A passenger traffic assignment model with capacity constraints for transit networks

Fabien Leurent, Ektoras Chandakas, Alexis Poulhès

XVth EWGT Meeting, Paris 2012

A passenger traffic assignment model with capacity constraints for transit networks

Fabien Leurent[a*], Ektoras Chandakas[a], Alexis Poulhès[a]

[a]Université Paris-Est, Laboratoire Ville Mobilité Transport, Ecole des Ponts ParisTech
6-8 avenue Blaise Pascal, 77455, Marne la Vallée, Cedex 2, France

Abstract

A model is provided to capture capacity phenomena in passenger traffic assignment to a transit network. These pertain to the interaction of passenger traffic and vehicle traffic: vehicle seat capacity drives the internal comfort, vehicle total capacity determines internal comfort and also platform waiting, passenger flows at vehicle egress and access interplay with dwell time, dwell time drives track occupancy and in turn the period frequency of any service that passes the station along the line of operations, and then service frequency influences service capacity and platform waiting. These phenomena are dealt with by line of operations on the basis of a set of local models yielding specific flows or costs. The topological order of the line is used to devise two line models of, respectively, flow loading and cost setting, each of which calls its local sub-models. The pair of line algorithms amounts to a complex cost-flow relationship at the level of the line. The line model is used as a sub-model in passenger assignment to network hyperpaths, where line pairs of access-egress stations constitute leg links. The properties of static traffic equilibrium for both vehicles and passengers are established.

Keywords: Traffic equilibrium; Bi-layered assignment; Seat Capacity; Vehicle Capacity; Transit Bottleneck; Platform Stock; Wait Time; Vehicle Load; Frequency Modulation; Trach Occupancy, Line Model

1. Introduction

Background. On a transit network, passengers and service vehicles interact in a number of ways. Vehicle traffic determines the in-vehicle travel times and also the waiting time on platform. Passenger traffic influences the dwelling time, hence also track occupancy and maybe even service frequency. Furthermore, the interplay of passenger flows and vehicle capacity determines the in-vehicle comfort and the residual capacity for access at a station. A systemic analysis of capacity constraints in transit systems is available in [1], building upon the Transit Capacity and Quality of Service Manual [2] and other previous works e.g. [3]. [4] have provided a static network assignment model, dubbed CapTA for Capacitated Transit Assignment, to deal with (i) vehicle total capacity, (ii) passenger waiting on a station platform where one or more transit services are available, (iii) vehicle egress and access flows of passengers and their interplay with vehicle dwell time and (iv) track occupancy by vehicles and its effect on the operations frequency of any service that uses the track. The CapTA model addresses passenger traffic in two layers: the lower layer deals with service operations by line and the local interactions of vehicles and passengers, whereas the upper layer pertains to passenger route choice and network flows on links either pedestrian or "line leg" – a line leg being a bundle of transit services used from an access station to an egress station along a given line.

Objective. The paper has a twofold objective: first, it extends the CapTA model by capturing also the seat capacity of vehicles and its specific effects on the in-vehicle comfort of passengers; second, it provides a high-level presentation of the modeling framework and states the outreach of the line model to capture transit

[*] Corresponding author. Tel.: +33-164-152-111; fax: +33-164-152-140
E-mail address: fabien.leurent@enpc.fr

operations. The model of seat capacity is adapted from [5, 6], where the vector of passenger flows along the legs of a given transit service is faced to the seat capacity and the amounts of it that are available at any stage along the route: seated passengers that exit at a given station give up their seat, yielding residual capacity that can be used by on-board standees prior to newly incoming riders. Among a population of passengers willing to get a seat at a given stage, an equal probability of success is assumed. Here, the sitting model is addressed on a vehicle basis rather than the transit service, so as to deal with passenger loads in consistency with the bi-layered framework in CapTA.

Approach. A set of comfort states is introduced to distinguish sitting versus standing. By link of sojourn in station or of interstation run along the service route, the generalized time experienced by a passenger is modeled as the product of the physical time and a function by comfort state of the specific passenger occupancy – the ratio of the specific passenger load to the specific vehicle capacity. The sitting processes of, first, on-board standees and, second, new incomers, are local sub-models of flow along the line of operations. Their integration within the CapTA line model of flow loading is straightforward. To evaluate passenger generalized cost by service leg then by line leg, the CapTA line model of cost setting is adapted to deal with the comfort states; the additional computational complexity is limited to the squared number of comfort states, due to the efficient formation of leg costs by recursion from a given egress station.

Structure. The rest of the paper is organized in six parts. After recalling the CapTA framework on the upper layer of passenger route choice and its properties for static traffic equilibrium (section 2), the focus is restricted to the line model by providing first a systemic overview (section 3), then the physical model of line flow loading (section 4), next the economic model of line cost setting (section 5). Then, an instance is addressed (section 6). The conclusion points to various perspectives of application and development (section 7).

2. Passenger routing and traffic equilibrium

2.1. Basic assumptions

Supply representation. Assume that the transit assignment network is made up of transit lines that include one or several transit services, together with pedestrian links for terminal access, line transfers, vehicle alighting and boarding. Here a transit service is identified by a specific route and the sequence of stations where it stops along the line; it must be distinguished from a particular vehicle run.

Demand representation. The study area is divided into zones of travel demand: each zone, as a set of locations, can contain trip endpoints. Passenger demand is modeled as a set of trip-makers: each trip-maker chooses a network path so as to minimize his own cost of travel between his origin and destination points.

