefficient that defines hydrographs’ attenuation. It is thus possible, in theory, to study this process using one of these theoretical models. However, no analytical solution exists, in general, for these models. If inflows are slowly varied, a solution exists – the Hayami model (Chow, 1959) – but this hypothesis is not consistent with rapidly varying hydrographs for storm events. Thus, using this approach requires performing numerical simulations on specific sewer systems and does not allow finding general guidance for SC regulation.

Another approach, alternative to consider the whole flow routing problem, but just the attenuation in sewers, is that used by Ackers and Harrison (1964): they performed a series of experiment on a controlled setting to measure the attenuation entity for peaks superposed to a
uniform baseflow. Their result is an empirical equation linking inlet hydrograph height and pipes characteristics to the outlet hydrograph height.

In order to find an approximated model of the process, useful for a general estimation of its effects on SC, we will base our approach on Ackers and Harrison results, and use an approximation of their original formula. Then, in order to evaluate implications for scale-transition and SC, we perform a sensitivity analysis of the resulting equation.

Remark: we always refer, in this work, to the case of free surface flow in circular pipes, with no backward effect.

**Peak attenuation.**

In order to quantify attenuation, we consider the empirical results obtained by Ackers and Harrison (1964) for peak attenuation in partially-full circular conduits (notation in table 1). They observe that the attenuation of a peak superposed to a permanent, constant baseflow, depends on the peak’s volume. They also observe that a relationship subsists between the peak height at a distance \( x \) from the inlet, and the dimensionless distance \( \frac{x \sqrt{gD^3}}{\Delta V Q_f} \).

In (Ackers et al., 1965), a formula is suggested for this relationship, for \( 0 \leq h_0/D \leq 0.5 \) and

\[
\Delta h^*(0) = \frac{1}{2} \frac{(D-h_0)}{D}.
\]

\[
\Delta h^*(x) = \frac{1}{2 + \left( \frac{x \sqrt{gD^3}}{15 \Delta V Q_f} \right)^{1+\frac{h_0}{D}}}
\]

(eq. 1)

**Table 1. Notation. In the first and second part, respectively, conduit’s and hydrograph’s characteristics**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Units</th>
<th>Description</th>
<th>Notes / Calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L )</td>
<td>m</td>
<td>Length of the conduit</td>
<td></td>
</tr>
<tr>
<td>( x )</td>
<td>m</td>
<td>Position in the conduit</td>
<td>( x=0 ) for the inlet, ( x=L ) for the outlet</td>
</tr>
<tr>
<td>( D )</td>
<td>m</td>
<td>Diameter of the conduit</td>
<td></td>
</tr>
<tr>
<td>( i )</td>
<td>-</td>
<td>Slope of the conduit</td>
<td></td>
</tr>
<tr>
<td>( n )</td>
<td>-</td>
<td>Manning coefficient</td>
<td></td>
</tr>
<tr>
<td>( Q_f )</td>
<td>m(^3)/s</td>
<td>Maximal flow in the unpressurized conduit</td>
<td>Mannng formula: ( Q_f = \frac{1}{n} \frac{R^3}{4} \frac{1}{4} \frac{1}{S} \frac{D^2}{4} )</td>
</tr>
<tr>
<td>( h_0 )</td>
<td>m</td>
<td>Baseflow height</td>
<td></td>
</tr>
<tr>
<td>( h(x) )</td>
<td>m</td>
<td>Height of the perturbation at position ( x )</td>
<td></td>
</tr>
<tr>
<td>( \Delta h^*(x) )</td>
<td>-</td>
<td>Relative height of the perturbation at position ( x )</td>
<td>( \Delta h^*(x) = \frac{h(x)}{D} - \frac{h_0}{D} )</td>
</tr>
<tr>
<td>( \Delta V )</td>
<td>m(^3)</td>
<td>Volume of the peak</td>
<td></td>
</tr>
</tbody>
</table>

If we consider the whole length \( L \) of the conduit, we can define the term \( C \) (m\(^3\)), depending only on the conduit’s characteristics,

\[
C(L) = \frac{L \sqrt{gD^3}}{15 Q_f}
\]
Substituting $\alpha = 1 + 1.5 \frac{h_0}{D}$, eq. 1 can be rewritten:

$$\Delta h^*(L) = \frac{1}{\left(2 + C(L)/\Delta V\right)^{\alpha}}$$

(eq. 1b)