Let V denote the set of zones, indexed by v. Let W denote the set of zone pairs as origin and destination, also called origin-destination pairs, denoted $(u,v) \in V^2$.

2.2. Passenger routing

To a given trip-maker, each available path yields a generalized cost built sequentially on the basis of local generalized cost defined by network element, node or link or service. A rational, microeconomic behavior of cost minimization is assumed for path choice.

More precisely, the path options are hyperpaths – a hyperpath being an oriented, acyclic sub-graph connecting the origin and destination nodes – on the upper layer network made up of pedestrian links and line legs – a line leg being a bundle of services from an access station to an egress station along the line. Such a hyperpath involves two kinds of service bundling: intra-line combination of services yield line legs, whereas inter-line combination may take place from choice nodes on the upper layer network. The distinction between intra-line versus inter-line bundles is essential in CapTA.

2.3. The lower and upper network layers

Intra-line bundling of services takes place between access-egress pairs of stations along that line: it means that passengers are willing to board on any vehicle with available capacity that services their own station of egress.

On the lower layer of network, each transit line is modeled as a specific sub-network, with vehicle links either of passage in station or of line section between two adjacent stations, and pedestrian links for service boarding and alighting (fig. 1a). More details are provided in the next section.

There is a twofold relationship between the two layers, by line ℓ: top-down, a vector of passenger flows $\mathbf{x}_\ell = [x_a : a \in A_\ell]$ by leg a along the line is inputted to the line sub-model; bottom-up, the line sub-model yields three vectors to characterize the leg costs: $\mathbf{g}_\ell$ of average minimum generalized cost, $\mathbf{w}_\ell$ of waiting time and φ_ℓ of effective operating frequency. In other words, the line sub-model amounts to a sophisticated cost-flow relationship in vector form on the upper layer of network – the passenger network.

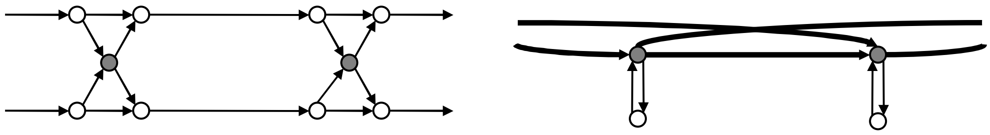

Fig. 1. Network representation: (a) service layer (lower), (b) the passenger layer (upper)

2.4. Upper layer path choice

On the upper layer, pedestrian links a have generalized cost g_a, zero waiting time w_a and infinite availability frequency φ_a. From node m, between pedestrian links $a \approx (m, n_a)$ leading to subsequent node n_a with generalized cost $g_{n_a v}$ to destination v, path choice is reduced to the selection of a link of minimum cost $g_a + g_{n_a v}$, yielding

$$g_{ms} = \min\{g_a + g_{n_a v} : a \approx (m, n_a)\} . \tag{2.1}$$

A line leg $a \approx (m,n)$ yields minimum generalized cost to destination of $g_a + g_{nv}$, plus random cost of waiting. In the basic model of line combination [7], the average waiting cost is evaluated to α / φ_a, and any service with minimum cost less than the average cost of the optimal path bundle must be included in it proportionally to its frequency.

In CapTA, the treatment is the same if $w_a = \alpha / \varphi_a$, but if $w_a > \alpha / \varphi_a$ due to crowding congestion on the line platform, then the passenger may not be able to board in the first incoming vehicle. In this case the leg option is akin to a pedestrian option evaluated at its average cost only. To integrate continuous (pedestrian) and discrete (uncongested transit) availability, let us define a “discontinuity attenuation function” denoted ψ that is continuous and decreases from 1 at 0 to $\psi(x) = 0$ whatever $x \geq \varepsilon$ a small positive parameter. The average waiting w_a and effective operating frequency φ_a delivered by the line model yield a revised frequency of

$$\hat{\varphi}_a = \varphi_a / \beta_a \text{ if } \beta_a \equiv \psi(w_a - \alpha / \varphi_a) > 0 \text{, or} \tag{2.2a}$$

$$\hat{\varphi}_a = \infty \text{ if } \beta_a = 0 . \tag{2.2b}$$

Then, line bundling on the upper layer is based on the revised frequencies and proceeds in the classical way, yielding route-based hyperpaths as in [8]. The discontinuity attenuation function is in fact a development of the line model, as it involves only results of that model.

2.5. Traffic equilibrium

On the upper layer network, a stationary user equilibrium of passenger traffic is defined as the conjunction of (i) passenger choice of minimum cost hyperpath, (ii) passenger assignment to upper layer links according to hyperpath bundling, (iii) flow conservation in the upper layer nodes and (iv) the dependency of leg generalized time, wait time and frequency on the vector of upper layer link flows.

The mathematical formulation is a fixed point problem with respect to the vector of flows by upper layer link and by destination node (or by commodity if there are several user classes), which is the main state variable. A regularization process has been designed to ensure that, based on a given set of link times and frequencies, there is a unique leg flow vector that minimizes the hyperpath costs of all passenger trips, and so in a continuous way: in other words, the mapping $(\mathbf{g}, \hat{\varphi}) \mapsto \mathbf{x}$ is continuous [10]. On the other hand, the cost-flow relationship can be made continuous by ad-hoc assumptions about discontinuity attenuation and the line model: then the mapping $\mathbf{x} \mapsto (\mathbf{g}, \hat{\varphi})$ is continuous also. Lastly, the composed mapping is continuous, which ensures the existence of a fixed point in the set of leg flows (which is convex and compact), hence of traffic equilibrium. Despite the static setting, the capacity constraints do not restrict the feasibility of the assignment problem, because feasibility is addressed and enforced within the line model.

Traffic equilibrium can be computed by a Method of Successive Averages, adapted from [6] by replacing the line model of seat capacity with the more comprehensive CapTA line model.

3. Line system

Let us define a line of operations as a connected, arborescent, acyclic network in a single direction of traffic. The link set includes track links either of interstation run or station sojourn, together with pedestrian links for egress and access at stations. Denote by $\ell \in L$ a line, $A^{(\ell)}$ its set of links, $A_I^{(\ell)}$ the subset of interstation track links and $A_S^{(\ell)}$ that of station track links. The line is operated by one or several transit services, denoted $z \in Z_\ell$: each service has a given track route i.e. an acyclic path denoted $P_z \subset A^{(\ell)}$ and a node set $N_z \subset N^{(\ell)}$ of stations serviced along the route.

The line and service topology of links and nodes is useful to model not only the topology of service legs and line legs, but also the chronological order of traffic operations. Fig. 2 depicts the process of operations: in fact there are five parallel and related processes of, first, passenger traffic within a vehicle, second, passenger alighting, third, passenger waiting on station platform and boarding vehicles with available capacity and servicing their egress station, fourth, dwell time and track occupancy that determine vehicle operations hence in turn service operations and their frequency during the period of reference, fifth, interaction with external traffic on interstation links.

A fundamental principle in CapTA is to address each line system as a particular subsystem in the transit network, on the basis of specific models.

There are two main models at the level of the line: a physical model of flow loading in vehicles and of service traffic, and an economic model of cost evaluation in the setting of the individual passenger that would use the line on a given leg.

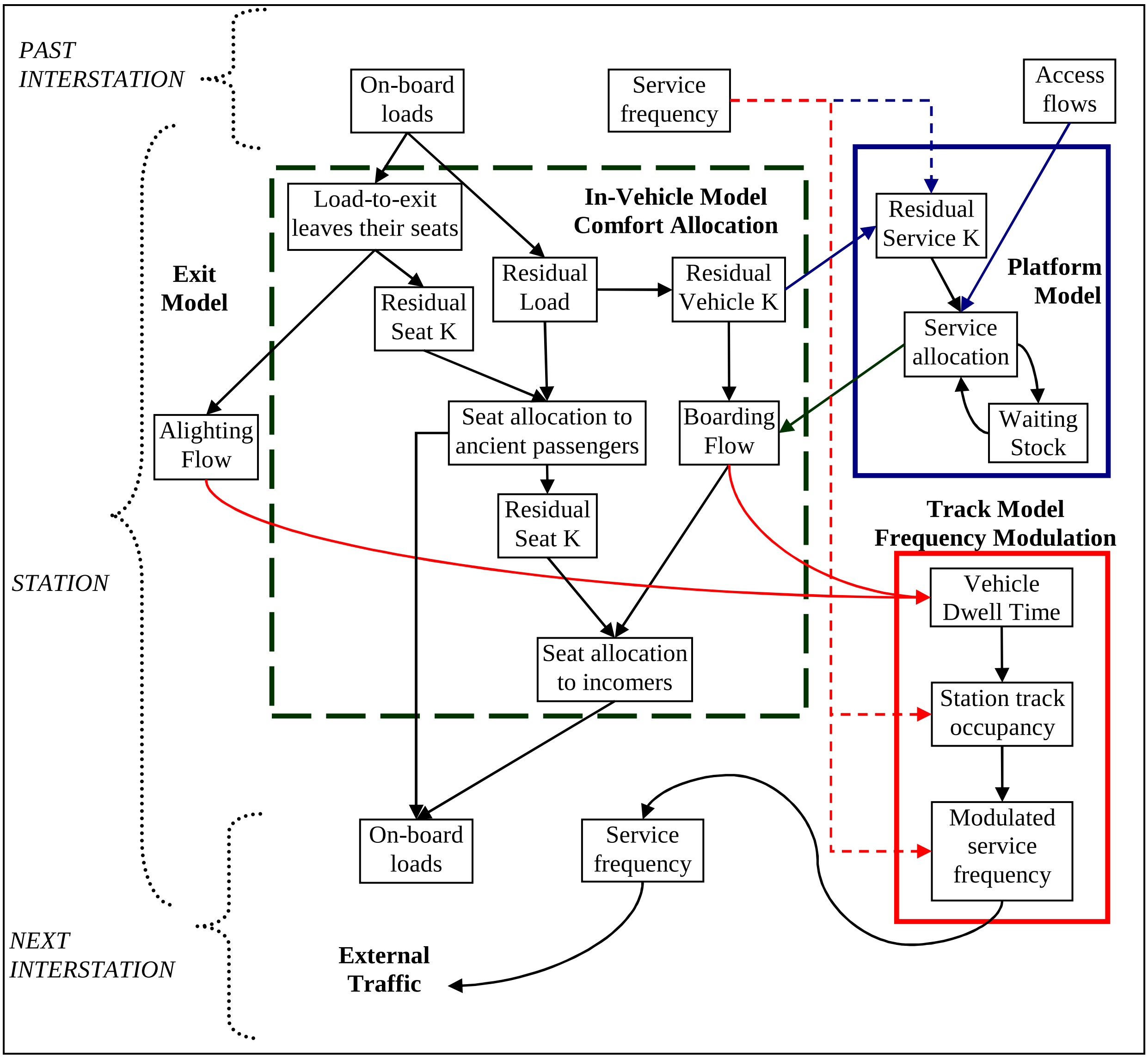

Fig. 2. Overview of the line system (K for capacity)

The main state variables for line traffic are twofold: let y_{za}^{sr} denote the passenger load in a vehicle of service z along link a with egress station s and comfort state r, and φ_{za} denote the frequency of service z on link $a \in P_z$, i.e. the number of vehicles that operate on service during the assignment period.