It is possible to show that Eq.1 and Eq.1b are not consistent for, respectively, $x = 0$ and $L = 0$, unless $h_0 = 0$. A second issue is that the two equations are defined for a specific value of $\Delta h^*(0)$. We can solve these two issues adopting an approximation of Eq.1b, explicitly involving $\Delta h^*(0)$ and consistent for $L = 0$. A possible approximation, having these characteristics is:

$$\Delta h^*(L) = \frac{1}{\left(\Delta h^*(0)^{-1/\alpha} + C(L)/\Delta V\right)^{\alpha}} \iff$$

$$\iff \Delta h^*(L)^{-1/\alpha} = \Delta h^*(0)^{-1/\alpha} + C(L)/\Delta V$$

(eq. 2)

Numerical tests showed that the difference between Eq.1b and Eq.2 is less than 1% for $h_0/D \leq 0.2$ (i.e. for low baseflow and high perturbations) and less than 10% for $C/\Delta V > 1.3$ (i.e. for long flowpaths). Thus, Eq.2 can be considered globally satisfying, considering that it approaches well Eq.1b in the most significant cases (long flowpaths, high perturbations) and it differs from Eq.1b where the latter shows consistency problems (short flowpaths).

The peak’s attenuation - the difference between the peak’s height at the inlet $\Delta h^*(0)$ and its height at the outlet $\Delta h^*(L)$ – does not appear explicitly in Eq.2. However, considering that $1 \leq \alpha \leq 2.5$, and thus $-1/\alpha < 0$, it is possible to infer from Eq.2 that the attenuation grows with the $C/\Delta V$ term.

Remark: physical interpretation of $C$. As underlined by Ackers and Harrison (1964), the dimensionless term $C/\Delta V$ has a physical interpretation. In fact, it can be rewritten as:

$$\frac{C}{\Delta V} = \frac{L \sqrt{gD^5}}{15\Delta VQ_f} = \frac{1}{15} \cdot \frac{LD^2}{\Delta V} \cdot \frac{\sqrt{gD^5}}{Q_f}$$

The factor $LD^2/\Delta V$ is proportional to the ratio between pipe’s volume ($LD^2 \cdot \pi/4$) and peak’s volume ($\Delta V$). The other factor, $\sqrt{gD^5}/Q_f$, is proportional to the inverse of the Froude number ($Q_f/\sqrt{gD^5} \cdot \pi/2$, for a full circular pipe). Thus, the effect of attenuation is influenced by both the volume and the characteristic flow conditions of the pipe.

**SENSITIVITY ANALYSIS**

To assess the orders of magnitude of the attenuation process, we plot in Figure 1 the values of $\Delta h^*(C)$ for different values of $\Delta h^*(0)$ and $\Delta V$. We observe that attenuation is strongly dependent on the $C/\Delta V$ term: considering the $\Delta V=100m^3$ set of curves,

- for $C=100m^3$ ($C/\Delta V=1$), peaks’ height ($\Delta h^*(C)$) are about halved $(0.5 \to 0.22; \ 0.3 \to 0.16; \ 0.1 \to 0.07)$;
- for $C=200m^3$ ($C/\Delta V=2$), peaks are strongly attenuated, for $\Delta h^*(0)=0.5$, $\Delta h^*(C)=0.125$;
• for \( C=300 \text{ m}^3 \) \((C/\Delta V=3)\), even for \( \Delta h^*(0)=0.5, \Delta h^*(C)<0.1; \)
• for \( C=500 \text{ m}^3 \) \((C/\Delta V=5)\), the values of \( \Delta h^*(C) \) are extremely close, comprised between 0.025 and 0.045. Attenuation makes nearly undistinguishable the initial differences in peaks’ height.

The same observations can be done on the other sets of curves, for the same values of \( C/\Delta V \).

High peaks are thus halved for \( C/\Delta V \approx 1 \), reduced to a quarter for \( C/\Delta V \approx 2 \) and to a tenth for \( C/\Delta V \approx 5 \). Smaller peaks incur in a slower attenuation, but approximately we can consider that attenuation is weak for \( C/\Delta V < 1 \) and strong for \( C/\Delta V > 2 \).