It would be possible to model one sub-network link by passenger state hence by link a, service z, comfort state r and egress station s. However, it is sufficient to model the service topology in relation to the line topology on the basis of the sets P_z and N_z, and to identify the relevant flow state variables y_{za}^{sr} and φ_{za}.

The probability of immediate boarding in a vehicle of service z arriving at station a for passengers destined to station s is an endogenous variable denoted π_{zs}^{a}. To model seat capacity, two comfort states of sitting versus standing are identified by index r : incoming passengers in service z at station link a have a probability p_{zas}^{+r} of getting comfort state r with respect to their destination station s. On the previous track link, on-board standees that do not exit at the station have a probability $p_{zas}^{o(\rho)r}$ of getting state r from previous state ρ. The three probability vectors are determined within the line flow model together with flows y_{za}^{sr} and frequencies φ_{za}. All these variables are taken as exogenous in the line cost model.

4. Line flow model

4.1. Exogenous conditions and state variables

The conditions external to the line pertain to the operational setting (by the operator) and the passenger load vector by line leg, $\mathbf{x}_\ell = [x_{is} : i, s \in S_\ell, s > i]$. By service z, denote φ_{z0} the nominal frequency, k_{z0}^R the vehicle capacity in passengers and k_{z0}^r the part of it with comfort state r. At the line level, denote by H the duration of the reference period, α_s the parameter of operations regularity at station s, by ω_{za} the operating margin between vehicles on track link a. Beside the state variables already mentioned, let k_{za}^r denote the residual capacity of comfort state r in a vehicle of service z that remains available on link a, and k_{za}^R the overall residual capacity.

4.2. Line flow algorithm

The Line Flow Algorithm proceeds in the direction of traffic along the line, by handling the track links or equivalently the line stations in forward topological order. It consists in one initialization step followed by the sequential treatment of stations. To initialize the algorithm, the residual capacities and the current service frequencies are set at their respective nominal values. The on-board flows $y_{z,a}^{s,r}$ are set to zero.

The treatment of each station i involves successive steps as indicated below, in which $< z,i >$ denotes the interstation link prior to i for service z, $a =< i,i >$ the sojourn link and $< i,z >$ the next link for that service:

- By service z such that $i \in N_z$, apply the Exit model to yield the exit load e_{zi} and update the residual vehicle capacity by comfort state, $k_{z,a}^r$. Apply the Comfort allocation model to the riders that remain on board and obtain updated $y_{z,a}^{s,r}$ and $k_{z,a}^r$.
- Apply the platform model at i to the incoming flows x_{si} for $s > i$ and the residual overall capacities $k_{z,a}^R$, so as to get the boarding flows b_{zis} by destination s and by vehicle of any service z stopping at i and serving s.
- By service z stopping at i and destination s served by z, apply the Comfort allocation model to get the $y_{z,<z,i>}^{s,r}$ for all stations $s > i$ in N_z, together with the residual capacities $k_{z,<i,z>}^r$.
- Apply the Track occupancy and Frequency modulation model to all services passing at i - i.e. using link $< i,i >$ or $< i,z >$ - so as to determine the service frequency $\varphi_{z,<i,z>}$.

4.3. Exit model

The Exit model is purported to evaluate the exit flow and update the residual capacities. It is applied to a service z that stops at a station $i \in N_z$. It consists in:

- Evaluating the passenger load that exits from any vehicle, $e_{zi} = \sum_{r \in R} y_{z,i}^{<z,i>,r}$.
- Updating the residual capacities, by letting $k_{z,a}^{r} = k_{z,<z,i>}^{r} + y_{z,<z,i>}^{i,r}$ for any $r \in R$.
- Applying the Comfort allocation model to the on-board riders and the currently available capacities, yielding loads $[y_{z,a}^{s,r} : s > i, r \in R]$ and probabilities $p_{z,a}^{(r)\rho}$ of getting state ρ for a candidate rider with previous state r.

4.4. Comfort allocation model

By vehicle, the comfort allocation model assigns a load of candidate passengers to the comfort states according to the available capacities. Assumedly, the comfort states are ranked in order of preference which is persistent along the line and homogeneous among the passengers. By rank from highest to lowest, the available capacity is faced to the number of candidates: either on-board riders of lower rank or newcomers. By assumption, every candidate has an equal chance of getting a place of the target rank, among the population of candidates at a given stage. So there are two priority rules: first, priority according to the chronological order of stages; second, equiprobability at a given stage [5]. In the case of two comfort states, sitting is preferred to standing: at each stage the number of candidates say y is faced to the available seat capacity say k. Then, the probability to get a seat amounts to

$$p_{z,a}^{r} = \min\{1,\ k/y\} \text{ (and } p_{z,a}^{r} = 1 \text{ if } y = 0\text{).} \qquad (4.1)$$

After the exiting passengers have left their seat, the updated seat capacity is available to the standees remaining on board, yielding an on-board sitting probability say $p_{z,i}^{o}$. Then the seat capacity is revised again, and it is available to the incoming boarders, in number of $b_{zi} = \sum_{s \in N_z, s > i} b_{zis}$, yielding a second, “at boarding” sitting probability say $p_{z,i}^{+}$. The y_{zs}^{ar} are updated accordingly [6].

4.5. Platform model of passenger waiting and boarding

The platform model of passenger traffic yields the number of passengers that board each vehicle of every service z which stops at i and that are destined to any subsequent station $s \in N_z$, b_{zis}. It also yields the average waiting time, w_{is}, the average size of the “passenger stock” waiting on platform by egress station, σ_{is}, and the probability of immediate boarding in z, π_{zi}. To that end, the transit bottleneck model of [9] is used: given exogenous flows x_{is} by egress station, service frequency φ_{zi} with available boarding capacity k_{zi} by vehicle, the number of passengers that candidate to board on a vehicle of service z when it arrives is $n_{zi} = \sum_{s \in N_z} \sigma_{is}$. Then the probability to board is

$$\pi_{zi} \equiv \min\{1, k_{zi}/n_{zi}\} \text{ (with } \pi_{zi} = 1 \text{ if } n_{zi} = 0\text{).} \qquad (4.2)$$

Assuming that waiting passengers are mingled, on average their waiting time is equivalent to that in a bottleneck model. Then, the vector of stock variables $\sigma = [\sigma_{is} : s > i]$ satisfies a Fixed Point Problem (FPP) as follows [9]:

$$\frac{2\sigma_{is}}{x_{is}H^2} = \frac{x_{is}}{\sigma_{is}} - (\varphi\pi)_{is}, \text{ in which } (\varphi\pi)_{is} \equiv \sum_{z:s \in N_z} \varphi_{zi}\pi_{zi}(\sigma). \qquad (4.3)$$

Some destinations may not be queued, if the exit flow $q_{is} = (\varphi\pi)_{is}\sigma_{is}$ matches the entry flow x_{is} : in such a case the FPP is only an approximation and yields $\sigma_{is} << Hx_{is}$.

The FPP has a solution which is unique. It can be solved using a Newton algorithm.

From the stock variables σ_{is} , the π_{zi} and q_{is} follow, as well as the vehicle inflows $b_{zis} = \pi_{zi}\sigma_{is}$. Furthermore, for destination s the queue duration is $H_{is} = x_{is}H / q_{is}$, yielding waiting cost as follows:

$$w_{is} = \frac{\sigma_{is}H_{is}}{x_{is}H} = \frac{\sigma_{is}}{q_{is}} = \frac{1}{(\varphi\pi)_{is}} . \qquad (4.4)$$

Lastly, $H_{is} > H$ induces $(\varphi\pi)_{is} < \varphi_{is}$, which may hold also when $H_{is} = H$ but some routes are saturated.

4.6. Track occupancy and frequency modulation

The flowing of both service vehicles and passengers at station i along line ℓ involves, first, the vehicle arrivals at upstream frequency $\varphi_{z,<z,i>}$ and the discharge from each vehicle of a number e_{zi} of passengers from upstream stations to the current station; second, the waiting and eventual boarding of those passengers incoming at i ; third, the modulation of service frequency; fourth, the propagation to downstream of the refilled vehicles and the revised service frequencies [4].

Let H' denote the time required to accommodate the intervehicle gaps, valued at ω_{zi} by vehicle of service z , plus the sojourn times on the platform, denoted by T_{zi} . For a service that passes at i without stopping, $T_{zi} = T_{zi}^0$ a small value. For a dwelling vehicle, let T_{zi}^0 denote a minimum sojourn time including slowing down, dwelling and re-accelerating, T_{zi}^1 denote a minimum time for vehicle movements at platform and θ_{zi}^- (resp. θ_{zi}^+) the equivalent alighting time (resp. boarding) by passenger, taking into account not only the elemental time by passenger but also the number of passengers that can alight (resp. board) simultaneously, in relation to the number and width of the vehicle doors. Then, $T_{zi} = \max\{T_{zi}^0, T_{zi}^1 + e_{zi}.\theta_{zi}^- + b_{zi}.\theta_{zi}^+\}$.

The respective service frequencies and occupation time requirement determine the accommodation time in the following way:

$$H' = \textstyle\sum_{z \text{ via } i} \varphi_{zi}\,(\omega_{zi} + T_{zi}) . \qquad (4.5)$$

If $H' > H$ the period of reference, then platform occupancy requires to modulate the service frequency by a reduction factor

$$\eta_i = \min\{1,\ H / H'\} . \qquad (4.6)$$

Frequency modulation is accomplished by letting

$$\varphi_{z,<z,i>} = \eta_i\ \varphi_{z,<i,z>} \quad \forall z \in Z_\ell \text{ such that } <i,i> \in P_z . \qquad (4.7)$$

4.7. External traffic

Here, external traffic refers to vehicle traffic out of the transit stations. The service vehicles contribute to interstation track occupancy, according to their frequency and the elementary “track load” imposed by each vehicle depending on its length and safety requirements. If the interstation track is exposed to interaction with other streams, for instance car traffic on a roadway, then the joint traffic will determine the interstation time of each vehicle type.