The additional x-axes in Figure 1 show the conduits lengths corresponding to \( C \) values in the main axis, for two diameters (1 and 2 m) of classical sewer pipes. It is possible to notice that the lengths’ orders of magnitude correspond well to typical lengths for a city-wide sewer system (~10\(^{-3}\)-10\(^{-4}\) m).

![Figure 1. Plot of \( \Delta h^*(C) \) for different values of \( \Delta h^*(0) \) (0.1; 0.3; 0.5) and of \( \Delta V \) (10; 100; 1,000 m\(^3\); respectively the dotted, continue and dashed lines). Additional axes show the corresponding length for two concrete conduits \((n=0.013)\) with slope \( i=0.01 \text{ m/m} \) and varying diameters (1; 2 m).](image)

**IMPLICATIONS FOR SOURCE CONTROL REGULATION**

To analyse the implications for SC regulation, we can consider the case of a typical urban catchment in which each parcel is connected to a “uniform” sewer network (constant \( D, i, n \)).

The hydrograph generated from each parcel is routed to the outfall, and the corresponding \( C \) value depends only on the parcel position in the catchment (i.e. the distance from the outlet). We search the best SC regulation to minimise the hydrograph’s height at the outfall.

The first case that we consider is that of a parcel close to the catchment’s outlet. In this case, the hydrograph routing will occur over a small distance, and the corresponding \( C \) will be...
small. To minimise the peak’s height at the outfall, it is necessary to keep its volume extremely low, or to limit the peak’s height at the sewer inlet. In this case, a parcel-scale flow-rate-based regulation is a pertinent option, while a volume-based one should be too restrictive.

The second case is that of a parcel in the upper part of the catchment, for which the hydrograph routing in sewer occurs for several km, with a high $C$ value. In this case, if the hydrograph volume is not of the same order of magnitude of $C$, the peak height at the parcel-scale is completely negligible at the catchment-scale. To minimise the peak’s height at the outfall, reducing hydrograph volume makes unneeded to control hydrograph’ peak at the parcel-scale. In this second case, a volume-based regulation seems more pertinent than a flow-rate one.

In a real catchment, the pertinence of a given regulation over an area depends on the catchment’s characteristics. However, these simple examples show how, as a consequence of transport processes, the contribution of different part of the catchment to its global output can vary. We can draw two general implications for SC regulation:

- Uniformity of the regulation: inside of a catchment, differences actually exist among its different parts. A hydrologically-sound regulation should consider this fact. Uniform regulations are not "wrong", as they can be justified in specific situations, but the initial framework of any SC regulation should be spatially varied.

- Volume-based regulations. At our knowledge, when these regulations are used, they are usually justified by water quality and natural water-cycle concerns. On the contrary, flow-rate-based limits are justified by overflows concerns. Our results on the attenuation process show that, except in proximity of the outlet (or of an overflow point), flow-rate regulations are poorly effective to solve overflow concerns, while runoff volume regulations can be. This seems to reduce the scope of flow-rate-based regulations to a relatively small set of situations and, conversely, to extend the scope of volume-based regulations.

CONCLUSIONS

In this paper, we studied the hydrographs’ attenuation in sewers. Although the results obtained take into account just one aspect of the water routing process in urban settings, and should be extended to more complex situations, like ramified sewers, they provide some guidance on how to regulate SC.

We derived a formulation for the attenuation of peaks and perturbations in partially full pipes. Smaller the volume of the peak or perturbation, stronger the attenuation. Then, we performed a sensitivity analysis to assess the weight of the different factors on attenuation. The analysis highlighted a strong link between the $C/\Delta V$ factor and the attenuation. Studies about the scope and applicability of a simplified rule to estimate attenuation in practical situations can make the object of further research.

We found that the driving factors of the attenuation for an hydrograph drained by a sewer system are different according to the distance from the outlet: if this distance is small, attenuation is mainly driven by hydrograph height, while for a large distance by hydrograph volume.

These results are clearly contrasting with the linear approach that most SC regulations follow today: transport processes create actual differences among the different parts of a catchment.
and their contributions to the catchment hydrological behaviour. If future SC regulations will integrate this fact, they could become more effective.

REFERENCES