4.8. Computational issues

Let us evaluate the computational complexity on a worst case basis where all stations in S_ℓ would be serviced by all services in Z_ℓ. In the line flow algorithm, the initialization step requires $\mathrm{O}(\overline{S}_\ell.\overline{Z}_\ell)$ elementary operations, with $\overline{X} = \# X$. Then, by station, the exit model requires $\mathrm{O}(\overline{Z}_\ell)$ first at alighting then at evaluating the residual capacities, plus $\mathrm{O}(\overline{S}_\ell.\overline{Z}_\ell)$ for updating the load by service and destination; the platform model requires $\mathrm{O}(\overline{S}_\ell.\overline{Z}_\ell)+\mathrm{O}(\overline{S}_\ell^2)$ for solving the fixed point problem by a fixed number of iterations of a Newton-Raphson algorithm; The comfort model involves $\mathrm{O}(\overline{R}^2.\overline{S}_\ell.\overline{Z}_\ell)$ for updating passenger loads and residual capacities, at each stage either from on-board or at-boarding. The frequency modulation model involves only $\mathrm{O}(\overline{Z}_\ell)$. By aggregation, the treatment of a station requires $\mathrm{O}(\overline{R}^2\overline{S}_\ell.\overline{Z}_\ell)+\mathrm{O}(\overline{S}_\ell^2)$, of which the dominant term will be $\mathrm{O}(\overline{S}_\ell^2)$.

The overall complexity amounts to $\mathrm{O}(\overline{S}_\ell^3)$. It drops down to $\mathrm{O}(\overline{S}_\ell^2)$ for a single-service line.

Another computational issue of even greater importance pertains to the smoothness of the probabilities and vehicle loads as functions of the vector of leg flows. This can be ensured by adding a very small tolerance ε to any vehicle capacity involved in an allocation relationship yielding a probability, as in the basic seat capacity model.

5. Lin e cost model

5.1. Exogenous conditions and state variables

The line cost model is purported to yield cost vectors by line leg as evaluated by a passenger on an individual basis. The cost vectors pertain to in-vehicle generalized time, g_a, platform waiting time, w_a, and modulated line leg frequency, φ_a. As both times, in-vehicle and on platform, are random variables, average values are basic outputs that can be complemented by information about variability – notably by values of variance.

The conditions external to the link cost model include local cost functions of travel time and discomfort cost functions by service and track link, say t_{za}^r and χ_{za}^r respectively, as well as traffic conditions inherited from the line flow model: vehicle loads, modulated frequencies, waiting times, probabilities of immediate boarding and of comfort improvement.

5.2. Line cost algorithm

The Line Cost Algorithm proceeds in the reverse direction of traffic along the line, by handling the track links or equivalently the line stations in backward topological order. It consists in one initialization step followed by the sequential treatment of stations as passenger destinations. To initialize the algorithm, by service and track link and comfort state the local travel time and the local discomfort cost are established on the basis of their respective models (see subsections 5.3 and 5.4). Then, by service, track link and comfort state, the local discomfort cost is set to $g_{za}^r = t_{za}.\chi_{za}^r$.

The treatment of each station s as a destination is purported to build the times and costs of all service legs that end in the station. It deals with the sequential accumulation of each endogenous variable along the track links handled in backward topological order. At the beginning, for every service z such that $s \in N_z$, $t_{z(s,s)}$ is set to the passenger time for vehicle alighting and platform exit at station s, and $g_{z(s,s)}^r$ to the corresponding generalized time according to the comfort state. Then, by track link $a \approx (i, j)$ along each service z stopping at

s, tentative leg costs are obtained by accumulation: $t_{z(i,s)} = t_{z(i,j)} + t_{z(j,s)}$ and $g^r_{z(i,s)}$ stems from $g^r_{z(i,j)}$ and the costs $g^\rho_{z(j,s)}$ of the comfort states $\rho \geq r$ based on the model of service leg cost (see subsection 5.5).

Lastly, by entry station i leading to s, the costs from i to s are evaluated by service z then at the level of the line, based on the model of line leg cost (see subsection 5.6).

5.3. Local vehicle time

By service z and dwelling station s, each vehicle spends a sojourn time T_{za} with $a = (s, s)$ that depends on the passenger flows at alighting and boarding, on the basis of the track occupancy and frequency modulation model. By interstation link, the travel time function t_{za} may depend on the "external" traffic (see subsection 4.7). Frequency modulation increases interstation time if the line operator regularizes the vehicle headways. On the average, if frequency is modulated from φ at the tail station to φ' at the head one, a relaxation delay of $(H/\varphi' - H/\varphi)/2$ is imposed on the basic travel time to yield a modulated interstation time, say t'_{za}. This can be interpreted as the average individual waiting time spent by a service vehicle in a traffic bottleneck due to capacity φ' insufficient to satisfy incoming flow φ.

5.4. Local generalized cost

By service, link and comfort state, passenger discomfort per time unit is a function χ^r_{za} with respect to flow. The relevant flow is y^r_{za}, in relation to the associated vehicle capacity k^r_{z0}. Other flows may also contribute to increase discomfort, for instance standing and sitting passengers may interfere and impede on one another. The travel time and the unit discomfort jointly determine the generalized time of travel to a passenger: $g^r_{za} = t'_{za}.\chi^r_{za}$.

This function is an approximation at least for sojourn links, since not only do the passenger loads vary during station sojourn but also the allocation of comfort states to the passengers. The evaluation of sojourn time for incoming passengers should differ from that for on-board riders, at least by a halving factor to account for average duration.

5.5. The cost of a service leg

Starting from a given station of egress, the backward accumulation of travel time by track link along a service is straightforward:

$$t_{z(i,s)} = t_{z(i,j)} + t_{z(j,s)} \text{ for } a \approx (i, j) . \qquad (5.1)$$

The corresponding accumulation of generalized time by comfort state is more complex as it involves the eventuality of getting a better comfort state. Denoting by r the comfort state at the tail of link a, by ρ that at the link head, by $p^{(r)\rho}_{za}$ the probability of getting state ρ from state r on the link, then

$$g^r_{z(i,s)} = g^r_{za} + \sum_{\rho \in R} p^{(r)\rho}_{za} g^\rho_{z(j,s)} \text{ for } a \approx (i, j) . \qquad (5.2)$$

On an interstation link this involves the comfort reallocation between on-board passengers due to passengers that prepare to alight. On a sojourn link, there is no comfort reallocation between prior on-board passengers; to an incoming passenger, the comfort state is allocated at boarding or immediately after and will not vary on that sojourn link.

To a passenger incoming at station i, the average generalized time of the service leg amounts to

$$\bar{g}_{z(i,s)} = \sum_{r\in R} p_{za}^{+r} g_{z(i,s)}^{r} \ . \tag{5.3}$$

Similar recursive formulae have been provided for cost variances [6].

5.6. The cost of a line leg

By line leg (i,s), both the travel time and the generalized time must be averaged over the services z such that $i,s \in N_z$. Denoting by φ_{zi} the frequency at i and π_{zi} the probability of immediate boarding in service z, the allocation of leg passengers to alternative services is proportional to $\varphi_{zi}\,\pi_{zi}$. Then,

$$\bar{t}_{\ell(i,s)} = [\textstyle\sum_{z\in \ell\cap(i,s)} \varphi_{zi}\,\pi_{zi}\,\bar{t}_{z(i,s)}] \,/\, (\varphi\pi)_{\ell(i,s)}\text{, where } (\varphi\pi)_{\ell(i,s)} = \textstyle\sum_{z\in \ell\cap(i,s)} \varphi_{zi}\,\pi_{zi} \ . \tag{5.4}$$

A similar formula yields $\bar{g}_{\ell(i,s)}$ with respect to the $\bar{g}_{z(i,s)}$.

The line leg is evaluated by a passenger in terms of both average in-vehicle generalized time, $\bar{g}_{\ell(i,s)}$, and average platform waiting time, $w_{\ell(i,s)}$ that stems from the platform model.

The composite frequency $\varphi_{\ell(i,s)} = \sum_{z\in \ell\cap(i,s)} \varphi_{zi}$ is also useful as a reference for the available frequency $(\varphi\pi)_{\ell(i,s)}$: an increasing discrepancy between them indicates that the leg passenger traffic tends to continuous rather than discrete regime (see subsection 2.4).

5.7. Computational issues

The computational complexity of the Line Cost Algorithm is in $O(\bar{R}^2 \bar{S}_\ell^2 \bar{Z}_\ell)$: the initialization step involves $O(\overline{RA_\ell})$ with $O(\bar{A}_\ell) = O(\bar{S}_\ell . \bar{Z}_\ell)$, whereas the treatment of each destination station amounts to $O(\bar{R}^2 \bar{S}_\ell . \bar{Z}_\ell)$, leading to $O(\bar{R}^2 \bar{S}_\ell^2 \bar{Z}_\ell)$ over the set of stations.

The computations mostly consist in products and additions on quantities that depend on the leg flow vector $\mathbf{x}_\ell$ in a continuous way. The only exception pertains to the division by the available frequency in the line cost model: there, the regularized treatment of capacity constraints in the Line Flow Model will yield strictly positive available frequency.

6. Application instance

A classroom instance is used to demonstrate the effects of the model. The case mimics the busiest railway line in the Paris metropolitan area, named RER A – the RER being the Regional Express Network. At the morning peak the directional passenger flow through the central trunk amounts to about 50,000 persons per hour; the nominal frequency of 30 trains per hour is frequently decreased to 25 on average, due to congestion which is particularly acute at central stations where many transfers take place.

In the East to West direction, there are two main service routes which link the two Northern (resp. Southern) branches to the central trunk (Fig. 3). Between North-East and Centre another railway line, RER E, competes with RER A. Table 1 indicates the main characteristics of the transit services at the morning peak hour. The minimum dwelling time is planned as 40 s on RER A and 50 s on RER E which is less congested.

The upper layer network contains 6 stations as origin and destination zones, with 35 nodes and 50 arcs. A simplified OD trip matrix was built to match the reference conditions on the central trunks.

Three models have been applied alternatively, namely (1) with no capacity constraints, (2) with total vehicle capacity, platform waiting and track occupancy but no comfort (old CapTA), (3) with all capacity and comfort effects (new CapTA). Table 2 reveals a significant increase in generalized time per trip, as well as some tradeoff between wait time and in-vehicle cost in the full model, in which the OD pair from NorthEast to Auber is assigned to RER E at 86%, compared to 27% only in model 2.

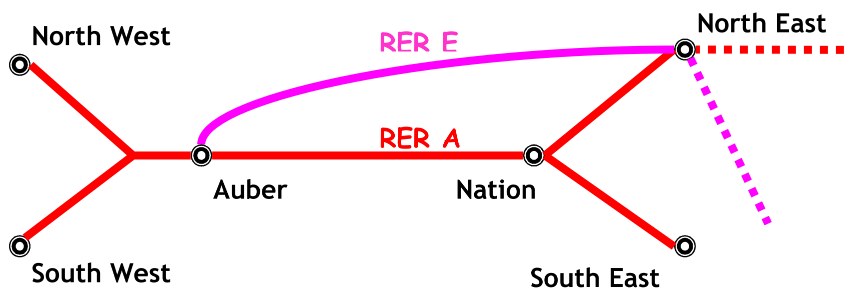

Figure 3. Abstraction of the selected network, emphasizing on line legs

Table 1. The characteristics of the transit routes

Route	Frequency (veh/h)	Seat Capacity (per veh.)	Total Capacity (per veh.)
RER A north	18	432	1760
RER A south	12	600	1888
RER E	8	1100	2564

Table 2. The Generalized Time (in minutes) for each model variable

Model Variable	Optimal GT	Waiting cost	In-vehicle cost	Transfer cost	Access – Egress cost
Without Capacity	52,8	7,8	31	0	14
Without Comfort	67	22	31	0	14
With all capacity constraints	82,8	19,7	48,5	0	14

Table 3. Operation results of RER A under capacity constraints

Transit Route	Dwell Time at Nation (s)	Dwell Time at Auber (s)	Frequency at Auber	Expected Waiting time at Auber (min)
RER A north	61,6	40	15,80	60,3
RER A south	48,9	40	10,55	59,9

At the lower level of vehicle and service operations, Table 3 displays the main characteristics along the two services of line RER A. Both of them dwell longer at station Nation. This entails a modulation of service frequency from 30 down to 26,4 vehicles/h for the downstream station Auber. This contributes in turn to insufficient total capacity there, yielding the formation of a persistent stock of passengers and a large increase in waiting time.

Aside from the instance, the CapTA model was applied to the transit network of the Paris Metropolitan area. The transit network is composed of 95 directional railway lines (as defined in section 2.3 for the guided transit modes – Train, RER, Metro and Light Rail) yielding 259 guided transit services, and 4483 bus services. Demand at the morning peak hour involves 1.23 million trips between 1,305 travel demand zones. The network contains about 159,800 nodes and 307,700 arcs and line legs. The model was programmed in C++ and run on a 2,66 GHz PCwith 4 GB of RAM. The average run time per iteration amounts to 10 minutes in Model 2, compared to 8 minutes for Model 1. Each iteration in Model 3 requires 13 minutes more than Model 2, i.e. about 23 minutes. An acceptable level of convergence was reached after 30 iterations (with a duality gap reduced to 5‰ of the initial value).

7. Conclusion

The line model constitutes a framework to represent a variety of features and phenomena in a transit system, of physical as well as microeconomic nature. The representation is mesoscopic as it gathers the microscopic features of individual passengers and vehicles and the macroscopic features of passenger flow and service frequency. The framework is essentially systemic and modular: some parts of it may be replaced by more appropriate sub-models, for instance about track occupancy, or the interaction of access and egress flows in station dwelling. Many developments can be thought of: (i) a complementary model for vehicle alighting and platform access, (ii) door allocation to only one direction of traffic flow, (iii) control of dwell time and restriction of boarding flow, instead of endogenous dwell time, (iv) external traffic on interstation links, (v) featuring the platform layout and its effects on the passenger flows and waiting stock, (vi) the return trip of service vehicles relate the two directions of traffic on a given line, (vii) stochastic features in the formation not only of travel time and generalized cost, but also of flows and stocks.

Acknowledgement

This research was supported by the Research and Education Chair on “The socio-economics and modeling of urban public transport”, operated by Ecole des Ponts ParisTech in partnership with the Transport Organizing Authority in the Paris Region (STIF), to which we are grateful.

References

[1] Leurent F (2011) Transport capacity constraints on the mass transit system: a systemic analysis, *European Transport Research Review, 3/1: 11-21.*

[2] TRB (2003) *Transit Capacity and Quality of Service Manual*. On-line report prepared for the Transit Cooperative Research Program, available on-line at the following website address: http://gulliver.trb.org/publications/tcrp/tcrp_webdoc_6-a.pdf . First edition 1999.

[3] Vuchic V.R. (2005) Urban Transit : Operations, Planning and Economics. Wiley. 665 p.

[4] Leurent F., Chandakas E., Poulhes A. (2011) *User and service equilibrium in a structural model of traffic assignment to a transit network*, in Zak J. (ed) The State of the Art in the European Quantitative Oriented Transportation and Logistics Research – 14th Euro Working Group on Transportation & 26th Mini Euro Conference & 1st European Scientific Conference on Air Transport. Elsevier Procedia Social and Behavioral Sciences, 20: 495-505.

[5] Leurent F (2006) Confort et qualité de service en transport collectif urbain de voyageurs: analyse, modélisation et évaluation. Proceeding of the ATEC Congress 2006, CD Rom edition.

[6] Leurent F (2012) On Seat Capacity in Traffic Assignment to a Transit Network. Journal of Advanced Transportation, 46/2: 112-138

[7] Chriqui C., Robillard P. (1975). Common bus lines, Transportation Science, 9: 115–121.

[8] De Cea J, Fernández JE (1993) Transit Assignment for Congested Public Transport Systems: An Equilibrium Model. *Tptn. Sc*., 27(2), 133-147.

[9] Leurent F. (2012b) *A regularization process to yield smooth upper layer link flows with respect to link costs and frequencies in the CapTA model*, Working Document, LVMT.

[10] Leurent F. (2011) The transit bottleneck model. *Working document*, Ecole Nationale des Ponts et Chaussées, Université Paris Est.